TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue II, June, 2013

CONTRIBUTIONS TO THE DYNAMIC STUDY OF ONE DEGREE OF FREEDOM MECHANICAL SYSTEM, ACTUATED BY A HARMONIC FORCE, WITH DIFFERENT ELASTIC AND VISCOUS ELEMENTS

Nicolae URSU-FISCHER, Ioan RADU, Ioana Alexandra MUSCĂ

Abstract. *The one degree of freedom mechanical system with a mass subjected to the harmonic force and elastic and viscous elements other than in Kelvin-Voigt rheological model are studied.*

The third order differential equations that model the mass displacements were established and also the general solution expression.

The steady-state regimes were studied and the frequency responses were determined.

Based on the frequency response plots one may find the rheological model that has as result the desired movement amplitude during the steady-state period.

Key words*: Rheological models, differential equations, mechanical vibrations, displacement magnification factor*

1. INTRODUCTION

In the dynamic studies of mechanical systems movements usually the Kelvin-Voigt model is considered, as a link between two masses belonging to the system or between one mass and a fixed support [2], [7], [13], [15], [17].

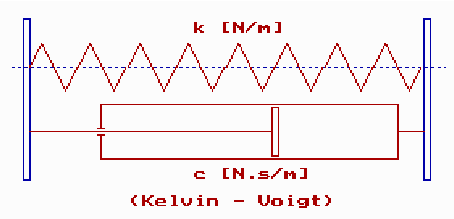


Fig. 1. The Kelvin-Voigt elasto-viscous model

If this rheological model is considered, the force is a sum between the elastic force kx and the damping force $c\dot{x}$ because the two elements are disposed in parallel.

The use of the Kelvin-Voigt modes has a great advantage regarding the solving of obtained differential equations.

The differential equations will be linear and with constant coefficients, easy to solve, analytically and also numerically.

The Kelvin-Voigt model is one of possible combinations of Hooke's (elastic body) and Newton' s (viscous liquid) models (figure 2), disposed in parallel.

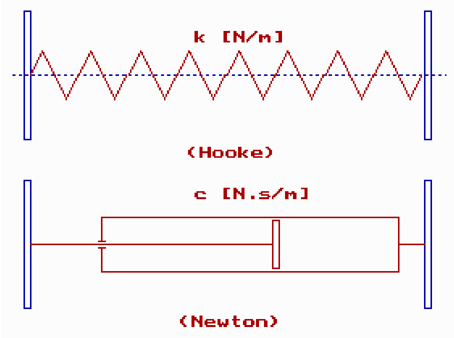


Fig. 2. The Hooke's and Newton's elements

If these elements are disposed in series on obtains the Maxwell model (figure 3).

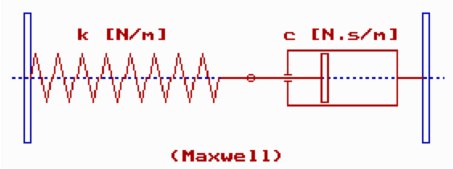
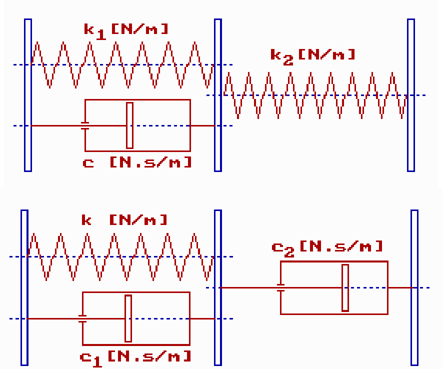
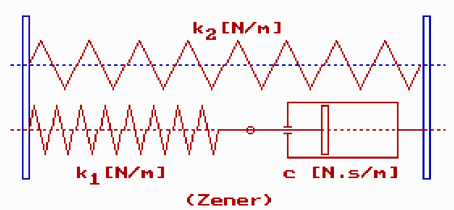


Fig. 3. The Maxwell's model

If exists three elements of Hooke type and Newton type between two discrete masses or between some support and a mass there are four such combinations, three of them being presented in figure 4 [12], [14], [[15], [21].



“Standard Solid I” and “Standard Fluid” model types



“Standard Solid II – Zener” model type

Fig. 4. Rheological models with three elements

A lot of different rheological models may be obtained if four elements of Hooke and Newton type are considered. In figure 5 are presented two of them, more important and most used. The first is composed of two Kelvin-Voigt models linked in series and the second, the so called Burgers model, contains the Kelvin-Voigt model linked in series with a Maxwell model.

