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THE DYNAMIC EQUATIONS OF THE TRTTRR1 ROBOT USING THE NEWTON-EULER FORMALISM

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Abstract: *To determine the robot's dynamic equations using the Newton-Euler formalism, is primarily needed the geometric and kinematic modeling. Secondly are required the distribution parameters mass and certain simplifying assumptions of choosing the mass centers C_i but also the mechanical centrifugal moments of inertia. With these are determined the mass centers accelerations and the reduction torsos elements for the external forces. The next step is to determine the torsos of the contact forces and the moments of these contact forces. The last step is to determine the driving generalized forces from the couplers robots; their expressions represent the dynamic equations of the TRTTRR1 robot.*

Key words: *generalized forces, contact forces, center of mass, reduction torsos, Newton-Euler formalism.*

1. INTRODUCTION

To determine the dynamic equations of the robot, it's used an iterative method, which emphasizes the generalized variables, the driving generalized forces and the contact forces that arise between linked components of the robot. The calculation algorithm is based on the Luth-Walker-Paul method [1] and consists of two parts namely:

1. Iterations to the exterior structure of the robot.

Using the Newton-Euler dynamic equations, is determined for each element i, (i=1÷n), the linear and angular velocities and accelerations, the forces and moments exterior forces.

2. Iterations inside the mechanical structure of the robot.

Under this case, is determined for each element i (i=1÷n) of the robot, the torsos contact forces between the elements i, i+1, respectively the generalized driving forces of kinematic axes.

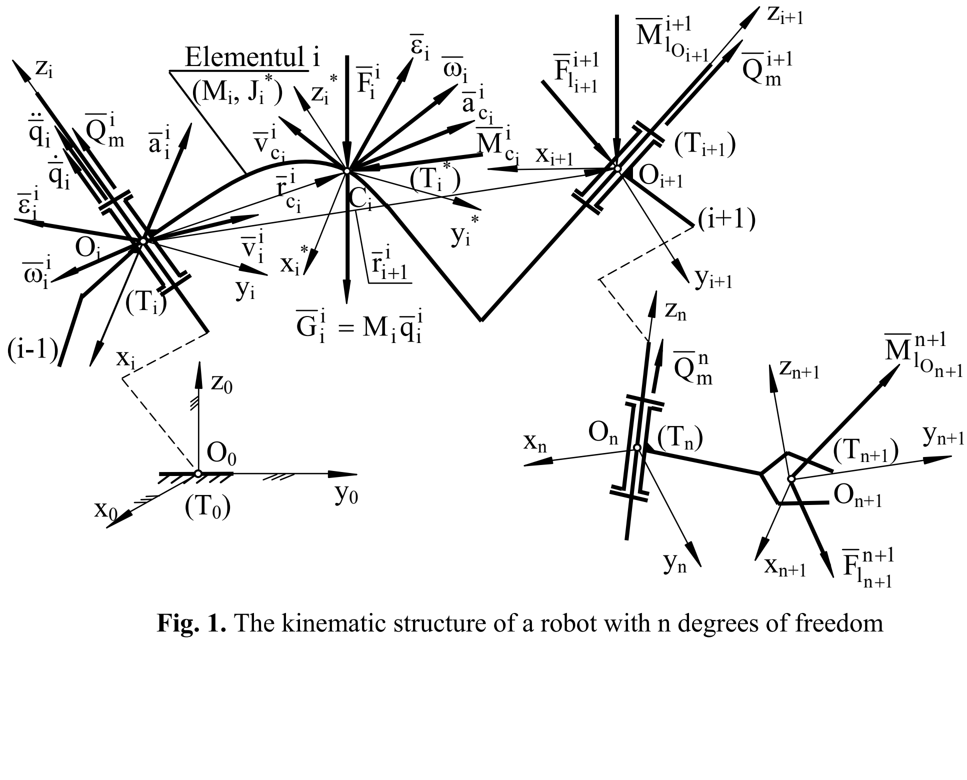

Fig. 1. The kinematic structure of a robot with n degrees of freedom

In accordance with the kinematic structure of a robot with n degrees of freedom (figure 1), [2] and [3], it can be established the dynamic equations of a robot. The kinematic structure of the robot it's geometric modeling, using one of the methods described in [4]. Thus, for each item is determined the homogeneous transformation matrix:

$$[T]_i^{i-1}=\begin{bmatrix} [R]_i^{i-1} & | & [\bar{r}]_i^{i-1} \\ --- & | & --- \\ 0\ 0\ 0 & | & 1 \end{bmatrix}, \qquad (1)$$

If the method of compounds operators DH in the second variant is applied, the matrix elements (1) have the following meanings:

$[R]_i^{i-1}$ - is the rotation matrix defining the orientation of each axis of the system (T_i) with the system (T_{i-1}), and has the expression:

$$[R]_i^{i-1}=\begin{bmatrix} c\theta_i & -s\theta_i & 0 \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} \end{bmatrix}; \qquad (2)$$

$\bar{r}_i^{i-1}$ - is the column vector which defines the position of origin O_i of the system (T_i) in relation with the origin O_{i+1} of the system (T_{i+1}), with the matrix expression:

$$[\bar{r}]_i^{i-1}=\begin{bmatrix} a_i & -d_i s\alpha_{i-1} & d_i c\alpha_{i-1} \end{bmatrix}^T. \qquad (3)$$

It's determine the inverse rotation matrix $[R]_i^{i-1}$:

$$[R]_{i-1}^{i}=\left[R_{i-1}^{\ i}\right]^{-1}=\left[R_{i-1}^{\ i}\right]^T, \qquad (4)$$

$$[R]_{i-1}^{i}=\begin{bmatrix} c\theta_i & s\theta_i c\alpha_{i-1} & s\theta_i s\alpha_{i-1} \\ -s\theta_i & c\theta_i c\alpha_{i-1} & c\theta_i s\alpha_{i-1} \\ 0 & -s\alpha_{i-1} & c\alpha_{i-1} \end{bmatrix}. \qquad (5)$$

For each item i are determined the following parameters:

M_i – the mass of the element I with the relation $M_i=\sum_{j=1}^{k_i}\sigma_j m_j$, where

$$\sigma_j=\begin{cases} +1, & \text{if the item j remains in the item i composition;} \\ -1, & \text{if the item j is eliminated;} \end{cases}$$

$\bar{r}_{c_i}^{i}$ - the position vector of the mass center C_i in relation with the origin O_i of the reference system (T_i), with the relation:

$$[\bar{r}_c]_i^{iD}=[T]_i^{iD}\cdot[\bar{r}_c]_i^i=\begin{bmatrix} x_{c_i}^{iD} & y_{c_i}^{iD} & z_{c_i}^{iD} & 1 \end{bmatrix}^T, \qquad (6)$$

where $[T]_i^{iD}$ is the matrix that defines the position and the orientation of each axis of the reference system (T_i) in relation to the reference system DH, (T_{iD}), which can be determined, according to [5], with the relation:

$$[T]_i^{iD}=\begin{bmatrix} \begin{bmatrix} \bar{x}_{iD}^T \\ \bar{y}_{iD}^T \\ \bar{z}_{iD}^T \end{bmatrix} \cdot \begin{bmatrix} \bar{x}_i & \bar{y}_i & \bar{z}_i \end{bmatrix} & \begin{matrix} \bar{x}_{iD}^T(\bar{p}_i-\bar{p}_{iD}) \\ \bar{y}_{iD}^T(\bar{p}_i-\bar{p}_{iD}) \\ \bar{z}_{iD}^T(\bar{p}_i-\bar{p}_{iD}) \end{matrix} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}; \qquad (7)$$

J_i^{*i} – the inertial tensor of the element i compared with the reference system (T_i^*) with the origin in the mass center C_i. This is determined according to [5], with the following relation:

$$J_i^{*iD}=\begin{bmatrix} J_x^{*iD} & -J_{xy}^{*iD} & -J_{xz}^{*iD} \\ -J_{yx}^{*iD} & J_y^{*iD} & -J_{yz}^{*iD} \\ -J_{zx}^{*iD} & J_{zy}^{*iD} & J_z^{*iD} \end{bmatrix}, \qquad (8)$$

where the matrix elements are the axial and centrifugal mechanical moments of inertia of the element i determined in relation with the reference system (T_{iD}), with the origin in the mass center C_i.

On the robot acts the system of the weight forces for every element i and a system of external forces situated at the end of the robot.

The system of external forces is reduced compared to the origin O_{n+1} of the reference system (T_{n+1}), jointly with the gripper and the manipulated object caught in the grip handle. Thus, it's obtain the reduction torsos of the external forces, with the elements: the resulting vector $\overline{F}_{n+1}^{n+1}$ and the resulting moment $\overline{M}_{O_{n+1}}^{n+1}$, expressed in the reference system (T_{n+1}).

If the robot is in motion, then on each axis of motion the generalized variables $q_i, \dot{q}_i, \ddot{q}_i$, i=1÷n are highlighted.

