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SIMULATION OF NONLINEAR POSITION AND ATITUDE CONTROL FOR MICRO QUADCOPTER WITH LOW DISTURBANCE CONTROLLED BY SLIDING-MODE AND BACKSTEPPING METHODS

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Abstract: *The paper addresses the issues regarding the development of a robust assisted quad-rotor aerial robot (also known as quad-copter) with stable flight capacity. After detailing a dynamical flight model final results will be presented. For position control and attitude of the unmanned aerial vehicle (UAV) different nonlinear control laws are investigated. Back-stepping and sliding-mode controllers are compared with regards to stability and performance. Theoretical analysis will be validated by experiments on the implemented platform.*
Key words: *quad-copter, control, sliding-mode, back-stepping, low disturbance*

1. INTRODUCTION

Aerial applications have been increasing in the last decade. Traffic surveillance, area mapping, remote inspections, remote access in areas with no or poor ground access like accident areas or polluted zones, have been increasing. One particular example would be the recent Fukushima disaster where aerial inspection proved to be the best means of damage report. Also military development applications are high in aerospace domain. Creating a military drone with stealth and maneuverability would be a great tactical advantage for ground mapping and enemy detection. High risk areas could be accessed without human participation and thus reducing the risk of casualties. Injured soldiers pinned down could be fueled with medical kits, food and water or even weapons. Such drones are already used in some parts of the globe but they are only in an incipient state.

High performance unmanned aerospace vehicles (UAV) have seen a rise in the necessity for precise and robust controllers that can cope with aggressive flight maneuvers in situations where the dynamics of the platform and the medium are poorly known and also the need for precise stability and fine control in low disturbance situations.

Taking all this into consideration it is only logical that a mathematical model needs to be created. Basically a mathematical model of a quad-copter is an approximation if the real physical structure dynamics. This is more or less accurate depending on the depth and detail of the mathematical model. Depending on the structure configuration there are forces, inertias and other variables that can be neglected if their value in considered not to modify the behavior of the system, or to modify it but within acceptable limits. Ignoring certain characteristics can greatly decrease the complexity of the system, increasing performance and making the control techniques simpler to implement on various platforms.

Recent development in battery capacity, processor power and sensor performance enable us to build reliable easy to control and user friendly unmanned aerial vehicles (UAV).

This paper focuses on micro quad-copter vehicles evolving towards a full autonomy in low disturbance environments. In this paper we present two control methods and compare them both in simulation. Firstly we will develop a dynamic model of the platform so that we will be able to develop both controllers with precision and performance.

2. DINAMIC MODEL

Micro-copters can be considered a highly dynamic system, given this a suitable model should include gyro effects from both rigid body in space and the four propulsion group's rotation. The latter effect includes propellers, gearbox and motor rotation if inertia that must not be neglected.

Notation of frames is:

Sf - Structure frame

If - Inertial frame

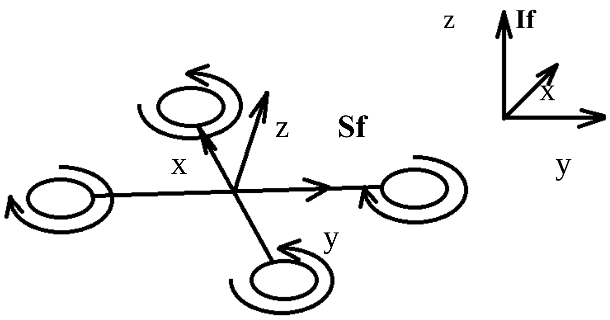

Fig.1 Structure Frame and Inertial Frame

Using Euler angles parameterization, orientation in space is determined by rotation matrix R from *Sf* to *If where R* is considered the rotation matrix. Newton-Euler formalism will dictate the dynamic model that can be derived. The dynamics of the rigid body under forces applied to center that are external (1) are formulated in [1]:

$$\begin{bmatrix} \omega \times mL \\ \omega \times I\omega \end{bmatrix} + \begin{bmatrix} mI_{3x3} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{L} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} F \\ \tau \end{bmatrix} \quad (1)$$

where the inertial matrix is noted with I, L the body linear speed vector and ω the body angular speed. In the frame system Fig.1, the equations of motion for the Quad-copter become [4]:

$$\begin{aligned} \dot{Pv} &= \nu \\ m\dot{v} &= RF_b \\ \dot{R} &= R\hat{\omega} \\ J\dot{\omega} &= -\omega \times J\omega + Tb \end{aligned} \quad (2)$$

