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THE GEOMETRIC AND KINEMATIC MODEL OF RTTRR SMALL-SIZED MODULAR ROBOT

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Abstract: *The first part of the paper describes the mechanical structure of RTTRR small-sized modular robot. The second part is reserved for the determination of the geometric model of the robot, using the method of rotation matrices [2] and the third part presents the kinematic model, determined using the iterative method for kinematic modeling [2]. The equations of the geometric and kinematic models are determined with the help of the first three modules of the generalized modeling application Robot_Symbolic [1] and they are useful in the simulation of the geometric and kinematic behavior of the analyzed robot.*
Key words: *robot, serial, modular, geometric and kinematic model, symbolic computation.*

1. THE RTTRR SMALL-SIZED MODULAR ROBOT

The kinematic scheme of RTTRR small-sized robot is presented in fig.1. The robot is designed as a modular structure, with interchangeable modules, and by their arrangement it is possible to build different configurations, having a maximum number of five degrees of freedom, of which maximum three rotations and a maximum number of two translations.

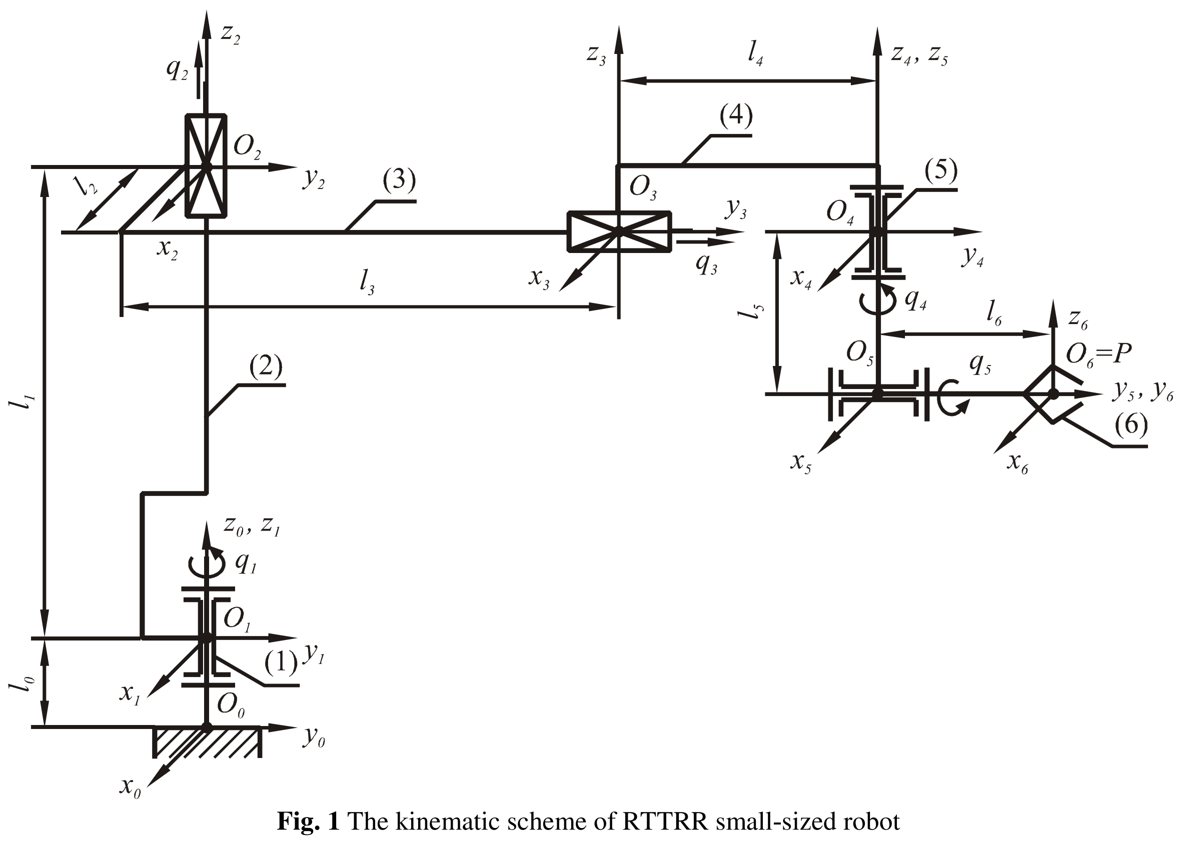

Fig. 1 The kinematic scheme of RTTRR small-sized robot

The mechanical structure has, according to fig.1, the rotation module (1) from the base of the robot (vertical rotation), the vertical translation module (2), the horizontal translation module (3) and the gripper's orientation mechanism, consisting in two rotation joints: (4), achieving a vertical rotation and (5), a horizontal rotation. The gripper (6) is attached to the final element (5) and it has the purpose of catching (grasping) the manipulated object.

The notations from the figure, representing the nest position of the robot (the zero configuration), are:

l_i – constructive parameters of the robot;

O_i – $\{i\}$ frames origins;

x_i, y_i, z_i – $\{i\}$ frames axes ($i = \overline{0,6}$);

q_k – the generalized coordinates ($k = \overline{1,5}$).

2. THE MECHANICAL STRUCTURE DEFINITION OF RTTRR SMALL-SIZED MODULAR ROBOT

The geometric modeling is accomplished using the first two generalized modules of the *Robot_Symbolic* application, described in [1], [3], [5].