The kinematic parameters that characterize the movement of the element i at a time, are: $\bar{\omega}_i^i, \bar{\varepsilon}_i^i, \bar{v}_{c_i}, \bar{a}_{c_i}$, i=1÷n, to which are added the kinematic parameters of the fixed element (0):

$$[\bar{\omega}]_0^0=[\dot{\bar{\omega}}]_0^0=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$$
$$[\dot{\bar{v}}]_0^0=\begin{bmatrix}0 & 0 & g\end{bmatrix}^T, \qquad (9)$$

where g is the gravitational acceleration.

By applying the Newton-Euler method the robot's dynamic equations are determined, from which are obtained the generalized driving forces, according to [2] and [3].

These results are obtained covering the two stages of the calculation algorithm and by introducing the notation:

$$\Delta_i = \begin{cases} 1, & \text{if the coupling is rotation} \\ 0, & \text{if the coupling is translation.} \end{cases} \quad (i = 1 \div n) \tag{10}$$

1. Iterations to the exterior of the robot are in the mechanical structure. Applying the calculation algorithm of the iterative method presented in the kinematic modeling, in [6], determine the following kinematic parameters:

$\overline{\omega}_i^i$ - the angular velocity of the element i relative to the fixed system (T_0) from the base of the robot, expressed in the system (T_i) with the relation:

$$\overline{\omega}_i^i = [R]_{i-1}^i \cdot \overline{\omega}_{i-1}^{i-1} + \Delta_i \dot{q}_i \overline{k}_i^i ; \tag{11}$$

$\overline{\varepsilon}_i^i$ - the angular acceleration of the element i relative to the system (T_0), expressed in the system (T_i) with the relation:

$$\overline{\varepsilon}_i^i = [R]_{i-1}^i \cdot \overline{\varepsilon}_{i-1}^{i-1} + \Delta_i \left\{ [R]_{i-1}^i \overline{\omega}_{i-1}^{i-1} \times \dot{q}_i \overline{k}_i^i + \ddot{q}_i \overline{k}_i^i \right\} ; \tag{12}$$

$\overline{a}_i^i$ - the linear acceleration of the origin of system (T_i) relative to the fixed system (T_0), expressed in the system (T_i) with the relation:

$$\begin{aligned} \overline{a}_i^i = [R]_{i-1}^i \left\{ \overline{a}_{i-1}^{i-1} + \overline{\varepsilon}_{i-1}^{i-1} \times \overline{r}_i^{i-1} + \overline{\omega}_{i-1}^{i-1} \times \left(\overline{\omega}_{i-1}^{i-1} \times \overline{r}_i^{i-1} \right) \right\} + \\ + (1 - \Delta_i)\left(2 \overline{\omega}_i^i \times \dot{q}_i \overline{k}_i^i + \ddot{q}_i \overline{k}_i^i \right); \end{aligned} \tag{13}$$

$\overline{a}_{ci}$ - the mass center acceleration of the element i, determined in relation to the fixed system (T_0) and expressed relative to the system (T_i) by the relation:

$$\overline{a}_{c_i}^i = \overline{a}_i^i + \overline{\varepsilon}_i^i \times \overline{r}_{c_i}^i + \overline{\omega}_i^i \times \left(\overline{\omega}_i^i \times \overline{r}_{c_i}^i \right). \tag{14}$$

The reduction torsos elements of the external forces, are obtained by applying to each element i, the dynamic equations of the Newton-Euler and their expressions are:

$$\begin{aligned} \overline{R}_i^i &= M_i \overline{a}_{c_i}^i \\ \overline{M}_{c_i}^i &= J_i^{*i} \overline{\varepsilon}_i^i + \overline{\omega}_i^i \times J_i^{*i} \overline{\omega}_i^i . \end{aligned} \tag{15}$$

2. Iterations inside the mechanical structure of the robot. Using the equations

$$\begin{aligned} \overline{F}_{l_i}^i &= M_i \overline{a}_{c_i}^i - \overline{F}_i^i - \overline{F}_{l_{i+1}}^i \\ \overline{M}_{l_{o_i}}^i &= \overline{r}_{c_i}^i \times M_i \overline{a}_{c_i}^i + J_i^* \overline{\varepsilon}_i + \overline{\omega}_i \times J_i^* \overline{\omega}_i - \overline{M}_{c_i}^i - \\ &\quad - \overline{r}_{c_i}^i \times \overline{F}_i^i - \overline{M}_{l_{o_{i+1}}}^i - \overline{r}_{i+1}^i \times \overline{F}_{l_{i+1}}^i , \end{aligned} \tag{16}$$

for each element i, it can be determined the reduction torsos of the contact forces. The elements of these torsos have the expressions:

$$\begin{aligned} \overline{F}_{l_i}^i &= M_i \overline{a}_{c_i} - \overline{F}_i^i - [R]_{i+1}^i \overline{F}_{l_{i+1}}^{i+1} \\ \overline{M}_{l_{o_i}}^i &= \overline{r}_{c_i} \times M_i \overline{a}_{c_i} + J_i^* \overline{\varepsilon}_i + \overline{\omega}_i \times J_i^* \overline{\omega}_i - \overline{M}_{c_i}^i - \\ &\quad - \overline{r}_{c_i} \times \overline{F}_i^i - [R]_{i+1}^i \overline{M}_{l_{o_{i+1}}}^{i+1} - \overline{r}_{i+1}^i \times [R]_{i+1}^i \overline{F}_{l_{i+1}}^i . \end{aligned} \tag{17}$$

From relations (17) by transforming vectors $\overline{F}_{l_{i+1}}^i$ and $\overline{M}_{l_{o_{i+1}}}^i$ in the vectors:

$$\overline{F}_{l_{i+1}}^i = [R]_{i+1}^i \overline{F}_{l_{i+1}}^{i+1} ; \quad \overline{M}_{l_{o_{i+1}}}^i = [R]_{i+1}^i \overline{M}_{l_{o_{i+1}}}^{i+1} , \tag{18}$$

are obtained the relations (18).

Given the relation below

$$Q_m^i = \begin{cases} \overline{F}_{l_i}^i \cdot \overline{k}_i^i , \textit{ if the coupling is translation} \\ \overline{M}_{l_{O_i}}^i \cdot \overline{k}_i^i , \textit{ if the coupling is rotation.} \end{cases}$$

the generalized driving forces Q_m^i are determined, which actually represents the dynamic model of the robot:

$$Q_m^i = \Delta_i \left[M_{l_{O_i}}^i \right]^T \cdot \overline{k}_i^i + (1 - \Delta_i) \left[\overline{F}_{l_i}^i \right]^T \cdot \overline{k}_i^i + Q_f^i , \tag{19}$$

where Q_f^i is, according to [1], the generalized force due to friction and has the following expressions:

$$Q_f^i = b_i \dot{q}_i + Q_{f_c}^i . \tag{20}$$

The parameters b_i and $Q_{f_c}^i$ from the relation (20) are:

b_i – the viscous friction coefficient;

$Q_{f_c}^i$ - the generalized force due to dry friction (Coulomb friction) and has the expression:

$$\begin{aligned} Q_{f_c}^i = \Delta_i c_i \frac{d_i}{2} \left| \overline{k}_i^i \times \overline{F}_{l_i}^i \right| \operatorname{sgn} \dot{q}_i + \\ + (1 - \Delta_i) c_i \left| \overline{k}_i^i \times \overline{F}_{l_i}^i \right| \operatorname{sgn} \dot{q}_i . \end{aligned} \tag{21}$$

In the relation above, c_i is the dry friction coefficient, and d_i is the diameter of spindle torque. The dynamic equations system (19) can be written as:

$$\overline{Q}_m(t) = \left[Q_m^i(t) = f^{-1}\left(q_j(t), \ j = 1 \div n\right), \ i = 1 \div n \right]^T \tag{22}$$

and represents the dynamic model of the robot with n degrees of freedom.

In the direct problem of robot dynamics are known the column vector of the generalized driving forces. Thus, the functions can be deduced:

$$\overline{q}(t) = f\{\overline{Q}_m(t)\} = [q_i(t),\ i = 1 \div n]^T, \quad (23)$$

which is the law of motion of the robot in configuration space of states.

In the inverse problem of robot dynamics, called inverse dynamic model, are known functions $\overline{q}(t)$ and with the relation (19) the generalized driving forces $\overline{Q}_m(t)$ are determined. Using Newton-Euler iterative method, with relation (19) can be determined the elements of torsos of contact forces between the components of the robot. In conclusion, the Newton-Euler iterative method includes the following steps:

1. It's shape geometrical the mechanical structure of the robot with n degrees of freedom and is determined for each i=1÷n, the matrices (1) and their inverse.

2. Is calculated for each element i, i=1÷n parameters which characterizing the mass distribution, namely: the mass M_i, the position vector $\overline{r}_{c_i}^{\,i}$ of mass center C_i relative with O_i, (6), (7) and with the tensor J_i^{*i} with relation (8).