Considering:

Pv– Position vector

R – Rotation matrix

$\hat{\omega}$ – skew symmetric matrix

Angles (Φ - Roll, θ - Pitch, ψ - Yaw)

Inertia (J_r - Rotor, J_p - Prop, J_m - Motor)

$I_{x,y,z}$ − Body inertia

Tb − Torque on body

tf – Thrust factor

df – Drag factor

al – Arm length

rs − Rotor speed

First-level model approximation results:

$$\begin{aligned} \dot{Pv} &= \nu \\ \dot{\nu} &= -ge_3 + R_{e3}\left(\frac{b}{m}\sum \mathrm{rs}_i^2\right) \\ \dot{R} &= R\hat{\omega} \\ I\dot{\omega} &= -\omega \times I\omega - \sum J_r(\omega \times e_3)\mathrm{rs}_i + Tb \end{aligned} \quad (3)$$

Applied torque on the vehicle's body along an axis represents difference between the torques generated by each propeller on the other axis.

$$Tb = \begin{pmatrix} al * tf(\mathrm{rs}_4^2 - \mathrm{rs}_2^2) \\ al * tf(\mathrm{rs}_3^2 - \mathrm{rs}_1^2) \\ df(\mathrm{rs}_2^2 + \mathrm{rs}_4^2 - \mathrm{rs}_1^2 - \mathrm{rs}_3^2) \end{pmatrix} \quad (4)$$

Considering motor inertia and a reversing gearbox with negligible inertia, the rotor inertia results as:

$$\begin{aligned} \ddot{x} &= (\cos\Phi\sin\theta\cos\psi + \sin\Phi\sin\psi)\frac{1}{m}IP1 \\ \ddot{y} &= (\cos\Phi\sin\theta\sin\psi - \sin\Phi\cos\psi)\frac{1}{m}IP1 \\ \ddot{z} &= -g + (\cos\Phi\cos\theta)\frac{1}{m}IP1 \end{aligned} \quad (5)$$

$$\begin{aligned} \ddot{\Phi} &= \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\mathrm{rs} + \frac{1}{I_x}IP2 \\ \ddot{\theta} &= \dot{\Phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) + \frac{J_r}{I_y}\dot{\Phi}\mathrm{rs} + \frac{1}{I_y}IP3 \\ \ddot{\psi} &= \dot{\Phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}IP4 \end{aligned} \quad (6)$$

The gyroscopic effect resulting from the rigid body rotation in space is the first term in the orientation subsystem (Φ, θ, ψ) is and the second one is due to the propulsion group rotation. System's inputs are: *IP1, IP2, IP3, IP4* and rs a disturbance, resulting:

$$\begin{gathered} IP1 = tf(\mathrm{rs}_1^2 + \mathrm{rs}_2^2 + \mathrm{rs}_3^2 + \mathrm{rs}_4^2) \\ IP2 = tf(\mathrm{rs}_4^2 - \mathrm{rs}_2^2) \\ IP3 = tf(\mathrm{rs}_3^2 - \mathrm{rs}_1^2) \\ IP4 = df(\mathrm{rs}_2^2 + \mathrm{rs}_4^2 - \mathrm{rs}_1^2 - \mathrm{rs}_3^2) \\ \mathrm{rs} = \mathrm{rs}_2 + \mathrm{rs}_4 - \mathrm{rs}_3 - \mathrm{rs}_1 \end{gathered} \tag{6}$$