The first step is the definition of the robot's kinematic structure, running the module *Robot_Definition* [3]. This module will request the robot's name (RTTRR) and the number of degrees of freedom (5). For each joint and the gripper, the following data will be input: joint type, joint axes unit vector, given by the directing cosines and the position vector of the joint (i) origin with respect to the previous frame $\{i\text{-}1\}$.

The program generates the rotation matrices, according to table 1. The data are saved into the file *RTTRR_intro.mat* in order to be passed to the second module of the software application.

3. THE GEOMETRIC MODEL OF RTTRR SMALL-SIZED MODULAR ROBOT

By calling the module *Robot_Geometry* [5], the program determines further the rotation matrices with respect to the fixed frame {0}:

$$ {}^0_1[R] = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\ {}^0_2[R] = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad (1) $$

Table 1

The mechanical structure definition for RTTRR robot

Joint	Type	Unit vector	Position vector	Rotation matrix
1	R	${}^1\bar{k}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}^0\bar{r}_1 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}$	${}^0_1[R] = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2	T	${}^2\bar{k}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}^1\bar{r}_2 = \begin{bmatrix} 0 \\ 0 \\ q_2 + l_1 \end{bmatrix}$	${}^1_2[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3	T	${}^3\bar{k}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	${}^2\bar{r}_3 = \begin{bmatrix} l_2 \\ q_3 + l_3 \\ 0 \end{bmatrix}$	${}^2_3[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4	R	${}^4\bar{k}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	${}^3\bar{r}_4 = \begin{bmatrix} 0 \\ l_4 \\ 0 \end{bmatrix}$	${}^3_4[R] = \begin{bmatrix} cq_4 & -sq_4 & 0 \\ sq_4 & cq_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
5	R	${}^5\bar{k}_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	${}^4\bar{r}_5 = \begin{bmatrix} 0 \\ 0 \\ -l_5 \end{bmatrix}$	${}^4_5[R] = \begin{bmatrix} cq_5 & 0 & sq_5 \\ 0 & 1 & 0 \\ -sq_5 & 0 & cq_5 \end{bmatrix}$
EF	-	${}^6\bar{k}_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$	${}^5\bar{r}_6 = \begin{bmatrix} 0 \\ l_6 \\ 0 \end{bmatrix}$	${}^5_6[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$ {}^{0}_{3}[R]=\begin{bmatrix} cq_1 & -sq_1 & 0\\ sq_1 & cq_1 & 0\\ 0 & 0 & 1\end{bmatrix}; \quad (2)$$

$$ {}^{0}_{4}[R]=\begin{bmatrix} c(q_1+q_4) & -s(q_1+q_4) & 0\\ s(q_1+q_4) & c(q_1+q_4) & 0\\ 0 & 0 & 1\end{bmatrix}; \quad (3)$$