3. It is calculated, by iteration outward, the kinematics parameters (9)-(14) and the torsos of external forces (15).

4. Is determined by iterations inward, the torsos of contact forces whose elements are given by (17) and the generalized driving forces using the dynamic equations (19).

2. THE DYNAMIC EQUATIONS OF THE TRTTRR1 ROBOT USING THE NEWTON-EULER FORMALISM

According to [7], [8], [9] and [10], the dynamic model of the TRTTRR1 robot (figure 2), will be achieved by applying the Newton-Euler method, implemented in the symbolic modeling program Robot_Symbolic, Robot_Dynamics module of the program Matlab 7.1, [7].

For applying the formalism is required the geometric and kinematic model and the mass distribution parameters. It also required some simplifying assumptions: - are chosen the masses C_i in the origins O_i of the Cartesian reference system $O_i x_i y_i z_i$, i=1÷6, and so the position vectors of mass center are void;

- choosing the mobile reference system so that their axes coincide with the main directions of inertia associated with the origins of these systems, result that the mechanical centrifugal moments of inertia are void. Below are presented the mass distribution parameters: masses of the element i, centers of mass and inertial tensors:

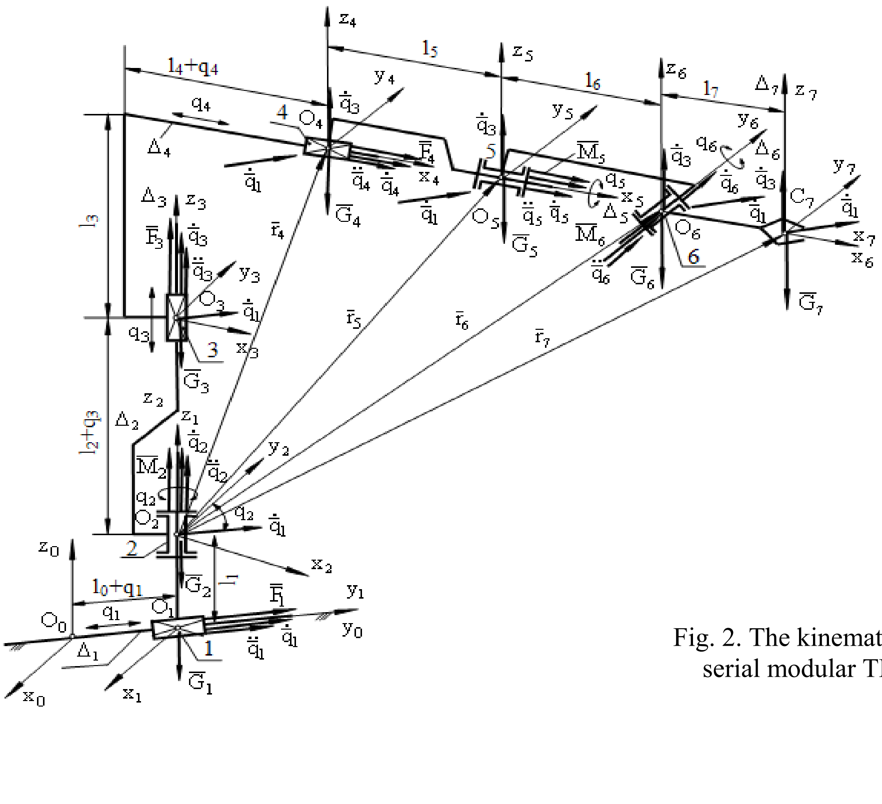

Fig. 2. The kinematic structure of the serial modular TRTTRR1 robot

- the masses: $M_1, M_2, M_3, M_4, M_5, M_6$;
- centers of mass: $\bar{r}_{c_1}^1 = [0\ 0\ 0]^T$,

$\bar{r}_{c_2}^2 = [0\ 0\ 0]^T$, $\bar{r}_{c_3}^3 = [0\ 0\ 0]^T$, $\bar{r}_{c_4}^4 = [0\ 0\ 0]^T$,

$\bar{r}_{c_5}^5 = [0\ 0\ 0]^T$, $\bar{r}_{c_6}^6 = [0\ 0\ 0]^T$;

- inertial tensors:

$$J_1^{*1} = \begin{bmatrix} J_x^{*1} & 0 & 0 \\ 0 & J_y^{*1} & 0 \\ 0 & 0 & J_z^{*1} \end{bmatrix}, \quad J_2^{*2} = \begin{bmatrix} J_x^{*2} & 0 & 0 \\ 0 & J_y^{*2} & 0 \\ 0 & 0 & J_z^{*2} \end{bmatrix},$$

$$J_3^{*3} = \begin{bmatrix} J_x^{*3} & 0 & 0 \\ 0 & J_y^{*3} & 0 \\ 0 & 0 & J_z^{*3} \end{bmatrix}, \quad J_4^{*4} = \begin{bmatrix} J_x^{*4} & 0 & 0 \\ 0 & J_y^{*4} & 0 \\ 0 & 0 & J_z^{*4} \end{bmatrix},$$

$$J_5^{*5} = \begin{bmatrix} J_x^{*5} & 0 & 0 \\ 0 & J_y^{*5} & 0 \\ 0 & 0 & J_z^{*5} \end{bmatrix}, \quad J_6^{*6} = \begin{bmatrix} J_x^{*6} & 0 & 0 \\ 0 & J_y^{*6} & 0 \\ 0 & 0 & J_z^{*6} \end{bmatrix}.$$

Where J_x^{*i}, J_y^{*i}, J_z^{*i}, i=1, 2, 3, 4, 5, 6 are the mechanical axial moments of inertia relative to the system i, with the origin in the mass center C_i and having the same guidance with the system attached to each element of the robot.

Accelerations corresponding to mass centers, are determine according to relation (14).

Since between the robots TRTTR1 and TRTTRR1 are similarities up to the translation module, the following accelerations results:

$$[\bar{a}_c]_1^1 = \begin{bmatrix} 0 \\ 0 \\ g + \ddot{q}_1 \end{bmatrix}; \quad [\bar{a}_c]_2^2 = \begin{bmatrix} sq_2\ddot{q}_1 \\ cq_2\ddot{q}_1 \\ g \end{bmatrix}; \quad [\bar{a}_c]_3^3 = \begin{bmatrix} sq_2\ddot{q}_1 \\ cq_2\ddot{q}_1 \\ g + \ddot{q}_3 \end{bmatrix};$$

$$[\bar{a}_c]_4^4 = \begin{bmatrix} sq_2\ddot{q}_1 - \dot{q}_2^2 q_4 - \dot{q}_2^2 l_4 + \ddot{q}_4 \\ cq_4\ddot{q}_1 + \ddot{q}_2 q_4 + \ddot{q}_2 l_4 + 2\dot{q}_4\dot{q}_2 \\ g + \ddot{q}_3 \end{bmatrix}; \tag{24}$$

$$\bar{a}_{c_5}^5 = \bar{a}_5^5 + \bar{\varepsilon}_5^5 \times \bar{r}_{c_5}^5 + \bar{\omega}_5^5 \times \left(\bar{\omega}_5^5 \times \bar{r}_{c_5}^5\right), \tag{25}$$

$$[\bar{a}_c]_5^5 = \begin{bmatrix} \ddot{q}_1 sq_2 - \dot{q}_2^2 q_4 - \dot{q}_2^2 l_4 + \ddot{q}_4 - \dot{q}_2^2 l_5 \\ \ddot{q}_1 cq_5 cq_2 + \ddot{q}_2 q_4 cq_5 + \ddot{q}_2 l_4 cq_5 + \\ + 2\dot{q}_2\dot{q}_4 cq_5 + \ddot{q}_2 l_5 cq_5 + gsq_5 + \ddot{q}_3 sq_5 \\ -\ddot{q}_1 sq_5 cq_2 - \ddot{q}_2 q_4 sq_5 - \ddot{q}_2 l_4 sq_5 - \\ -2\dot{q}_2\dot{q}_4 sq_5 - \ddot{q}_2 l_5 sq_5 + gcq_5 + \ddot{q}_3 sq_5 \end{bmatrix}; \tag{26}$$

$$\bar{a}_{c_6}^6 = \bar{a}_6^6 + \bar{\varepsilon}_6^5 \times \bar{r}_{c_6}^6 + \bar{\omega}_6^6 \times \left(\bar{\omega}_6^6 \times \bar{r}_{c_6}^6\right), \tag{27}$$