Micro DC motors are used for the rotors. For rotor dynamics we will take into consideration the next equations [5]:

$$\begin{gathered} J_r = J_p - J_m r \\ L\frac{di}{dt} = u - Ri - k_e \omega_m \\ J\frac{d\omega_m}{dt} = \tau_m - Ml \end{gathered} \tag{7}$$

DC motor dynamics of second order may be approximated (taking into consideration the ultra-low inductance):

$$J\frac{d\omega_m}{dt} = -\frac{k_m^2}{Rm}\omega_m - Ml + \frac{k_m}{Rm}u \tag{8}$$

With the propeller and gearbox models equation *(8)* is transformed in:

$$\begin{gathered} \dot{\omega_m} = -\frac{1}{\tau}\omega_m - \frac{d}{\eta r^3 J_t}\omega_m^2 + \frac{1}{k_m \tau}u \\ where \\ \frac{1}{\tau} = \frac{k_m^2}{Rm J_t} \end{gathered} \tag{9}$$

The resultant equation (9) can be linearized (see [2]) around $\dot{\omega}$ to the form $\dot{\omega} = -A\omega_m + Bu + C\ with:$

$$A = \left(\frac{1}{\tau} + \frac{2dw_0}{\eta r^3 J_t}\right), B = \frac{1}{k_m \tau}, C = \left(\frac{d\omega_0^2}{\eta r^3 J_t}\right) \tag{10}$$

where:
u – Motor input
τ – Time constant for motor
Rm – Motor inertial resistance
k_e – Back EMF constant
k_m − Torque constant
ω_m − Motor angular speed
τ_d Ml − Motor load
τ_m − Motor torque
r – Gearbox ratio
η − Gearbox yield
J_t − Total inertia on the motor

3. MICRO QUADCOPTER BACK-STEPPING MODE CONTROL

Given the high difficulty of control on UAV's a deep understanding of the system is needed. In this regard performing several open loop simulations is mandatory. The simulations were useful for the recognition of the contribution of each modeled effect to the dynamics of the system. Also, knowing the natural behavior of the system could be useful for establishing adapted control laws. As an example if the controller is too sensitive the system's oscillations are amplified, gaining energy and destabilizing the system even more.
The model (5) will be rewritten in a state-space form $X = \dot{f}(X, U)$ by introducing X=$(x_1 \ldots x_{12})^T \in R^{12}$ as state vector for the system.

$$\begin{gathered} x_1 = \Phi \\ x_2 = \dot{x_1} = \dot{\Phi} \\ x_3 = \theta \\ x_4 = \dot{x_3} = \dot{\theta} \\ x_5 = \psi \\ x_6 = \dot{x_5} = \dot{\psi} \\ x_7 = z \\ x_8 = \dot{x_7} = \dot{z} \\ x_9 = x \\ x_{10} = \dot{x_9} = \dot{x} \\ x_{11} = y \\ x_{12} = \dot{x_{11}} = \dot{y} \end{gathered} \tag{11}$$

From (5) and (11) results:

$$f(X,U) = \begin{pmatrix} x_2 \\ x_4 x_6 a_1 + x_4 a_2 \mathrm{rs} + b_1 IP2 \\ x_4 \\ x_2 x_6 a_3 + x_2 a_4 \mathrm{rs} + b_2 IP3 \\ x_6 \\ x_4 x_2 a_5 + b_3 U_4 \\ x_8 \\ -g + (\cos x_1 \cos x_3)\frac{1}{m} IP1 \\ x_{10} \\ u_x \frac{1}{m} U_1 \\ x_{12} \\ u_y \frac{1}{m} U_1 \end{pmatrix} \tag{12}$$

with:

(13)

It is important to observe that in the latter system, the angles and their time derivatives do not depend on translation components, but the translations do. The final image described by (12) is formed from two subsystems (the angular rotations and the linear translations)

The full control scheme for the overall system is divided 3 sub-systems:

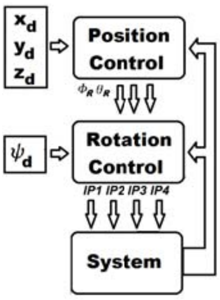

Fig.2 Full Control scheme

One set is desired ( ) , the position controller generates the required (to the rotation controller. The measured values are fed back to both controllers.

