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A LUMPED MASS EXHAUST SYSTEM MODEL

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Abstract: *In the present paper the dynamics of an exhaust system is under observation. In order to study the system vibrations, to perform modal analysis and the vibration isolation, the exhaust system is modeled by using lumped masses. The system of the differential equations of the exhaust system structure is derived and placed in matrix form. Natural frequencies and the modal vectors for the first modes of vibration can be determined. This approach has the target to rapidly find the modal and vibration parameters of an exhaust system to get a first feeling of the system dynamics getting rid, at this stage, of the gas flow through the pipes and the mufflers. This is a prerequisite for the hanger's parameters optimization in the effort to minimize the transferred forces to the chassis.*
Keywords: *exhaust system, lumped masses, system of differential equations.*

1. GENERAL CONSIDERATIONS

The exhaust system is mainly composed of a catalyst or catalytic converter and two acoustic volumes or mufflers, the intermediate or pre-silencer and the rear or main silencer. These functional components are interconnected by pipes. The system is connected to the engine exhaust manifold often by a flexible joint and to the car body by hangers. Finding the best hanging locations and characteristics can be seen as an optimization problem. The catalyst is connected often to the manifold by a flexible joint which has a nonlinear behavior. The joint allows the motion of the engine and filters the vibration transmitted to the exhaust system.

The exhaust system is an important noise source of the vehicle, due to the pressure pulsation of the exhaust gas. From the spectrum of the exhaust noise one get information on the firing frequency and associated harmonics.

In order to describe the dynamical behavior of the exhaust system, in general a dynamical simulation by using finite elements is to be performed. In parallel, the experimental approach is aiming to validate the finite element model. Considering the excitation of the system coming from the engine, the gas pressure pulsations, the turbulent flow and the road [6], a dynamic response of the exhaust system has to be observed. Therefore the

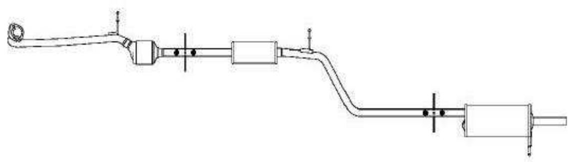

Fig. 1. The layout of the exhaust system

exhaust system eigenfrequencies, eigenmodes and damping are important in order to avoid resonances or to proper manage them. The system vibration generates forces which are transmitted to the car body through the attaching or hanging points. The vibration transmitted to the car body will induce structure born noise in the cabin, as well. Hence, a plan to decouple the exhaust system vibration from the car structure is of interest. A first step to decouple exhaust system from the chasses is to choose the hanger locations on the vibration nodes of the exhaust system. Further, the validated model can be used for optimization of the dynamical response of the whole structure. On the other side the structure shape design is important for the noise reduction, hence a decreased overall cross-section of the ducts is reducing the generated noise while this is enforcing the resistance to the gas flow and the engine energy consumption [10]. The large temperature variation along the system's length is influencing the noise speed propagation and the structure behavior. Numerous studies on the exhaust system dynamics, simulation and experiment are available in the literature [4], [5], [6], [10].

A physical exhaust system of Dacia Logan, has been measured accepting some approximations, which have to be overcome. A geometrical CAD model and the finite element model of the whole exhaust system is developed in a previous work [6]. The normal modal analysis and the EMA are set up resulting the estimated and respectively measured system modal parameters.

2. THE LUMPPED MASS MODEL OF THE SYSTEM

The exhaust system is connected to the engine, receiving the burned gas and the engine vibrations. Elastic supports are sustaining the engine body. The exhaust system body is hung from the chassis in three spots through rubber hungers. The dynamical model of the exhaust system is important in order to manage the system resonances.

A lumped mass simplified model is the target in this approach. The exhaust system mass is replaced by three lumped masses placed under the chassis (Fig. 2). The relative elasticity and the damping between two adjacent masses or between the engine and the closest to the engine mass is modeled by a pair of a spring and a viscous damper.

The dynamic equilibrium differential equation for the engine, following the vertical direction z_m is:

$$m_m\ddot{z}_m = \\ -k_m z_m - c_m\dot{z}_m + (z_1 - z_m)k_{m1} + (\dot{z}_1 - \dot{z}_m)c_{m1}$$

where m_m the engine mass, k_m and c_m are the total stiffness coefficient and the viscous damping of the engine supports and z_m is the generalized coordinate of the lumped mass replacing the engine. The pair k_{m1}, c_{m1} is

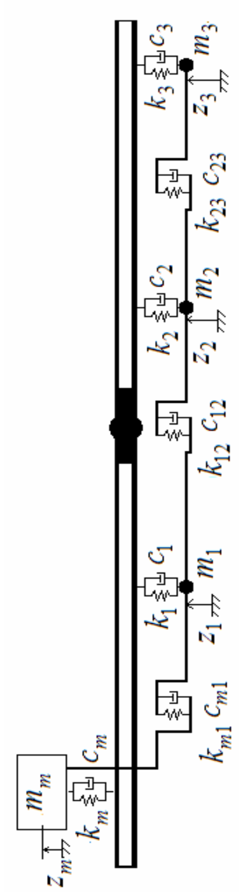

Fig. 2. The exhaust system discrete model

modeling the elasticity and the damping relative to the first lumped mass of the exhaust system. Rearranging the terms, the first differential equation results of the form:

$$m_m\ddot{z}_m + \dot{z}_m(c_m + c_{m1}) - \dot{z}_1 c_{m1} + \\ + z_m(k_m + k_{m1}) - z_1 k_{m1} = 0 \quad (1)$$