$$[\bar{a}_c]_6^6 = \begin{bmatrix} \ddot{q}_1 sq_2 cq_6 - \dot{q}_2^2 q_4 cq_6 - \dot{q}_2^2 l_4 cq_6 + \ddot{q}_4 cq_6 - \\ -\dot{q}_2^2 l_5 cq_6 - \dot{q}_2^2 l_6 cq_6 + \ddot{q}_1 sq_6 sq_5 cq_2 + \\ + \ddot{q}_2 q_4 sq_6 sq_5 + \ddot{q}_2 l_4 sq_6 sq_5 + \\ + 2\dot{q}_2\dot{q}_4 sq_6 sq_5 + \ddot{q}_2 l_5 sq_6 sq_5 - gcq_5 sq_6 + \\ -\ddot{q}_3 sq_6 cq_5 + \ddot{q}_2 l_6 sq_6 sq_5 \\ \ddot{q}_1 cq_5 cq_2 + \ddot{q}_2 q_4 cq_5 + \ddot{q}_2 l_4 cq_5 + 2\dot{q}_2\dot{q}_4 cq_5 + \\ + \ddot{q}_2 l_5 cq_5 + gsq_5 + \ddot{q}_3 sq_5 + \ddot{q}_2 l_6 cq_5 \\ \ddot{q}_1 sq_6 sq_2 - \dot{q}_2^2 q_4 sq_6 - \dot{q}_2^2 l_4 sq_6 + \ddot{q}_4 sq_6 - \\ -\dot{q}_2^2 l_5 sq_6 - \dot{q}_2^2 l_6 sq_6 - \ddot{q}_1 cq_6 sq_5 cq_2 - \\ -\ddot{q}_2 q_4 cq_6 sq_5 - \ddot{q}_2 l_4 cq_6 sq_5 - 2\dot{q}_2\dot{q}_4 cq_6 sq_5 - \\ -\ddot{q}_2 l_5 sq_5 cq_6 + gcq_6 cq_5 + \ddot{q}_3 cq_6 cq_5 - \\ -\ddot{q}_2 l_6 sq_6 cq_5 \end{bmatrix}. \tag{28}$$

According to the Newton-Euler formalism, first, the mechanical structure is walked by iteration to outward of the robot mechanical structure, resulting the external forces system:

$$[\bar{F}]_1^1 = M_1[\bar{a}_c]_1^1, \bar{F}_1^1 = \begin{bmatrix} 0 \\ M_1\ddot{q}_1 \\ M_1 g \end{bmatrix}; \tag{29}$$

$$[\bar{F}]_2^2 = M_2[\bar{a}_c]_2^2, \bar{F}_2^2 = \begin{bmatrix} M_2\ddot{q}_1 sq_2 \\ M_2\ddot{q}_1 cq_2 \\ M_2 g \end{bmatrix}; \tag{30}$$

$$[\bar{F}]_3^3 = M_3[\bar{a}_c]_3^3, [\bar{F}]_3^3 = \begin{bmatrix} M_3\ddot{q}_1 sq_2 \\ M_3\ddot{q}_1 cq_2 \\ M_3(g + \ddot{q}_3) \end{bmatrix}; \tag{31}$$

$$[\bar{F}]_4^4 = M_4[\bar{a}_c]_4^4,$$

$$[\bar{F}]_4^4 = \begin{bmatrix} -M_4\left(-sq_2\ddot{q}_1 + \dot{q}_2^2 q_4 + \dot{q}_2^2 l_4 - \ddot{q}_4\right) \\ M_4\left(cq_2\ddot{q}_1 + \ddot{q}_2 q_4 + \ddot{q}_2 l_4 + 2\dot{q}_4\dot{q}_2\right) \\ M_4\left(g + \ddot{q}_3\right) \end{bmatrix}; \tag{32}$$

$$[\bar{F}]_5^5 = M_5[\bar{a}_c]_5^5,$$

$$[\bar{F}]_5^5 = \begin{bmatrix} -M_5\left(-\ddot{q}_1 sq_2 + \dot{q}_2^2 q_4 + \dot{q}_2^2 l_4 - \ddot{q}_4 + \dot{q}_2^2 l_5\right) \\ M_5\begin{pmatrix} \ddot{q}_1 cq_2 cq_5 + \ddot{q}_2 q_4 cq_5 + \ddot{q}_2 l_4 cq_5 + \\ + 2\dot{q}_4\dot{q}_2 cq_5 + \ddot{q}_2 l_5 cq_5 + gsq_5 + \ddot{q}_3 sq_5 \end{pmatrix} \\ -M_5\begin{pmatrix} \ddot{q}_1 cq_2 sq_5 + \ddot{q}_2 q_4 sq_5 + \ddot{q}_2 l_4 sq_5 + \\ + 2\dot{q}_4\dot{q}_2 sq_5 + \ddot{q}_2 l_5 sq_5 - gcq_5 - \ddot{q}_3 cq_5 \end{pmatrix} \end{bmatrix}; \tag{33}$$

$$[\bar{F}]_6^6 = M_6[\bar{a}_c]_6^6,$$

$$[\overline{F}]_6^6=$$

$$=\begin{bmatrix} M_6\big(\ddot{q}_1 sq_2 cq_6 - \dot{q}_2^2 q_4 cq_6 - \dot{q}_2^2 l_4 cq_6 + \ddot{q}_4 cq_6 - \\ - \dot{q}_2^2 l_5 cq_6 - \dot{q}_2^2 l_6 cq_6 + \ddot{q}_1 sq_6 cq_2 sq_5 + \\ + \ddot{q}_2 q_4 sq_6 sq_5 + \ddot{q}_2 l_4 sq_6 sq_5 + 2\dot{q}_2 \dot{q}_4 sq_6 sq_5 + \\ + \ddot{q}_2 l_5 sq_6 sq_5 - g sq_6 cq_5 - \ddot{q}_3 sq_6 cq_5 + \\ + \ddot{q}_2 l_6 sq_6 sq_5\big) \\ \hline M_6\begin{pmatrix} \ddot{q}_1 cq_2 cq_5 + \ddot{q}_2 q_4 cq_5 + \ddot{q}_2 l_4 cq_5 + \\ + 2\dot{q}_4 \dot{q}_2 cq_5 + \ddot{q}_2 l_5 cq_5 + g sq_5 + \ddot{q}_3 sq_5 + \ddot{q}_2 l_6 cq_5 \end{pmatrix} \\ \hline - M_6\big(-\ddot{q}_1 sq_2 sq_6 + \dot{q}_2^2 q_4 sq_6 + \dot{q}_2^2 l_4 sq_6 - \ddot{q}_4 sq_6 + \\ + \dot{q}_2^2 l_5 sq_6 + \dot{q}_2^2 l_6 sq_6 + \ddot{q}_1 cq_6 cq_2 sq_5 + \\ + \ddot{q}_2 q_4 cq_6 sq_5 + \ddot{q}_2 l_4 cq_6 sq_5 + 2\dot{q}_2 \dot{q}_4 cq_6 sq_5 + \\ + \ddot{q}_2 l_5 cq_6 sq_5 - g cq_6 cq_5 - \ddot{q}_3 cq_6 cq_5 + \\ + \ddot{q}_2 l_6 cq_6 sq_5\big) \end{bmatrix}. \quad (34)$$

According to the relation (15), the moments of external forces are obtained and have the following form:

$$\overline{M}_{c_1}^1 = J_1^{*1}\overline{\varepsilon}_1^1 + \overline{\omega}_1^1 \times J_1^{*1}\overline{\omega}_1^1, [\overline{M}_c]_1^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad (35)$$

$$\overline{M}_{c_2}^2 = J_2^{*2}\overline{\varepsilon}_2^2 + \overline{\omega}_2^2 \times J_2^{*2}\overline{\omega}_2^2, [\overline{M}_c]_2^2 = \begin{bmatrix} 0 \\ 0 \\ J_z^{*2}\ddot{q}_2 \end{bmatrix}; \quad (36)$$

$$\overline{M}_{c_3}^3 = J_3^{*3}\overline{\varepsilon}_3^3 + \overline{\omega}_3^3 \times J_3^{*3}\overline{\omega}_3^3, [\overline{M}_c]_3^3 = \begin{bmatrix} 0 \\ 0 \\ J_z^{*3}\ddot{q}_2 \end{bmatrix}; \quad (37)$$

$$\overline{M}_{c_4}^4 = J_4^{*4}\overline{\varepsilon}_4^4 + \overline{\omega}_4^4 \times J_4^{*4}\overline{\omega}_4^4, [\overline{M}_c]_4^4 = \begin{bmatrix} 0 \\ 0 \\ J_z^{*4}\ddot{q}_2 \end{bmatrix}; \quad (38)$$

$$\overline{M}_{c_5}^5 = J_5^{*5}\overline{\varepsilon}_5^5 + \overline{\omega}_5^5 \times J_5^{*5}\overline{\omega}_5^5,$$

$$[\overline{M}_c]_5^5 = \begin{bmatrix} J_x^{*5}\ddot{q}_5 - J_y^{*5}\dot{q}_2^2 cq_5 sq_5 + J_z^{*5}\dot{q}_2^2 cq_5 sq_5 \\ \hline J_y^{*5}\ddot{q}_2 sq_5 + J_y^{*5}\dot{q}_2\dot{q}_5 cq_5 + J_x^{*5}\dot{q}_2\dot{q}_5 cq_5 - \\ - J_z^{*5}\dot{q}_2\dot{q}_5 cq_5 \\ \hline J_z^{*5}\ddot{q}_2 cq_5 - J_z^{*5}\dot{q}_2\dot{q}_5 sq_5 - J_x^{*5}\dot{q}_2\dot{q}_5 sq_5 + \\ + J_y^{*5}\dot{q}_2\dot{q}_5 sq_5 \end{bmatrix}; \quad (39)$$