The control law forcing the system to follow a desired path can be synthesized using the back-stepping method. See [2] and [3] for details. The first step is considering the tracking error:

(14)

Using Lyapunov theorem and using Lyapunov function positive definite and it's time derivative negative semi-definite [1]:

(15)

Stabilization for is obtained by introducing a virtual control :

(16)

The equation (15) is then:

(17)

Proceeding to a variable by masking:

(18)

For the second step we are considering the optimized Lyapunov function [3]:

(19)

Resulting in time derivative:

(20)

The control input is the extracted ( ) satisfying ( )<0 and IP3 and IP4 are extracted taking into consideration with

(21)

with :

(22)

$$z_5 = x_{5d} - x_5$$
$$z_6 = x_6 - \dot{x}_{5d} - \alpha_5 z_5$$

4. LINEAR SUBSYSTEM BACK - STEPPING CONTROL

The altitude control $IP1$ is obtained using next relations [4]:

$$IP1 = (z_7 + g - \alpha_7(z_8 + \alpha_7 z_7) - \alpha 8z8\ m\cos x1 \cos x3 \quad (23)$$

with:

$$z_7 = x_{7d} - x_7$$
$$z_8 = x_8 - \dot{x}_{7d} - \alpha_7 z_7 \quad (24)$$

Using (5) it can be observed that the motion on x and y axes depends on $IP1$. In fact $IP1$ is the total thrust vector. If u_x and u_y are considered the responsible orientations for the motions on x and y, we can deduce form (13) the roll and pitch angles necessary to commute the controls u_x and u_y satisfying $\dot{L}(z_1 z_2)$ <0.

The yaw control is the desired angle

$$u_x = \frac{m}{IP1}(z_9 - \alpha_9(z_{10} + \alpha_9 z_9) - \alpha_{10} z_{10})$$
$$u_y = \frac{m}{IP1}(z_{11} - \alpha_{11}(z_{12} + \alpha_{11} z_{11}) - \alpha_{12} z_{12}) \quad (25)$$

Using dynamic model (5) and (6) in Simulink we executed several simulations with the 12 parameters controller. The task was to reach the position $x_d = y_d = z_d = 1m$ and ψ_d = 0 rad.

The simulated performance is considered to be adequate. Before testing the controller on the real system witch only has the 3D IMU board, different simulations have been performed taking into consideration only the angular rotations subsystem and the controller.

This controller has only 6 parameters $(a_{1,} \ldots a_{6,})$.

5. MICRO QUADCOPTER SLIDING MODE CONTROL

The sliding mode controller is basically based on relation (8), but in (19), z_2 is replaced by s_2 for clearance.

$$s_2 = x_2 - \dot{x}_{1d} - \alpha_1 z_1 \quad (26)$$

Then, we consider the following Lyapunov function:

$$L(z_1, s_2) = \frac{z_1^2 + s_2^2}{2} \quad (27)$$

If we derive (26) by time, we obtain the law for the attractive surface:

$$\dot{s}_2 = a_1 x_4 x_6 + a_2 x_4 \mathrm{rs} + b_1 IP2 - \ddot{x}_{1d} + \alpha_1(z_2 + \alpha_1 z_1) \quad (28)$$

Resulting:

$$IP2 = \frac{-a_1 x_4 x_6 - a_2 x_4 \mathrm{rs} - \alpha_1^2 z_1}{b_1} - \frac{k_1 sign(s_2) - k_2 s_2}{b_1}$$

$$IP3 = \frac{-a_3 x_2 x_6 - a_4 x_2 \mathrm{rs} - \alpha_2^2 z_3}{b_2} - \frac{-k_3 sign(s_3) - k_4 s_3}{b_2}$$

$$IP4 = \frac{-a_5 x_2 x_4 - \alpha_3^2 z_5 -}{b_3} - \frac{k_5 sign(s_4) - k_6 s_4}{b_3} \quad (29)$$

with:

$$z_3 = x_{3d} - x_3$$
$$s_3 = x_4 - \dot{x}_{3d} - \alpha_2 z_3$$
$$z_5 = x_{5d} - x_5 \quad (30)$$
$$s_4 = x_6 - \dot{x}_{5d} - \alpha_3 z_5$$

The controller was tuned using the Nonlinear Control Design block-set.