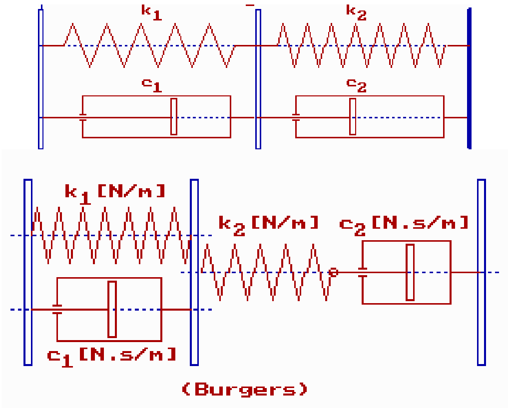


Fig. 5. Rheological models with four elements

At present in industry are used different materials with elastic and viscous properties, to minimize the vibrations effects.

In many situations these materials have a behavior that not coincides with the behavior of the Kelvin-Voigt model.

That in way is important to study the dynamics of mechanical systems with one or more degree of freedom, endowed with different rheological models, not only with Kelvin-Voigt models, especially when the experimental obtained values did not fit with the analytical obtained values, considering the Kelvin-Voigt models.

In literature are used many rheological models, that differ of the Kelvin-Voigt model [3], [5], [9], [16], a. o.

Rheological model with three elements are considered in the cases of modeling the human body movements, standing or sitting, subjected to vibrations.

The same models are used in the modeling of driver seat suspensions and in other many cases [5], [16], a. o..

In the following will be presented a detailed study of one degree of freedom mechanical system movements, the mass being linked with the support with the rheological model composed by two Kelvin-Voigt models linked in series or a Kelvin-Voigt model and a Hooke model also linked in series.

2. THEORETICAL BACKGROUND

The second order differential equation having the form

$$m\,\ddot{x} + c\,\dot{x} + k\,x = F_0 \cos\omega\, t$$

is obtained when between the mass and the support there exists a Kelvin-Voigt rheological model and the mass is actuated by a harmonic force. Usually this differential equation is established based on the Newton second law of dynamics.

In the case of the mechanical system presented in figure 6, the second Newton law has to be applied two times, considering the existing mass and also the fictive mass of intermediate support, between the two Kelvin-Voigt models.

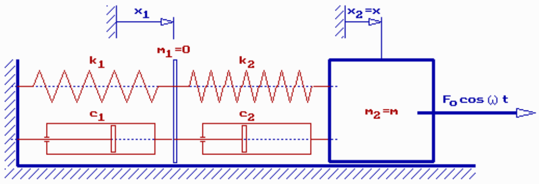


Fig. 6. The mechanical system contains two models of Kelvin-Voigt type, in series, and a mass subjected to the harmonic force.

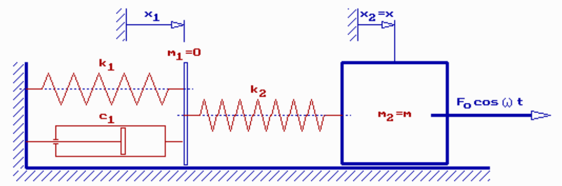


Fig. 7 The mechanical system contains two models of Kelvin-Voigt and Hooke type, in series, and a mass subjected to the harmonic force.

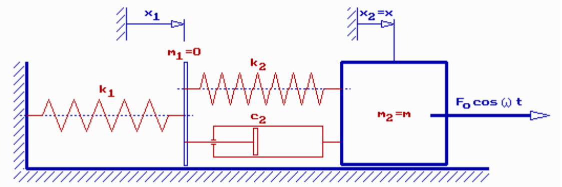


Fig. 8 The mechanical system contains two models of Hooke and Kelvin-Voigt type, in series, and a mass subjected to the harmonic force.

In figures 7 and 8 are considered two particular cases of the mechanical system.

The following matrix form may be used for writing the two differential equations that model the movements of the two considered masses m_1 and m_2 [2], [13], [15], [17], [21],

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+ + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ F_0\cos\omega t \end{bmatrix} \tag{1}$$

The other form of simultaneous differential equation is

$$\begin{cases} m_1\ddot{x}_1+(c_1+c_2)\dot{x}_1-c_2\dot{x}_2+ \\ \qquad +(k_1+k_2)x_1-k_2x_2=0 \\ m_2\ddot{x}_2-c_2\dot{x}_1+c_2\dot{x}_2-k_2x_1+k_2x_2= \\ \qquad =F_0\cos\omega t \end{cases} \tag{2}$$

Because $m_1 = 0$, $m_2 = m$ the system (2) becomes

$$\begin{cases} m\ddot{x}_2-c_2\dot{x}_1+c_2\dot{x}_2-k_2x_1+k_2x_2=F_0\cos\omega t \\ (c_1+c_2)\dot{x}_1-c_2\dot{x}_2+(k_1+k_2)x_1-k_2x_2=0 \end{cases} \tag{3}$$