$$\overline{M}_{c_6}^6 = J_6^{*6}\overline{\varepsilon}_6^6 + \overline{\omega}_6^6 \times J_6^{*6}\overline{\omega}_6^6,$$

$$[\overline{M}_c]_6^6=$$

$$=\begin{bmatrix} J_x^{*6}\ddot{q}_5 cq_6 - J_x^{*6}\ddot{q}_2 cq_5 sq_6 + J_x^{*6}\dot{q}_2\dot{q}_5 sq_6 sq_5 - \\ - J_x^{*6}\dot{q}_2\dot{q}_6 cq_6 cq_5 - J_x^{*6}\dot{q}_5\dot{q}_6 sq_6 - \\ - J_y^{*6}\dot{q}_2^2 cq_6 cq_5 sq_5 - J_y^{*6}\dot{q}_2\dot{q}_6 cq_6 cq_5 - \\ - J_y^{*6}\dot{q}_2\dot{q}_5 sq_6 sq_5 - J_y^{*6}\dot{q}_5\dot{q}_6 sq_6 + \\ + J_z^{*6}\dot{q}_2\dot{q}_5 sq_6 sq_5 + J_z^{*6}\dot{q}_2^2 cq_6 cq_5 sq_5 + \\ + J_z^{*6}\dot{q}_5\dot{q}_6 sq_6 + J_z^{*6}\dot{q}_2\dot{q}_6 cq_6 cq_5 \\ \hline J_y^{*6}\ddot{q}_2 sq_5 + J_y^{*6}\dot{q}_2\dot{q}_5 cq_5 + J_y^{*6}\ddot{q}_2 - \\ - J_x^{*6}\dot{q}_2\dot{q}_5 cq_5 - 2J_x^{*6}\dot{q}_2\dot{q}_5 c^2 q_6 cq_5 + \\ + J_x^{*6}\dot{q}_5^2 cq_6 sq_6 - J_x^{*6}\dot{q}_2^2 c^2 q_5 cq_6 sq_6 + \\ + J_z^{*6}\dot{q}_2\dot{q}_6 cq_5 - 2J_z^{*6}\dot{q}_2\dot{q}_5 c^2 q_6 cq_5 + \\ + J_z^{*6}\dot{q}_2^2 c^2 q_5 cq_6 sq_6 - J_z^{*6}\dot{q}_5^2 cq_6 sq_6 \\ \hline J_z^{*6}\ddot{q}_5 sq_6 + J_z^{*6}\ddot{q}_2 cq_5 cq_6 - J_z^{*6}\dot{q}_2\dot{q}_5 cq_6 sq_5 - \\ - J_z^{*6}\dot{q}_2\dot{q}_6 sq_6 cq_5 + J_z^{*6}\dot{q}_5\dot{q}_6 cq_6 + \\ + J_x^{*6}\dot{q}_2^2 sq_6 cq_5 sq_5 - J_x^{*6}\dot{q}_2\dot{q}_5 cq_6 sq_5 + \\ + J_x^{*6}\dot{q}_2\dot{q}_6 sq_6 cq_5 - J_x^{*6}\dot{q}_5\dot{q}_6 cq_6 - \\ - J_y^{*6}\dot{q}_2^2 sq_6 cq_5 sq_5 - J_y^{*6}\dot{q}_2\dot{q}_6 sq_6 cq_5 + \\ + J_y^{*6}\dot{q}_2\dot{q}_5 cq_6 sq_5 + J_y^{*6}\dot{q}_5\dot{q}_6 cq_6 \end{bmatrix}. \quad (40)$$

In the second part of the Newton-Euler method, the mechanical structure is walked by iteration to inward of the robot mechanical structure.

Thus, the contact forces torsos between elements and their moments are determined, respectively the generalized driving forces from the robot's couplers.

The contact forces, according to (16) and [2], have the following expressions:

$$\overline{F}_{l_6}^6 = [R]_7^6 \cdot \overline{F}_{l_7}^7 + \overline{F}_6^6,$$

$$\left[\overline{F}_l\right]_6^6=\begin{bmatrix} F_{l_x}^7+M_6\ddot{q}_1 sq_2 cq_6-M_6\dot{q}_2^2 q_4 cq_6-M_6\dot{q}_2^2 l_4 cq_6+M_6\ddot{q}_4 cq_6-M_6\dot{q}_2^2 l_5 cq_6-M_6\dot{q}_2^2 l_6 cq_6+M_6\ddot{q}_1 sq_6 cq_2 sq_5+M_6\ddot{q}_2 q_4 sq_6 sq_5+M_6\ddot{q}_2 l_4 sq_6 sq_5+2M_6\dot{q}_2\dot{q}_4 sq_6 sq_5+M_6\ddot{q}_2 l_5 sq_6 sq_5-M_6 g sq_6 cq_5-M_6\ddot{q}_3 sq_6 cq_5+M_6\ddot{q}_2 l_6 sq_6 sq_5 \\ F_{l_y}^7+M_6\ddot{q}_1 cq_2 cq_5+M_6\ddot{q}_2 q_4 cq_5+M_6\ddot{q}_2 l_4 cq_5+2M_6\dot{q}_4\dot{q}_2 cq_5+M_6\ddot{q}_2 l_5 cq_5+M_6 g sq_5+M_6\ddot{q}_3 sq_5+M_6\ddot{q}_2 l_6 cq_5 \\ F_{l_z}^7+M_6\ddot{q}_1 sq_2 sq_6-M_6\dot{q}_2^2 q_4 sq_6-M_6\dot{q}_2^2 l_4 sq_6+M_6\ddot{q}_4 sq_6-M_6\dot{q}_2^2 l_5 sq_6-M_6\dot{q}_2^2 l_6 sq_6-M_6\ddot{q}_1 cq_6 cq_2 sq_5-M_6\ddot{q}_2 q_4 cq_6 sq_5-M_6\ddot{q}_2 l_4 cq_6 sq_5-2M_6\dot{q}_2\dot{q}_4 cq_6 sq_5-M_6\ddot{q}_2 l_5 cq_6 sq_5+M_6 g cq_6 cq_5+M_6\ddot{q}_3 cq_6 cq_5-M_6\ddot{q}_2 l_6 cq_6 sq_5 \end{bmatrix}; \quad (41)$$

$$\overline{F}_{l_5}^5=\left[R\right]_6^5\cdot\overline{F}_{l_6}^6+\overline{F}_5^5,$$

$$\left[\overline{F}_l\right]_5^5=\begin{bmatrix} M_5\ddot{q}_1 sq_2-M_5\dot{q}_2^2 q_4-M_5\dot{q}_2^2 l_4-M_5\dot{q}_2^2 l_5+M_6\ddot{q}_1 sq_2-M_6\dot{q}_2^2 q_4-M_6\dot{q}_2^2 l_4-M_6\dot{q}_2^2 l_5-M_6\dot{q}_2^2 l_6+F_{l_x}^7 cq_6+F_{l_z}^7 sq_6+M_5\ddot{q}_4+M_6\ddot{q}_4 \\ F_{l_y}^7+M_6\ddot{q}_1 cq_5 cq_2+M_6\ddot{q}_2 q_4 cq_5+M_6\ddot{q}_2 l_4 cq_5+2M_6\dot{q}_2\dot{q}_4 cq_5+M_6\ddot{q}_2 l_5 cq_5+M_6 g sq_5+M_6\ddot{q}_3 sq_5+M_6\ddot{q}_2 l_6 cq_5+M_5\ddot{q}_1 cq_5 cq_2+M_5\ddot{q}_2 q_4 cq_5+M_5\ddot{q}_2 l_4 cq_5+2M_5\dot{q}_2\dot{q}_4 cq_5+M_5\ddot{q}_2 l_5 cq_5+M_5 g sq_5+M_5\ddot{q}_3 sq_5 \\ -M_5\ddot{q}_1 sq_5 cq_2-M_5\ddot{q}_2 q_4 sq_5-M_5\ddot{q}_2 l_4 sq_5-2M_5\dot{q}_2\dot{q}_4 sq_5-M_5\ddot{q}_2 l_5 sq_5-F_{l_x}^7 sq_6+F_{l_z}^7 cq_6+M_5 g cq_5+M_5\ddot{q}_3 cq_5+M_6 g cq_5+M_6\ddot{q}_3 cq_5-M_6\ddot{q}_1 cq_2 sq_5-M_6\ddot{q}_2 q_4 sq_5-M_6\ddot{q}_2 l_4 sq_5-2M_6\dot{q}_2\dot{q}_4 sq_5-M_6\ddot{q}_2 l_5 sq_5-M_6\ddot{q}_2 l_6 sq_5 \end{bmatrix}; \quad (42)$$