The initial values for the three angles were approximate 40 degrees, as it can be observed in figures 3, 4 and 5.

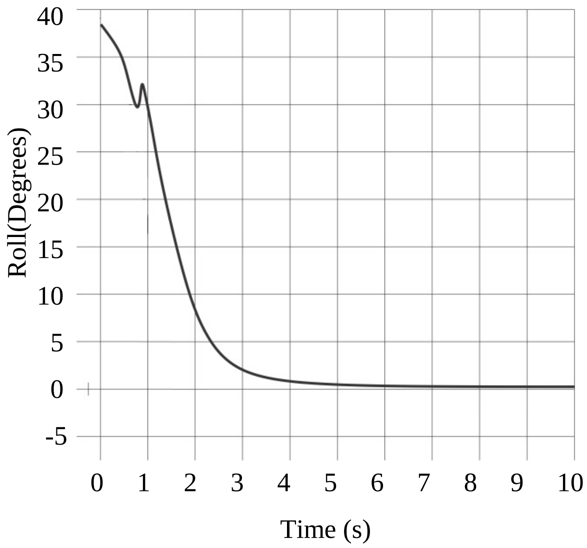

Fig.3 Back-stepping controller Roll simulation

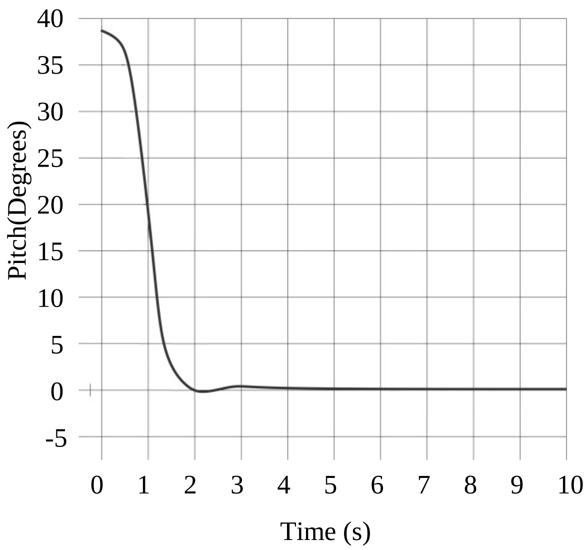

Fig.4 Back-stepping controller Pitch simulation

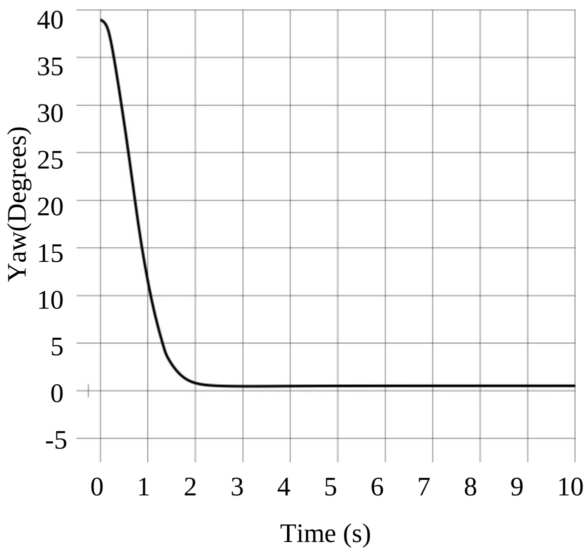

Fig.5 Back stepping controller Yaw simulation

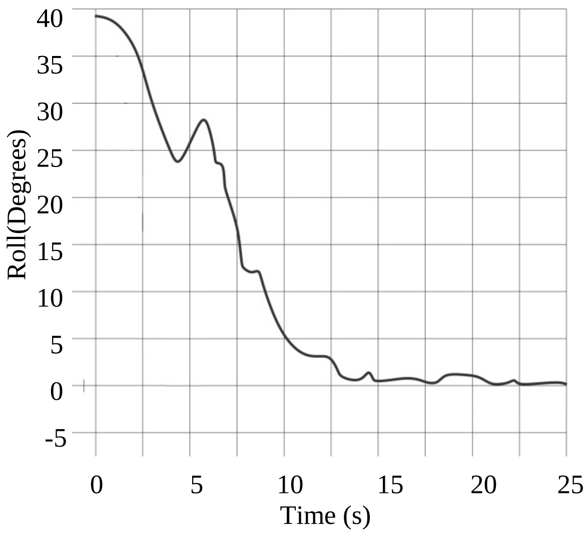

Fig.6 Sliding mode controller roll simulation

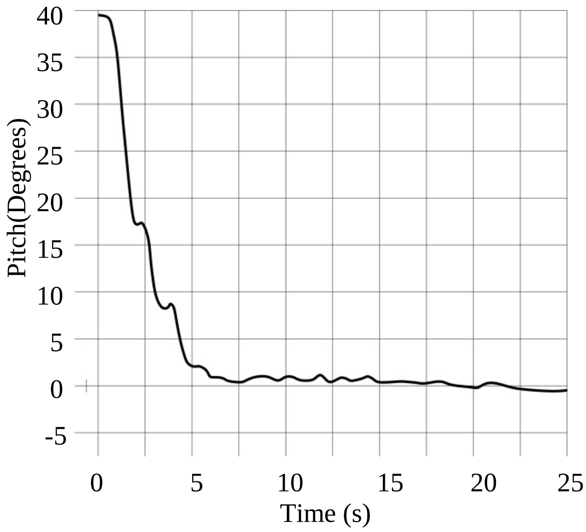

Fig.7 Sliding mode controller Pitch simulation

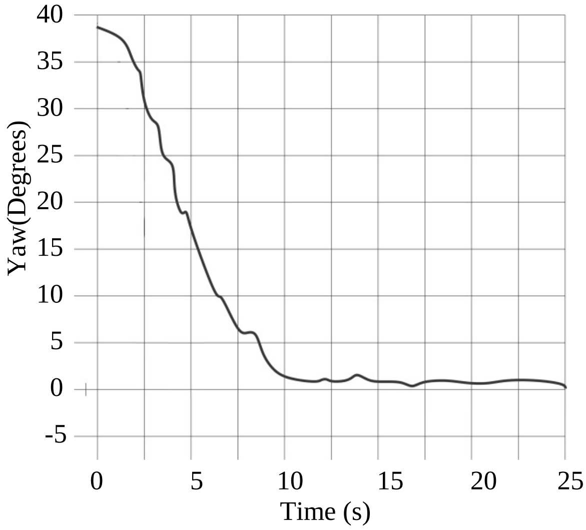

Fig.8 Sliding mode controller Yaw simulation

6. CONCLUSIONS

In this paper two methods of control have been presented. “Sliding mode “and “Back-stepping mode” implemented on a micro quad-copter simulation.

It can be deduced from simulated plots that the sliding mode controller yields average results due to it’s switching nature witch introduces high frequency vibrations causing sensor noise and drift, thus for low noise applications it is not an acceptable alternative. From a time point of view it is also mediocre to bad, getting within the stability area in a much longer time than the previous controller. 10 to 15 seconds is not an acceptable result taking into consideration the low external noise.