The system (3) may be considered as a simultaneous two linear algebraic equation with x_1 and $\dot{x}_1$ as unknown,

$$\begin{cases} -c_2\dot{x}_1-k_2x_1=F_0\cos\omega t-m\ddot{x}_2-c_2\dot{x}_2-k_2x_2 \\ (c_1+c_2)\dot{x}_1+(k_1+k_2)x_1=c_2\dot{x}_2+k_2x_2 \end{cases} \tag{4}$$

As a result of solving this equations with respect of unknowns x_1 and $\dot{x}_1$ on obtains

$$\dot{x}_1=\frac{\begin{vmatrix} F_0\cos\omega t-m\ddot{x}_2-c_2\dot{x}_2-k_2x_2 & -k_2 \\ c_2\dot{x}_2+k_2x_2 & k_1+k_2 \end{vmatrix}}{\begin{vmatrix} -c_2 & -k_2 \\ c_1+c_2 & k_1+k_2 \end{vmatrix}} \tag{5}$$

$$x_1=\frac{\begin{vmatrix} -c_2 & F_0\cos\omega t-m\ddot{x}_2-c_2\dot{x}_2-k_2x_2 \\ c_1+c_2 & c_2\dot{x}_2+k_2x_2 \end{vmatrix}}{\begin{vmatrix} -c_2 & -k_2 \\ c_1+c_2 & k_1+k_2 \end{vmatrix}} \tag{6}$$

The purpose is the obtaining the differential equation having only one unknown function x_2. After deriving with respect of time the expression (6) of x_1,

$$\dot{x}_1=\frac{\begin{vmatrix} -c_2 & -F_0\omega\sin\omega t-m\dddot{x}_2-c_2\ddot{x}_2-k_2\dot{x}_2 \\ c_1+c_2 & c_2\ddot{x}_2+k_2\dot{x}_2 \end{vmatrix}}{\begin{vmatrix} -c_2 & -k_2 \\ c_1+c_2 & k_1+k_2 \end{vmatrix}} \tag{7}$$

and equaling the two expressions of the x_1 derivative, given in (5) and (7), will result the desired relation, containing only x_2 and it derivatives,

$$\begin{vmatrix} -c_2 & -F_0\omega\sin\omega t-m\dddot{x}_2-c_2\ddot{x}_2-k_2\dot{x}_2 \\ c_1+c_2 & c_2\ddot{x}_2+k_2\dot{x}_2 \end{vmatrix}=$$

$$=\begin{vmatrix} F_o\cos\omega t-m\ddot{x}_2-c_2\dot{x}_2-k_2x_2 & -k_2 \\ c_2\dot{x}_2+k_2 & k_2x_2+k_2 \end{vmatrix}$$

If on notes $x_2 = x$, the determinants are evaluated, after some calculi will result the three order nonhomogeneous differential equation,

$$\begin{aligned} &m(c_1+c_2)\dddot{x}+[m(k_1+k_2)+c_1c_2]\ddot{x}+ \\ &\qquad +(k_1c_2+k_2c_1)\dot{x}+k_1k_2x= \\ &=F_0(k_1+k_2)\cos\omega t-F_0\omega(c_1+c_2)\sin\omega t \end{aligned} \tag{8}$$

The second part of this equation may be written in the following form

$$F_0(k_1+k_2)\cos\omega t-F_0\omega(c_1+c_2)\sin\omega t= \\ =F\cos(\omega t+\varphi)$$

where

$$F=F_0\sqrt{(k_1+k_2)^2+\omega^2(c_1+c_2)^2} \tag{9}$$

$$\varphi=\mathrm{atan}\left[\frac{\omega(c_1+c_2)}{k_1+k_2}\right] \tag{10}$$

and the final form of this differential equation is

$$\begin{aligned} &m(c_1+c_2)\dddot{x}+[m(k_1+k_2)+c_1c_2]\ddot{x}+ \\ &+(k_1c_2+k_2c_1)\dot{x}+k_1k_2x=F\cos(\omega t+\varphi) \end{aligned} \tag{11}$$

The particular solution, corresponding to the steady-state period has the similar form as the second term, being harmonic.