$$\overline{F}_{l_4}^4=\left[R\right]_5^4\cdot\overline{F}_{l_5}^5+\overline{F}_4^4;$$

$$\left[\overline{F}_l\right]_4^4=\begin{bmatrix} M_5\ddot{q}_1 sq_2-M_5\dot{q}_2^2 q_4-M_5\dot{q}_2^2 l_4-M_5\dot{q}_2^2 l_5+M_6\ddot{q}_1 sq_2-M_6\dot{q}_2^2 q_4-M_6\dot{q}_2^2 l_4-M_6\dot{q}_2^2 l_5-M_6\dot{q}_2^2 l_6+F_{l_x}^7 cq_6+F_{l_z}^7 sq_6+M_5\ddot{q}_4+M_6\ddot{q}_4+M_4\ddot{q}_1 sq_2-M_4\dot{q}_2^2 q_4-M_4\dot{q}_2^2 l_4+M_4\ddot{q}_4 \\ F_{l_x}^7 sq_5 sq_6-F_{l_z}^7 sq_5 cq_6+M_4\ddot{q}_1 cq_2+M_4\ddot{q}_2 q_4+M_4\ddot{q}_2 l_4+2M_4\dot{q}_2\dot{q}_4+M_5\ddot{q}_2 q_4+M_5\ddot{q}_1 cq_2+M_5\ddot{q}_2 l_4+M_5\ddot{q}_2 l_5+M_6\ddot{q}_1 cq_2+F_{l_y}^7 cq_5+M_6\ddot{q}_2 q_4+M_6\ddot{q}_2 l_4+M_6\ddot{q}_2 l_5+M_6\ddot{q}_2 l_6+2M_5\dot{q}_2\dot{q}_4+2M_6\dot{q}_2\dot{q}_4 \\ F_{l_y}^7 sq_5+M_4 g+M_4\ddot{q}_3+M_6 g+M_6\ddot{q}_3+M_5 g+M_5\ddot{q}_3-F_{l_x}^7 cq_5 sq_6+F_{l_z}^7 cq_5 cq_6 \end{bmatrix}; \quad (43)$$

$$\overline{F}_{l_3}^3=\left[R\right]_4^3\cdot\overline{F}_{l_4}^4+\overline{F}_3^3, \quad (44)$$

$$\left[\overline{F}_l\right]_3^3=\begin{bmatrix} M_5\ddot{q}_1 sq_2-M_5\dot{q}_2^2 q_4-M_5\dot{q}_2^2 l_4-M_5\dot{q}_2^2 l_5+M_6\ddot{q}_1 sq_2-M_6\dot{q}_2^2 q_4-M_6\dot{q}_2^2 l_4-M_6\dot{q}_2^2 l_5-M_6\dot{q}_2^2 l_6+F_{l_x}^7 cq_6+F_{l_z}^7 sq_6+M_5\ddot{q}_4+M_6\ddot{q}_4+M_4\ddot{q}_1 sq_2-M_4\dot{q}_2^2 q_4-M_4\dot{q}_2^2 l_4+M_4\ddot{q}_4+M_3\ddot{q}_1 sq_2 \\ F_{l_x}^7 sq_5 sq_6-F_{l_z}^7 sq_5 cq_6+M_4\ddot{q}_1 cq_2+M_4\ddot{q}_2 q_4+M_4\ddot{q}_2 l_4+2M_4\dot{q}_2\dot{q}_4+M_5\ddot{q}_2 q_4+M_5\ddot{q}_1 cq_2+M_5\ddot{q}_2 l_4+M_5\ddot{q}_2 l_5+M_6\ddot{q}_1 cq_2+F_{l_y}^7 cq_5+M_6\ddot{q}_2 q_4+M_6\ddot{q}_2 l_4+M_6\ddot{q}_2 l_5+M_6\ddot{q}_2 l_6+2M_5\dot{q}_2\dot{q}_4+2M_6\dot{q}_2\dot{q}_4+M_3\ddot{q}_1 cq_2 \\ F_{l_y}^7 sq_5+M_4 g+M_4\ddot{q}_3+M_6 g+M_6\ddot{q}_3+M_5 g+M_5\ddot{q}_3-F_{l_x}^7 cq_5 sq_6+F_{l_z}^7 cq_5 cq_6+M_3 g+M_3\ddot{q}_3 \end{bmatrix}; \quad (45)$$

$$\overline{F}_{l_2}^2=\left[R\right]_3^2\cdot\overline{F}_{l_3}^3+\overline{F}_2^2, \quad (46)$$

$$[\overline{F}_l]_2^2 = \begin{bmatrix} M_5\ddot{q}_1 sq_2 - M_5\dot{q}_2^2 q_4 - M_5\dot{q}_2^2 l_4 - M_5\dot{q}_2^2 l_5 + \\ + M_6\ddot{q}_1 sq_2 - M_6\dot{q}_2^2 q_4 - M_6\dot{q}_2^2 l_4 - M_6\dot{q}_2^2 l_5 - \\ - M_6\dot{q}_2^2 l_6 + F_{l_x}^7 cq_6 + F_{l_z}^7 sq_6 + M_5\ddot{q}_4 + \\ + M_6\ddot{q}_4 + M_4\ddot{q}_1 sq_2 - M_4\dot{q}_2^2 q_4 - M_4\dot{q}_2^2 l_4 + \\ + M_4\ddot{q}_4 + M_3\ddot{q}_1 sq_2 + M_2\ddot{q}_1 sq_2 \\ \hdashline F_{l_x}^7 sq_5 sq_6 - F_{l_z}^7 sq_5 cq_6 + M_4\ddot{q}_1 cq_2 + M_4\ddot{q}_2 q_4 + \\ + M_4\ddot{q}_2 l_4 + 2M_4\dot{q}_2\dot{q}_4 + M_5\ddot{q}_2 q_4 + M_5\ddot{q}_1 cq_2 + \\ + M_5\ddot{q}_2 l_4 + M_5\ddot{q}_2 l_5 + M_6\ddot{q}_1 cq_2 + F_{l_y}^7 cq_5 + \\ + M_6\ddot{q}_2 q_4 + M_6\ddot{q}_2 l_4 + M_6\ddot{q}_2 l_5 + M_6\ddot{q}_2 l_6 + \\ + 2M_5\dot{q}_2\dot{q}_4 + 2M_6\dot{q}_2\dot{q}_4 + M_3\ddot{q}_1 cq_2 + M_2\ddot{q}_1 cq_2 \\ \hdashline F_{l_y}^7 sq_5 + M_4 g + M_4\ddot{q}_3 + M_6 g + M_6\ddot{q}_3 + \\ + M_5 g + M_5\ddot{q}_3 - F_{l_x}^7 cq_5 sq_6 + F_{l_z}^7 cq_5 cq_6 + \\ + M_3 g + M_3\ddot{q}_3 + M_2 g \end{bmatrix}; \tag{47}$$

$$\overline{F}_{l_1}^1 = [R]_2^1 \cdot \overline{F}_{l_2}^2 + \overline{F}_1^1, \tag{48}$$

$$\begin{aligned} \overline{F}_{l_x}^1 = & -M_5\dot{q}_2^2 q_4 cq_2 - M_5\dot{q}_2^2 l_4 cq_2 - M_5\dot{q}_2^2 l_5 cq_2 - \\ & - M_6\dot{q}_2^2 q_4 cq_2 - M_6\dot{q}_2^2 l_4 cq_2 - M_6\dot{q}_2^2 l_5 cq_2 - \\ & - M_6\dot{q}_2^2 l_6 cq_2 - M_4\dot{q}_2^2 q_4 cq_2 - M_4\dot{q}_2^2 l_4 cq_2 - \\ & - F_{l_x}^7 sq_2 sq_5 sq_6 + F_{l_z}^7 sq_2 sq_5 cq_6 - M_4\ddot{q}_2 q_4 sq_2 - \\ & - M_4\ddot{q}_2 l_4 sq_2 - 2M_4\dot{q}_2\dot{q}_4 sq_2 - M_5\ddot{q}_2 q_4 sq_2 - \\ & - M_5\ddot{q}_2 l_4 sq_2 - M_5\ddot{q}_2 l_5 sq_2 - M_6\ddot{q}_2 q_4 sq_2 - \\ & - M_6\ddot{q}_2 l_4 sq_2 - M_6\ddot{q}_2 l_5 sq_2 - M_6\ddot{q}_2 l_6 sq_2 - \\ & - 2M_5\dot{q}_2\dot{q}_4 sq_2 - 2M_6\dot{q}_2\dot{q}_4 sq_2 - F_{l_x}^7 cq_2 cq_6 + \\ & + F_{l_z}^7 cq_2 sq_6 + M_5\ddot{q}_4 cq_2 + M_6\ddot{q}_4 cq_2 + \\ & + M_4\ddot{q}_4 cq_2 - F_{l_y}^7 sq_2 cq_5 ; \end{aligned} \tag{49}$$