The second differential equation associated to the lumped mass m1 of the exhaust system is of the form:

$$m_1\ddot{z}_1 = -(z_1 - z_m)k_{m1} - (\dot{z}_1 - \dot{z}_m)c_{m1} + \\ + (z_2 - z_1)k_{12} + (\dot{z}_2 - \dot{z}_1)c_{12} - k_1\dot{z}_1 - c_1\dot{z}_1$$

Rearranging the terms, the second differential equation results:

$$m_1\ddot{z}_1 + \dot{z}_1(c_{m1} + c_1 + c_{12}) - \dot{z}_m c_{m1} - \dot{z}_2 c_{12} + \\ z_1(k_{m1} + k_1 + k_{12}) - z_m k_{m1} - z_2 k_{12} = 0 \quad (2)$$

For the next lumped mass m_2 the dynamic equilibrium equation is:

$$m_2\ddot{z}_2 = \\ -(z_2 - z_1)k_{12} - (\dot{z}_2 - \dot{z}_1)c_{12} - k_2 z_2 - c_2\dot{z}_2 + \\ + (z_3 - z_2)k_{23} + (\dot{z}_3 - \dot{z}_2)c_{23}$$

from which the third differential equation of the system results:

$$m_2\ddot{z}_2 - \dot{z}_1 c_{12} + (c_2 + c_{12} + c_{23})\dot{z}_2 - c_{23}\dot{z}_3 - \\ z_1 k_{12} + (k_{12} + k_2 + k_{23})z_2 - k_{23}z_3 - z_1 k_{12} = 0 \quad (3)$$

For the last lumped mass the dynamic equation is:

$$m_3\ddot{z}_3 = \\ -(z_3 - z_2)k_{23} - (\dot{z}_3 - \dot{z}_2)c_{23} - k_3 z_3 - c_3\dot{z}_3$$

from which the fourth differential equation of the whole exhaust system is of the form:

$$m_3\ddot{z}_3 + \dot{z}_3(c_{23} + c_3) - \dot{z}_2 c_{23} + \\ + z_3(k_{23} + k_3) - z_2 k_{23} = 0 \quad (4)$$

The system of the four differential equations of the whole system can be written in compact matrix form:

$$M\ddot{Q} + D\dot{Q} + KQ = 0 \quad (5)$$

Explicitly the inertial term is:

$$\begin{bmatrix} m_m & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_3 \end{bmatrix}$$

which is multiplied by the generalized acceleration vector:

$$\begin{bmatrix} \ddot{z}_m & \ddot{z}_1 & \ddot{z}_2 & \ddot{z}_3 \end{bmatrix}^T$$

The damping matrix is of the form:

$$+\begin{bmatrix} c_m + c_{m1} & -c_{m1} & 0 & 0 \\ -c_{m1} & c_{m1} + c_{12} + c_1 & -c_{12} & 0 \\ 0 & -c_{12} & c_2 + c_{12} + c_{13} & -c_{23} \\ 0 & 0 & -c_{23} & c_{23} + c_3 \end{bmatrix}$$

and is multiplied by the generalized velocity vector:

$$\begin{bmatrix} \dot{z}_m & \dot{z}_1 & \dot{z}_2 & \dot{z}_3 \end{bmatrix}^T$$

The stiffness matrix is of the form:

$$\begin{bmatrix} k_{m1} + k_m & -k_{m1} & 0 & 0 \\ -k_{m1} & k_{m1} + k_{12} + k_1 & -k_{12} & 0 \\ 0 & -k_{12} & k_{12} + k_2 + k_{23} & -k_{23} \\ 0 & 0 & -k_{23} & k_{23} + k_3 \end{bmatrix}$$

which is multiplied by the generalized coordinates vector:

$$\begin{bmatrix} z_m & z_1 & z_2 & z_3 \end{bmatrix}^T$$

3. CONCLUSIONS

An exhaust system is under observation. In order to perform the system analysis under specific functional conditions, a mathematical model of the system is proposed; this is a lumped mass simplified dynamical model. The linear differential equations with constant coefficients of the proposed lumped masses system have been derived and presented in compact matrix form. The total mass of the system is divided in three lumped masses which vibration is assumed and restricted to the vertical direction. The same restriction is valid for the engine vibration. Each mass of the exhaust system is hung from the chassis through a hanger modeled by a spring and a viscous damper. This approach has the target to rapidly find the vertical modal and vibration parameters of an exhaust system to get a first feeling of the system dynamics, getting rid at this stage of the gas flow through the pipes and the mufflers. The horizontal and torsional modes of vibrations can not be caught with this model. This is a prerequisite for the hanger's parameters optimization in the effort to minimize the transferred forces to the chassis.

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Un model de sistem de evacuare a unei mase neomogene

Rezumat: *Articolul abordează dinamica unui sistem de evacuare. Pentru a studia vibraţiile, a realiza analiza modală şi izolarea sistemului se propune un model dinamic simplificat bazat pe concentrarea masei continue în câteva puncte. Sunt scrise ecuaţiile diferenţiale de mişcare ale sistemului iar sistemul obţinut este pus în formă matriceală. Astfel pot fi determinate frecvenţele şi formle de modale ale sistemului sismplificat. Scopul modelului simplificat este de se putea realiza o modelare rapidă aproximativă a sistemului.*

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