It is necessary to use the complex variables in the computation of amplitude and phase of the mass movements, during the vibrations, as follows,

$$F\cos(\omega t+\varphi) \rightarrow F e^{j(\omega t+\varphi)},$$

$$x \rightarrow Z = x_{max}\, e^{j(\omega t+\varphi+\psi)}$$

Deriving the complex displacement Z with respect of time

$$\dot{Z} = j\omega Z,\quad \ddot{Z} = -\omega^2 Z,\quad \dddot{Z} = -j\omega^3 Z$$

and introducing these values in the complex form of differential equation given in (11), will result

$$\{-[m(k_1+k_2)+c_1c_2]\omega^2+k_1k_2+$$
$$+j\omega\,[-m(c_1+c_2)\omega^2+k_1c_2+k_2c_1]\}\,Z=F e^{j(\omega t+\varphi)}$$

and finally the expression of Z,

$$Z=\frac{F}{\begin{array}{c}k_1k_2-[m(k_1+k_2)+c_1c_2]\omega^2+\\ +j\omega\,[-m(c_1+c_2)\omega^2+k_1c_2+k_2c_1]\end{array}}\,e^{j(\omega t+\varphi)}$$

The modulus and phase of this complex variable Z are determined as follows

$$|Z|=\frac{F}{\sqrt{\begin{array}{l}\{k_1k_2-[m(k_1+k_2)+c_1c_2]\omega^2\}^2+\\ +\omega^2\,[-m(c_1+c_2)\omega^2+k_1c_2+k_2c_1]^2\end{array}}} \tag{12}$$

$$\psi = \mathrm{atan}\left\{\frac{\omega\,[-m(c_1+c_2)\omega^2+k_1c_2+k_2c_1]}{k_1k_2-[m(k_1+k_2)+c_1c_2]\omega^2}\right\} \tag{13}$$

The expressions of complex amplitude Z and of real amplitude are,

$$Z=|Z|\,e^{j(\omega t+\varphi)}e^{j\psi}=|Z|e^{j(\omega t+\varphi+\psi)}$$

$$x_{part}(t)=|Z|\cos(\omega t+\varphi+\psi) \tag{14}$$

The characteristic equation of differential equation (11) is an algebraic equation of third degree,

$$\begin{array}{r}m(c_1+c_2)r^3+[m(k_1+k_2)+c_1c_2]r^2+\\ +(k_1c_2+k_2c_1)r+k_1k_2=0\end{array} \tag{15}$$

The three roots r_1, r_2 and r_3 (all three real or one real and two complex) are determined using the Cardano-Tartaglia's formula [6]. The form of the general solution of the homogeneous differential equation are as follows

$$x_{gen\ omog}(t)=C_1e^{r_1t}+C_2e^{r_2t}+C_3e^{r_3t}$$

The general solution of the nonhomogeneous differential equation is obtained as a sum between the two solutions,

$$x_{gen\ neomog}(t)=x_{gen\ omog}(t)+x_{part}(t)$$

$$\begin{array}{r}x(t)=C_1e^{r_1t}+C_2e^{r_2t}+C_3e^{r_3t}+\\ +|Z|\cos(\omega t+\varphi+\psi)\end{array} \tag{16}$$

the values of the constants C_1, C_2 and C_3 result according to the three imposed initial conditions.

Because the general solution of homogeneous differential equation decreases rapidly our interest will be focused on the study of the particular solution, determining the amplitude and phase.

In the "classic" case of the differential equation

$$m\ddot{x}+c\dot{x}+kx=F_0\cos\omega t$$

corresponding to the mechanical system having the Kelvin-Voigt rheological model and the mass being actuated by a harmonic perturbing force, is studied the displacement magnification factor A that depends on the forcing frequency $r=f/f_0$, having the following expression [[2], [17],

$$\begin{array}{c}A=\dfrac{1}{\sqrt{(1-r^2)^2+4\varsigma^2r^2}},\\ r=\dfrac{\omega}{\omega_0}=\dfrac{f}{f_0},\quad \varsigma=\dfrac{c}{c_0}=\dfrac{c}{2\sqrt{mk}}\end{array} \tag{17}$$

where ς is the viscous damping factor.

În the figure 9 are shown the plots of the displacement magnification factor considering different values for the parameter $\varsigma = c/c_0$, [2], [17].