$$\begin{aligned} \overline{F}_{l_y}^1 = & F_{l_x}^7 sq_5 sq_6 + F_{l_z}^7 sq_2 sq_6 + M_5\ddot{q}_4 sq_2 + M_6\ddot{q}_4 sq_2 + \\ & + M_4\ddot{q}_4 sq_2 + F_{l_y}^7 cq_2 cq_5 + M_6\ddot{q}_1 + M_4\ddot{q}_2 q_4 cq_2 + \\ & + M_4\ddot{q}_2 l_4 cq_2 - F_{l_z}^7 cq_2 sq_5 cq_6 + M_5\ddot{q}_2 q_4 cq_2 + \\ & + M_5\ddot{q}_2 l_4 cq_2 + M_6\ddot{q}_2 q_4 cq_2 + M_6\ddot{q}_2 l_4 cq_2 + \\ & + M_6\ddot{q}_2 l_6 cq_2 + 2M_5\dot{q}_2\dot{q}_4 cq_2 + M_5\ddot{q}_2 l_5 cq_2 - \\ & - M_5\dot{q}_2^2 q_4 sq_2 + 2M_6\dot{q}_2\dot{q}_4 cq_2 + M_6\ddot{q}_2 l_5 cq_2 - \\ & - M_5\dot{q}_2^2 l_4 sq_2 - M_5\dot{q}_2^2 l_5 sq_2 - M_6\dot{q}_2^2 q_4 sq_2 - \\ & - M_6\dot{q}_2^2 l_4 sq_2 - M_6\dot{q}_2^2 l_5 sq_2 - M_6\dot{q}_2^2 l_6 sq_2 - \\ & - M_4\dot{q}_2^2 q_4 sq_2 - M_4\dot{q}_2^2 l_4 sq_2 + F_{l_x}^7 cq_2 sq_5 sq_6 + \\ & + 2M_4\dot{q}_2\dot{q}_4 cq_2 + M_1\ddot{q}_1 + M_5\ddot{q}_1 + M_2\ddot{q}_1 + \\ & + M_3\ddot{q}_1 + M_4\ddot{q}_1 ; \end{aligned} \tag{50}$$

$$\begin{aligned} \overline{F}_{l_z}^1 = & F_{l_y}^7 sq_5 + M_4 g + M_4\ddot{q}_3 + M_6 g + M_6\ddot{q}_3 + \\ & + M_5 g + M_5\ddot{q}_3 - F_{l_x}^7 cq_5 sq_6 + F_{l_z}^7 cq_5 cq_6 + \\ & + M_3 g + M_3\ddot{q}_3 + M_2 g + M_1 g . \end{aligned} \tag{51}$$

Given the complexity of the moments of contact forces, they will not be presented in the paper. According to [7] and [2], the generalized driving forces are determined. Their expressions represent the differential dynamic equations system which characterized the dynamic model of the robot TRTTRR1.

Having the relation (19), the generalized driving forces have the following expressions:

$$Q_m^1 = \left[\overline{F}_{l_1}^1\right]^T \cdot \overline{j}_1^1 = \left[F_{l_x}^1 \quad F_{l_y}^1 \quad F_{l_z}^1\right] \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = F_{l_y}^1 , \tag{52}$$

$$\begin{aligned} Q_m^1 = & F_{l_x}^7 sq_5 sq_6 + F_{l_z}^7 sq_2 sq_6 + M_5\ddot{q}_4 sq_2 + \\ & + M_6\ddot{q}_4 sq_2 + M_4\ddot{q}_4 sq_2 + F_{l_y}^7 cq_2 cq_5 + M_6\ddot{q}_1 + \\ & + M_4\ddot{q}_2 q_4 cq_2 + M_4\ddot{q}_2 l_4 cq_2 - F_{l_z}^7 cq_2 sq_5 cq_6 + \\ & + M_5\ddot{q}_2 q_4 cq_2 + M_5\ddot{q}_2 l_4 cq_2 + M_6\ddot{q}_2 q_4 cq_2 + \\ & + M_6\ddot{q}_2 l_4 cq_2 + M_6\ddot{q}_2 l_6 cq_2 + 2M_5\dot{q}_2\dot{q}_4 cq_2 + \\ & + M_5\ddot{q}_2 l_5 cq_2 - M_5\dot{q}_2^2 q_4 sq_2 + 2M_6\dot{q}_2\dot{q}_4 cq_2 + \\ & + M_6\ddot{q}_2 l_5 cq_2 - M_5\dot{q}_2^2 l_4 sq_2 - M_5\dot{q}_2^2 l_5 sq_2 - \\ & - M_6\dot{q}_2^2 q_4 sq_2 - M_6\dot{q}_2^2 l_4 sq_2 - M_6\dot{q}_2^2 l_5 sq_2 - \\ & - M_6\dot{q}_2^2 l_6 sq_2 - M_4\dot{q}_2^2 q_4 sq_2 - M_4\dot{q}_2^2 l_4 sq_2 + \\ & + F_{l_x}^7 cq_2 sq_5 sq_6 + 2M_4\dot{q}_2\dot{q}_4 cq_2 + M_1\ddot{q}_1 + \\ & + M_5\ddot{q}_1 + M_2\ddot{q}_1 + M_3\ddot{q}_1 + M_4\ddot{q}_1 ; \end{aligned} \tag{53}$$

$$Q_m^2 = \left[\overline{M}_{l_{O_2}}^2\right]^T \cdot \overline{k}_2^2 = \left[M_{l_x}^2 \quad M_{l_y}^2 \quad M_{l_z}^2\right] \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = M_{l_z}^2 , \tag{54}$$

$$\begin{aligned} Q_m^2 &= M_5\ddot{q}_2 q_4^2 + M_4\ddot{q}_2 q_4^2 + 2M_4\ddot{q}_2 q_4 l_4 + F_{l_y}^7 q_4 cq_5 + \\ &+ F_{l_y}^7 l_4 cq_5 + M_6\ddot{q}_2 l_4^2 + M_5\ddot{q}_2 l_4^2 + M_4\ddot{q}_2 l_4^2 + M_6\ddot{q}_2 q_4^2 + \\ &+ 2M_4\dot{q}_2\dot{q}_4 l_4 + 2M_6\dot{q}_2\dot{q}_4 q_4 + J_y^{*6}\ddot{q}_2 + J_y^{*5}\ddot{q}_2 - \\ &- J_y^{*6}\ddot{q}_2 c^2 q_5 - J_y^{*5}\ddot{q}_2 c^2 q_5 + 2M_5\dot{q}_2\dot{q}_4 l_4 + 2M_6\dot{q}_2\dot{q}_4 l_4 + \\ &+ J_z^{*4}\ddot{q}_2 + F_{l_x}^7 q_4 sq_5 sq_6 - F_{l_z}^7 q_4 sq_5 cq_6 + M_4\ddot{q}_1 q_4 cq_2 + \\ &+ 2M_6\ddot{q}_2 l_5 l_6 + 2M_5\dot{q}_2\dot{q}_4 l_5 + 2M_6\dot{q}_2\dot{q}_4 l_5 + 2M_6\dot{q}_2\dot{q}_4 l_6 + \\ &+ 2M_5\ddot{q}_2 q_4 l_5 + 2M_5\ddot{q}_2 l_4 l_5 + J_z^{*2}\ddot{q}_2 + 2M_6\ddot{q}_2 q_4 l_6 + \\ &+ 2M_6\ddot{q}_2 l_4 l_6 + 2M_6\ddot{q}_2 q_4 l_5 + 2M_6\ddot{q}_2 l_4 l_5 + M_5\ddot{q}_1 l_4 cq_2 + \\ &+ J_z^{*3}\ddot{q}_2 + F_{l_x}^7 l_4 sq_5 sq_6 - F_{l_z}^7 l_4 sq_5 cq_6 + M_4\ddot{q}_1 l_4 cq_2 + \\ &+ M_5\ddot{q}_1 q_4 cq_2 + M_6\ddot{q}_1 q_4 cq_2 + M_6\ddot{q}_1 l_4 cq_2 - F_{l_z}^7 l_6 sq_5 cq_6 + \\ &+ F_{l_x}^7 l_6 sq_5 sq_6 - J_x^{*6}\ddot{q}_2 c^2 q_5 c^2 q_6 + M_5\ddot{q}_2 l_5^2 + M_6\ddot{q}_2 l_5^2 + \\ &+ M_6\ddot{q}_2 l_6^2 + M_5\ddot{q}_1 l_5 cq_2 + M_6\ddot{q}_1 l_6 cq_2 + J_z^{*6}\ddot{q}_2 c^2 q_5 c^2 q_6 + \\ &+ F_{l_y}^7 l_7 cq_5 cq_6 + J_y^{*6}\dot{q}_5\dot{q}_6 cq_5 - J_z^{*6}\dot{q}_5\dot{q}_6 cq_5 + J_x^{*6}\dot{q}_5\dot{q}_6 cq_5 + \\ &+ F_{l_x}^7 l_5 sq_5 sq_6 - F_{l_z}^7 l_5 sq_5 sq_6 - M_{l_x}^7 cq_5 sq_6 - F_{l_z}^7 l_7 sq_5 + \\ &+ F_{l_y}^7 l_5 cq_5 + J_y^{*6}\ddot{q}_6 sq_5 + J_x^{*6}\ddot{q}_2 c^2 q_5 + J_z^{*5}\ddot{q}_2 c^2 q_5 + \\ &+ M_{l_z}^7 cq_5 cq_6 + F_{l_y}^7 l_6 cq_5 + F_{l_y}^7 sq_5 + 2J_x^{*6}\dot{q}_2\dot{q}_6 c^2 q_5 sq_6 cq_6 - \\ &- 2J_z^{*6}\dot{q}_2\dot{q}_6 c^2 q_5 sq_6 cq_6 - J_x^{*6}\ddot{q}_5 cq_5 cq_6 sq_6 + \\ &+ 2J_z^{*6}\dot{q}_5\dot{q}_6 c^2 q_6 cq_5 - 2J_x^{*6}\dot{q}_5\dot{q}_6 c^2 q_6 cq_5 + J_z^{*6}\ddot{q}_5 cq_5 cq_6 sq_6 + \\ &+ 2J_x^{*6}\dot{q}_2\dot{q}_5 c^2 q_6 sq_5 cq_5 - 2J_z^{*6}\dot{q}_2\dot{q}_5 c^2 q_6 sq_5 cq_5 - \\ &- J_z^{*6}\dot{q}_5^2 sq_5 cq_6 sq_6 - 2J_z^{*5}\dot{q}_2\dot{q}_5 sq_5 cq_5 + 2J_y^{*6}\dot{q}_2\dot{q}_5 sq_5 cq_5 + \\ &+ J_x^{*6}\dot{q}_5^2 sq_5 cq_6 sq_6 + 2J_y^{*5}\dot{q}_2\dot{q}_5 sq_5 cq_5 - 2J_x^{*6}\dot{q}_2\dot{q}_5 sq_5 cq_5 ; \end{aligned} \tag{55}$$