Back-stepping controller on the other hand is able to control the angles of orientation in an acceptable time span and with no or ultra-low internal noise, unlike the sliding method. It also performs well in the presence of relatively high perturbations confirming previous studies. The average stabilization time for back-stepping controller is 2 seconds, 5 to 7 times faster than the sliding method.

Future development will be directed towards experimental validation of the simulated results in this article for both back-stepping and sliding mode control techniques using the test bench presented below. This system is composed out of a sturdy base (5), a metal pole (4), data acquisition and Flexboard development board (3) [7], the power train with all the 4 motors (2a, 2b, 2c, 2d) and an inertial motion unit (IMU) with built in motor controllers. The flexfoard has been developed especially for the Flexform project . It is based on a Atmega family chips, and it can be used on a large variety of projects given that it has built in four dc motor controllers, 4 digit 7segment display ,various buttons and expansion slots . From the connectivity point of view is is plug and play on USB port and it also has an external power supply for high current applications .The coupling between the metal pole and the actual platform is made with a universal low friction ball joint. Three degrees of freedom are locked. The IMU board has a built in I2C communication driver capable of 115Kb/s baud-rates. The angular velocities are processed by the IMU board and sent to the development board. The data is processed and orientation angles are estimated with the help of a Kalman filter. If the position needs to be corrected the motor speeds are calculated and sent back to the IMU board so that the orientation in space is changed. The full weight of the quad-copter is 76 grams. The four propulsion groups are formed out of a 4g propeller 6g gearbox and a 9g micro-motor, in addition to this we also have the center body with a 10g heavy and the IMU board with 12 grams, totaling 98g. The maximum motor pull force has been measured to be around 41 grams.

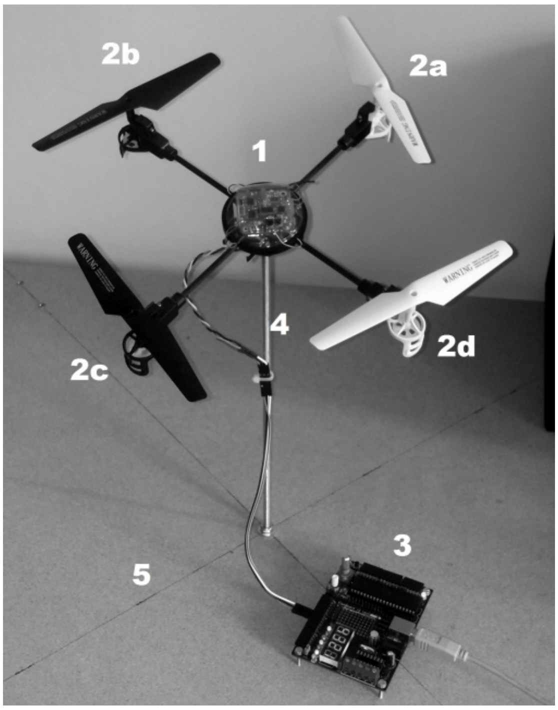

Fig. 9 Test-bench with quad-copter mounted.

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[5] Nonlinear Attitude and Position Control of a MicroQuadrotor using Sliding Mode and Backstepping Techniques Patrick Adigbli, Technische Universit at Munchen, 80290 Munchen, Germany
[6]www.flexform.ro
[7]www.synapseandsilicon.ro

Controlul neliniar de pozitie si aditudine al unui micro quadcopter cu perturbatii minore prin metodele backstepping si sliding-mode

Rezumat: *Lucrarea prezintă aspectele dezvoltării unei platforme de zbor cu patru rotoare (cunoscută ca și quadcopter), cu capacitatea de a-și menține stabilitatea automat. După prezentarea modelului de zbor, rezultatele vor fi prezentate. Pentru controlul în poziție și altitudine al aparatului de zbor, s-au folosit tehnici de control non-liniare. Tehnici de control de tipurile “Sliding-Mode” și “Backstepping” au fost folosite și s-au comparat rezultatele, ținându-se cont de stabilitate și performanță. Analiza teoretică va fi susținută de experimente practice, făcute pe platforma folosită.*

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