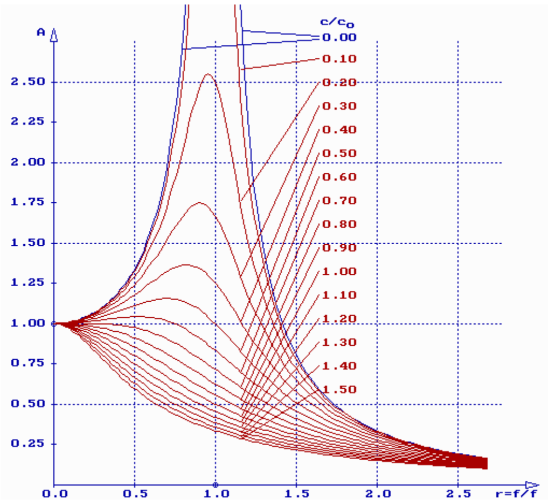


Fig. 9 Diagrams of magnification factor [17]

Considering the mechanical system from the figure 6 the problem of great interest will be the determination of a family of such curves of displacement magnification factor A, defined as follows

$$A=\frac{|Z|}{a_{st}},\qquad a_{st}=\frac{F}{k_{eq}},\qquad k_{eq}=\frac{k_1\,k_2}{k_1+k_2}$$

where k_{eq} is the stiffness of the equivalent spring, after some substitutions resulting the expression

$$A=\frac{k_1\,k_2\sqrt{(k_1+k_2)^2+\omega^2\,(c_1+c_2)^2}}{(k_1+k_2)\sqrt{\begin{aligned}&\{k_1\,k_2-[m\,(k_1+k_2)+c_1\,c_2\,]\,\omega^2\,\}^2+\\&+\omega^2\,[-m\,(c_1+c_2)\,\omega^2+k_1c_2+k_2c_1\,]^2\end{aligned}}} \tag{18}$$

Considering the mechanical systems from the figures 7 and 8 will result the simplified expressions of this factor

$$A_{(c_1\neq 0,\,c_2=0)}=$$

$$=\frac{k_1\,k_2\sqrt{(k_1+k_2)^2+\omega^2\,c_1^2}}{(k_1+k_2)\sqrt{\begin{aligned}&\{k_1\,k_2-m\,\omega^2\,(k_1+k_2)\,\}^2+\\&+\omega^2\,[-m\,c_1\,\omega^2+k_2\,c_1\,]^2\end{aligned}}} \tag{19}$$

$$A_{(c_1=0,\,c_2\neq 0)}=$$

$$=\frac{k_1\,k_2\sqrt{(k_1+k_2)^2+\omega^2\,c_2^2}}{(k_1+k_2)\sqrt{\begin{aligned}&\{k_1\,k_2-m\,\omega^2\,(k_1+k_2)\,\}^2+\\&+\omega^2\,[-m\,c_1\,\omega^2+k_1\,c_2\,]^2\end{aligned}}} \tag{20}$$

Will be very interesting to compare the shape of diagrams from the figure 9 with the diagrams obtained using the formula (18), corresponding to the considered mechanical system.

3. NUMERICAL TREATMENT

Considering the mechanical system having the characteristics showed in the figure 10,

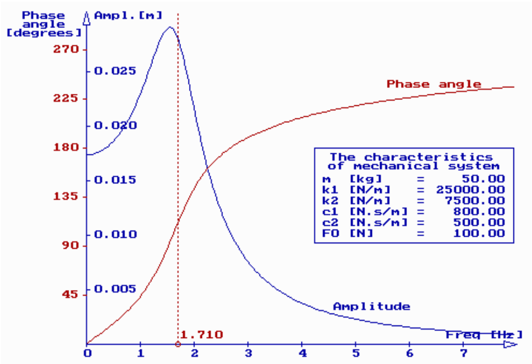


Fig. 10 Amplitude and phase angle

and solving the characteristic equation (15) will result the following roots values [6],

```
        Re( r )              Imag( r )
-3.41257390e+00    -1.17898088e+01
-3.41257390e+00     1.17898088e+01
-2.43286983e+01     0.00000000e+00
```

thus the general solution of differential equation is as follows

$$x(t)=C_1\,e^{-24.329\,t}+C_2\,e^{(-3.413-j\,11.790)\,t}+$$
$$+C_3\,e^{(-3.413+j\,11.790)\,t}+a_{st}\,A\cos(\omega\,t+\varphi+\psi)$$
$$x(t)=C_1\,e^{-24.329\,t}+e^{-3.413\,t}\,(C_2\,e^{-j\,11.790\,t}+$$
$$+C_3\,e^{j\,11.790\,t})+a_{st}\,A\cos(\omega\,t+\varphi+\psi)$$

We may notice that first three terms of the solution are rapidly decreasing therefore only the particular solution is important when the mass movements are studied.

The diagrams of the displacement magnification factor A and also of the phase angle ψ are presented in figure 10, using the formulas (18) and (13).

The formulas being established and having also a computer program we may compare the shape of the diagrams corresponding to the mechanical systems from the figure 6 with the well known diagrams presented in the figure 9.