$$Q_m^3 = \left[\overline{F}_{l_3}^3\right]^T \cdot \overline{k}_3^3 = \left[F_{l_x}^3 \quad F_{l_y}^3 \quad F_{l_z}^3\right] \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = F_{l_z}^3, \tag{56}$$

$$\begin{aligned} Q_m^3 &= F_{l_y}^7 sq_5 + M_4 g + M_4\ddot{q}_3 + M_6 g + M_6\ddot{q}_3 + \\ &+ M_5 g + M_5\ddot{q}_3 - F_{l_x}^7 cq_5 sq_6 + F_{l_z}^7 cq_5 cq_6 + \\ &+ M_3 g + M_3\ddot{q}_3 ; \end{aligned} \tag{57}$$

$$Q_m^4 = \left[\overline{F}_{l_4}^4\right]^T \cdot \overline{i}_4^4 = \left[F_{l_x}^4 \quad F_{l_y}^4 \quad F_{l_z}^4\right] \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = F_{l_x}^4, \tag{58}$$

$$\begin{aligned} Q_m^4 &= M_5\ddot{q}_1 sq_2 - M_5\dot{q}_2^2 q_4 - M_5\dot{q}_2^2 l_4 - M_5\dot{q}_2^2 l_5 + \\ &+ M_6\ddot{q}_1 sq_2 - M_6\dot{q}_2^2 q_4 - M_6\dot{q}_2^2 l_4 - M_6\dot{q}_2^2 l_5 - \\ &- M_6\dot{q}_2^2 l_6 + F_{l_x}^7 cq_6 + F_{l_z}^7 sq_6 + M_5\ddot{q}_4 + M_6\ddot{q}_4 + \\ &+ M_4\ddot{q}_1 sq_2 - M_4\dot{q}_2^2 q_4 - M_4\dot{q}_2^2 l_4 + M_4\ddot{q}_4 ; \end{aligned} \tag{59}$$

$$Q_m^5 = \left[\overline{M}_{l_{O_5}}^5\right]^T \cdot \overline{i}_5^5 = \left[M_{l_x}^5 \quad M_{l_y}^5 \quad M_{l_z}^5\right] \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = M_{l_x}^5, \tag{60}$$

$$\begin{aligned} Q_m^5 &= -J_x^{*6}\dot{q}_2^2 c^2 q_6 sq_5 cq_5 + J_x^{*6}\dot{q}_2\dot{q}_6 cq_5 - J_y^{*6}\dot{q}_2^2 sq_5 cq_5 - \\ &- J_y^{*6}\dot{q}_2\dot{q}_6 cq_5 - J_z^{*6}\dot{q}_2\dot{q}_6 cq_5 + M_{l_x}^7 cq_6 + M_{l_z}^7 sq_6 + \\ &+ J_z^{*6}\ddot{q}_5 - J_z^{*6}\ddot{q}_5 c^2 q_6 - J_x^{*6}\ddot{q}_2 sq_5 cq_6 sq_6 + J_z^{*5}\dot{q}_2^2 sq_5 cq_5 - \\ &- J_y^{*5}\dot{q}_2^2 sq_5 cq_5 + J_x^{*6}\ddot{q}_5 c^2 q_6 + F_{l_y}^7 l_7 sq_6 - \\ &- 2J_x^{*6}\dot{q}_5\dot{q}_6 c^2 q_6 cq_5 + J_z^{*6}\dot{q}_2^2 c^2 q_6 sq_5 cq_5 + \\ &+ 2J_z^{*6}\dot{q}_2\dot{q}_6 c^2 q_6 cq_5 - 2J_x^{*6}\dot{q}_5\dot{q}_6 cq_6 sq_6 + \\ &+ 2J_z^{*6}\dot{q}_5\dot{q}_6 cq_6 sq_6 + J_z^{*6}\ddot{q}_2 cq_5 cq_6 sq_6 + \\ &+ J_x^{*5}\ddot{q}_5 + J_x^{*6}\dot{q}_2^2 sq_5 cq_5 ; \end{aligned} \tag{61}$$

$$Q_m^6 = \left[\overline{M}_{l_{O_6}}^6\right]^T \cdot \overline{j}_6^6 = \left[M_{l_x}^6 \quad M_{l_y}^6 \quad M_{l_z}^6\right] \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = M_{l_y}^6, \tag{62}$$

$$\begin{aligned} Q_m^6 &= M_{l_y}^7 - F_{l_z}^7 l_7 + J_y^{*6}\ddot{q}_2 sq_5 + J_y^{*6}\dot{q}_2\dot{q}_5 cq_5 + J_y^{*6}\ddot{q}_6 - \\ &- J_x^{*6}\dot{q}_2\dot{q}_5 cq_5 + 2J_x^{*6}\dot{q}_2\dot{q}_5 c^2 q_6 cq_5 + J_x^{*6}\dot{q}_5^2 sq_6 cq_6 - \\ &- J_x^{*6}\dot{q}_2^2 c^2 q_5 sq_6 cq_6 + J_z^{*6}\dot{q}_2\dot{q}_5 cq_5 - \\ &- 2J_z^{*6}\dot{q}_2\dot{q}_5 c^2 q_6 cq_5 + J_z^{*6}\dot{q}_2^2 c^2 q_5 sq_6 cq_6 - \\ &- J_z^{*6}\dot{q}_5^2 sq_6 cq_6 . \end{aligned} \tag{63}$$

These generalized driving forces represents the system of differential dynamic equations characterizing the dynamic model of serial modular TRTTRR1 robot.

3. CONCLUSION

By using Newton-Euler, the dynamic equations are highlight in the engine couplers the forces and moments of the contact forces.

Their expressions are complicated and sometimes very lengthy, which is why this method is recommended only in cases that have made calculations for the coupling strength.

If such calculations are not reguired, for the dynamic study of the robot then are used the Lagrange formalism and the principle of virtual displacements.

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Ecuațiile dinamice ale robotului TRTTRR1 utilizând formalismul Newton-Euler

Rezumat:. Pentru a determina ecuațiilor dinamice ale robotului TRTTRR1 folosind formalismul lui Newton-Euler, este nevoie în primul rând de modelarea geometrică şi cinematică. În al doilea rând sunt necesari parametrii de distribuție a maselor şi anumite ipoteze simplificatoare legate de alegerea centrelor de masă C_i şi de momentele de inerție mecanice centrifugale. Cu acestea sunt determinate accelerațiile corespunzătoare centrelor de masă, apoi se determină elementele torsorului de reducere pentru sistemul forțelor exterioare. Următorul pas îl constituie determinarea torsorului forțelor de legătură şi a momentelor forțelor de legătură.

Ultimul pas este determinarea forțelor generalizate motoare din cuplele robotului, expresiile acestora reprezentând ecuațiile dinamice ale robotului TRTR.

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