The first example is presented in the figure 11. Considering the specified numerical values of mechanical system elements, the diagram of the displacement magnification factor was computed with formula (18) and plotted (with thick line) superposed to some diagrams taken from to the figure 9. One can notice that are difference between thick diagram and the others.

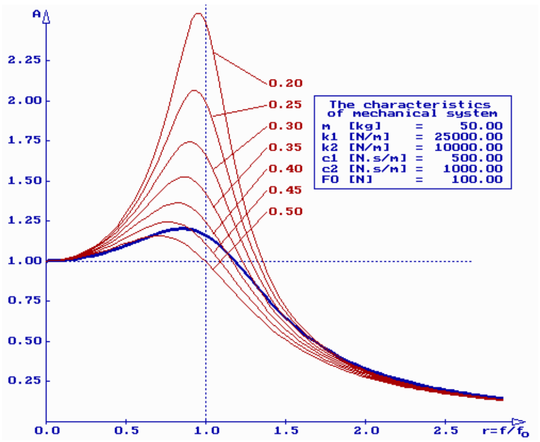


Fig. 11 Diagram of magnification factor considering the specified values

Considering different other values of mechanical system elements is possible to obtain diagrams that fit to those existing in figure 9. Two such examples are presented in figures 12 and 13, the diagrams plotted with thick line being very close to one of the diagrams from figure 9.

In figure 12 the diagram computed according to the formula (18), for specified numerical values, is superposed to the diagram corresponding to the value of viscous damping factor ς=0.25, the similar situation we may observe in figure 13, where the thick diagram is superposed to the diagram corresponding to the value of viscous damping factor ς=0.35.

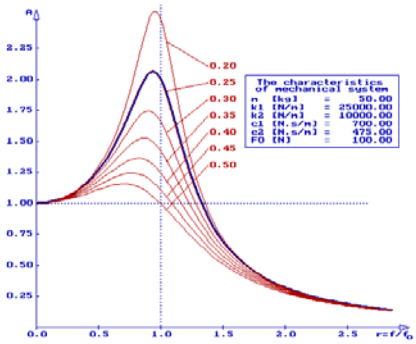


Fig. 12 Diagram of magnification factor for specified numerical values

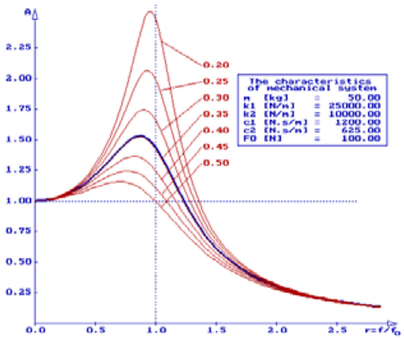


Fig. 13 Diagram of magnification factor for specified numerical values

Other problem of interest is the study of mechanical systems having the same equivalent stiffness, computed with formula $k_{eq} = \dfrac{k_1 k_2}{k_1 + k_2}$, the springs stiffness k_1 and k_2 being variable and the damping coefficient c_1=c has different values (1000, 2000 and 3000 [N.s/m] in the performed calculi), the second damping coefficient c_2 having zero value.

If the equivalent stiffness constant is considered k_{eq} =8000 [N/m] the values of each spring stiffness may be computed with the formulas

$$k_1 = \frac{8000(N+1)}{N}, \; k_2 = N k_1 .$$

In the figures 14, 15 and 16 are presented the diagrams of the displacements magnification factor, considering the values of N in the interval 0.75 ~ 3.75.

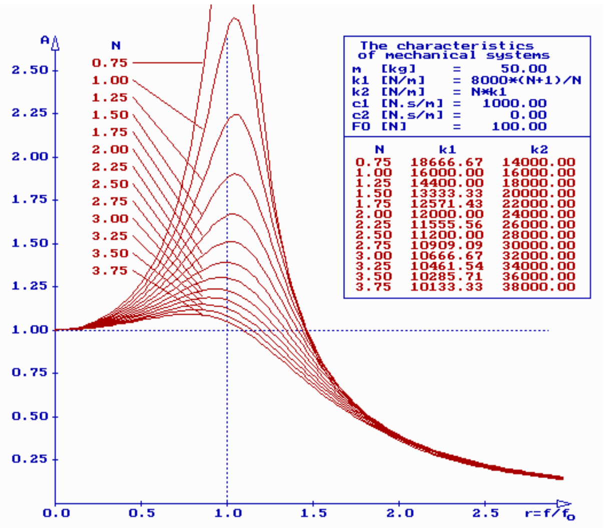


Fig. 14 The diagrams of magnification factor (c_1 =1000)

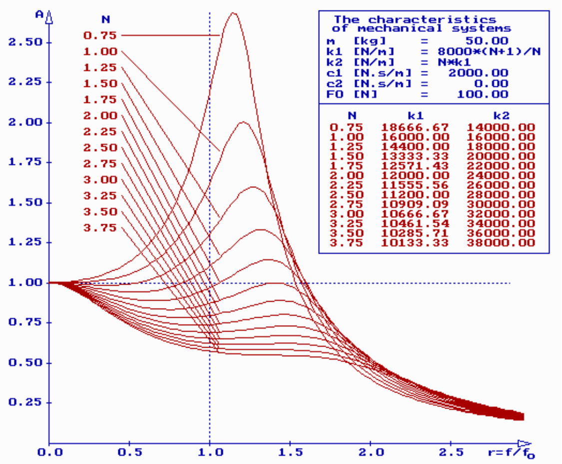


Fig. 15 The diagrams of magnification factor (c_1 =2000)

One can notice that some of the diagrams presented in the previous two figures, 15 and 16, have minimal values in the vicinity of the resonance, such a situations are not observed in the figure 9.

4. CONCLUSIONS

This theoretical study of mechanical systems having other rheological models than the Kelvin-Voigt model is important because many materials used as dampers, with elastic and viscous properties, have similar behavior as the previously considered rheological models.

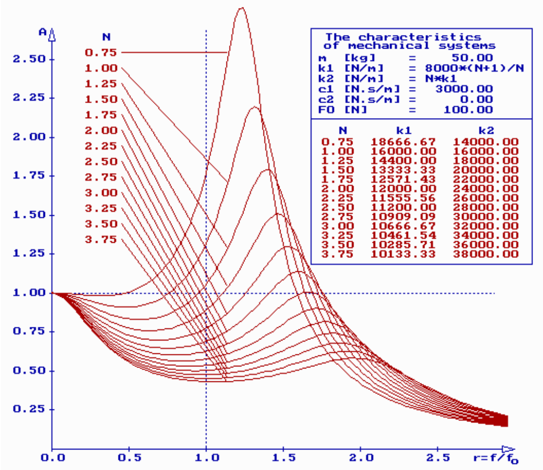


Fig. 16 The diagrams of magnification factor (c_1 =3000)

The movement of the mechanical system mass and having rheological models with three of four elements during the steady-state period was determined and also the general solution of third order differential equation.

Our attention was paid especially to the steady-state period because all the three terms of the homogeneous differential equation general solution decrease rapidly, all characteristic equation roots having negative real parts.

Other problem consist in fact that only two initial condition are known (the initial position and velocity of the mass) and the number of unknown constant are three, that's way it is necessary to consider the third condition, regarding the initial acceleration value, practically very difficult to know exactly.

5. REFERENCES

[1] Ascher, U. M., Petzold, Linda R., *Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations*, SIAM, Philadelphia, 1998, 330 pp.

[2] Buzdugan, G., Fetcu, Lucia, Radeş, M., *Vibrations of Mechanical Systems (in Romanian)*, Editura Academiei, Bucureşti, 1975, 572 pg.

[3] Carrella, A., Waters, T. P., Brennan, M. J., *Free vibration characteristics of an isolated system with a spring-relaxed damper*, Twelfth International Congress on Sound and Vibration, ICSV12, 2005, Lisbon, 8 pp., www.soton.ac. uk/~alecar/documents/carrella_waters_brennan _ ICSV12.pdf

[4] Dawkins, P., *Differential Equations*, 2007, 504 pp., http://tutorial.math.lamar.edu/ terms.aspx

[5] Fritton, J. C., Rubin, C. T., Qin, Y. X., McLeod, K. J., *Whole-body vibration in the skeleton: development of a resonance-based testing device*, Annals of Biomedical Engineering, 1997, Vol. 25, pp. 831-839

[6] Gellert, W. ş. a., (ed.), *Mică enciclopedie matematică*, Editura Tehnică, Bucureşti, 1980, 926 pg.

[7] Genta, G., *Vibration of Structures and Machines. Practical Aspects*, 2nd ed., Springer, 1995, 474 pp.

[8] Von Gierke, H. E., Brammer, A. J., *Chapter 42. Effects of shock and vibration on humans*, in Harris C. M., Piersol A. G. (eds.), Shock and Vibration Handbook, 5th edition, McGraw Hill, New York, 2002

[9] Herterich, J., Crede, C., *Wirkungen vertikaler mechanischer Schwingungen auf den stehenden Menschen*, Lehrstuhl fur Arbeit-System-planung und gestaltung, Universität Bochum, 1990

[10] Ionescu, D. V., *Differential and Integral Equations (in Romanian)*, Editura Didactică şi Pedagogică, Bucureşti, 1964, 332 pg.

[11] Lewandowski, R., Chorażyczewski, B., *Identification of parameters of the fractional rheological model of viscoelastic dampers*, 6 pp., XXIV Symposium "Vibrations in Physical Systems", Poznan-Bedlewo, May 12-15, 2010, www.vibsys.put.poznan.pl/journal/2010-24/ lewandowski.pdf

[12] Marques, S. P. C., Creus, G. J., *Computational Viscoelasticity*, Springer, 2012 (Chapter 2 – Rheological models: integral and differential representations, pp. 11-21)

[13] Meirovitch, L., *Fundamentals of Vibrations*, McGraw Hill, Boston, 2001, 806 pp.

[14] Moczo, Peter, Kristek, Jozef, Franek, Peter, *Lecture Notes on Rheological Models*, version 26-10-2006, Comenius University, Bratislava, 2006, 41 pp.

[15] Radeş, M., *Mechanical Vibrations, I*, Ed. Printech, Bucureşti, 2006, 291 pp.

[16] Stein, G. J., Múčka, P., Gunston, T. P., Badura, S., *Modelling and simulation of locomotive driver's seat vertical suspension vibration isolation system*, International Journal of Industrial Ergonomics, 2008, Vol. 38, pp. 384-395

[17] Ursu-Fischer, N., *Vibrations of Mechanical Systems. Theory and Applications (in Romanian)*, Casa Cărţii de Ştiinţă, Cluj-Napoca, 1998, 452 pp.

[18] Ursu-Fischer, N., Ursu, M., *Prog-ramming with C in Engineering* (in Romanian), Casa Cărţii de Ştiinţă, Cluj-Napoca, 2001, 405 pp.

[19] Ursu-Fischer, N., Ursu, M., *Complements of Mathematics with Application in Engineering (in Romanian),*Casa Cărţii de Ştiinţă, Cluj-Napoca, 2010, 373 pp.

[20] Ursu-Fischer, N., *Numerical Methods in Engineering and Programs in C. Differential Equations and Systems with Initial Values and Boundary Values* (in Romanian), Casa Cărţii de Ştiinţă, Cluj-Napoca, 2013, 454 pp.

[21] Voinea, R., Voiculescu, D., Simion, F. P., *Introduction to the Solid Body Mechanics and Applications in Engineering (in Romanian),* Editura Academiei, Bucureşti, 1989, 1130 pg.

[22] Wan, Y., Schimmels, J. M., *A simple model that captures the essential dynamics of a seated human exposed to whole body vibration*, Advances in Bioengineering, ASME, 1995, Vol. 31, pp. 333-334

CONTRIBUŢII LA STUDIUL DINAMIC AL SISTEMELOR MECANICE CU UN GRAD DE LIBERTATE, ACŢIONATE DE O FORŢĂ ARMONICĂ, CU DIFERITE ELEMENTE ELASTICE ŞI DE AMORTIZARE

Se consideră un sistem mecanic cu un grad de libertate format dintr-o masă şi elemente elastice şi de amortizare care corespund unor modele reologice diferite de cel uzual folosit, al lui Kelvin-Voigt, masa sistemului fiind acţionată de o forţă cu variaţie armonică. Este dedusă ecuaţia diferenţială de ordinul trei care modelează funcţionarea acestui sistem şi expresia soluţiei generale. Se analizează funcţionarea sistemului în timpul regimului permanent de vibraţii, efectuându-se un studiu al factorului de amplificare. Pe baza diagramelor prezentate se poate face o alegere a elementelor modelului reologic care să aibă efectul scontat în ceea ce priveşte amplitudinea mişcărilor în timpul regimului permamnent de vibraţii.

Prof. dr. eng. math. Nicolae URSU-FISCHER, Technical University of Cluj-Napoca, Faculty of Machine Building, Department of Mechanical Systems Engineering, 103-105 Muncii Avenue, 400641 Cluj-Napoca, ☎+40-264-401656, e-mail: **nic_ursu@yahoo.com**

Prof. dr. eng. Ioan RADU, “Aurel Vlaicu” University Arad, Faculty of Engineering, Department of Automation, Industrial engineering, Textile production and Transport, 77 Revolutiei Avenue, 310130 Arad, ☎+40-257-283010, e-mail: **raduioanuav@gmail.com**

Ing. Ioana Alexandra MUSCĂ, PhD student, Technical University of Cluj-Napoca, Faculty of Machine Building, Department of Mechanical Systems Engineering, 103-105 Muncii Avenue, 400641 Cluj-Napoca, ☎+40-264-401781, e-mail: **sandy50113@yahoo.com**