$$ {}^{0}_{5}[R]=\begin{bmatrix} c(q_1+q_4)\cdot cq_5 & -s(q_1+q_4) & c(q_1+q_4)\cdot sq_5\\ s(q_1+q_4)\cdot cq_5 & c(q_1+q_4) & s(q_1+q_4)\cdot sq_5\\ -sq_5 & 0 & cq_5\end{bmatrix};(4)$$

$$ {}^{0}_{6}[R]=\begin{bmatrix} c(q_1+q_4)\cdot cq_5 & -s(q_1+q_4) & c(q_1+q_4)\cdot sq_5\\ s(q_1+q_4)\cdot cq_5 & c(q_1+q_4) & s(q_1+q_4)\cdot sq_5\\ -sq_5 & 0 & cq_5\end{bmatrix};(5)$$

the relative position vectors as functions of the generalized coordinates and the constructive parameters:

$$\overline{p}_{10}=\begin{bmatrix}0\\0\\l_0\end{bmatrix}; \quad \overline{p}_{21}=\begin{bmatrix}0\\0\\q_2+l_1\end{bmatrix}; \quad (6)$$

$$\overline{p}_{32}=\begin{bmatrix} cq_1\cdot l_2-sq_1\cdot(q_3+l_3)\\ sq_1\cdot l_2+cq_1\cdot(q_3+l_3)\\ 0\end{bmatrix}; \ \overline{p}_{43}=\begin{bmatrix}-sq_1\cdot l_4\\ cq_1\cdot l_4\\ 0\end{bmatrix};(7)$$

$$\overline{p}_{54}=\begin{bmatrix}0\\0\\-l_5\end{bmatrix}; \overline{p}_{65}=\begin{bmatrix}-s(q_1+q_4)\cdot l_6\\ c(q_1+q_4)\cdot l_6\\ 0\end{bmatrix}; \quad (8)$$

and the position vectors with respect to the fixed frame {0}:

$$\overline{p}_{1}=\begin{bmatrix}0\\0\\l_0\end{bmatrix}; \quad \overline{p}_{2}=\begin{bmatrix}0\\0\\l_0+q_2+l_1\end{bmatrix}; \quad (9)$$

$$\overline{p}_{3}=\begin{bmatrix} cq_1\cdot l_2-sq_1\cdot(q_3+l_3)\\ sq_1\cdot l_2+cq_1\cdot(q_3+l_3)\\ l_0+q_2+l_1\end{bmatrix}; \quad (10)$$

$$\overline{p}_{4}=\begin{bmatrix} cq_1\cdot l_2-sq_1\cdot(q_3+l_3)-sq_1\cdot l_4\\ sq_1\cdot l_2+cq_1\cdot(q_3+l_3)+cq_1\cdot l_4\\ l_0+q_2+l_1\end{bmatrix}; \quad (11)$$

$$\overline{p}_{5}=\begin{bmatrix} cq_1\cdot l_2-sq_1\cdot(q_3+l_3)-sq_1\cdot l_4\\ sq_1\cdot l_2+cq_1\cdot(q_3+l_3)+cq_1\cdot l_4\\ l_0+q_2+l_1-l_5\end{bmatrix}; \quad (12)$$

$$\overline{p}_{6}=\begin{bmatrix} cq_1\cdot l_2-sq_1\cdot(q_3+l_3)-sq_1\cdot l_4-s(q_1+q_4)\cdot l_6\\ sq_1\cdot l_2+cq_1\cdot(q_3+l_3)+cq_1\cdot l_4+c(q_1+q_4)\cdot l_6\\ l_0+q_2+l_1-l_5\end{bmatrix}.(13)$$

The Euler's angles are afterwards determined, expressed by the following equations:

$$\alpha_z=\operatorname{atan2}\Big(\frac{1}{2}s(q_1+q_4+q_5)-\frac{1}{2}s(q_1+q_4-q_5),\ -\frac{1}{2}c(q_1+q_4-q_5)+\frac{1}{2}c(q_1+q_4+q_5)\Big) \quad (14)$$

$$\beta_x=\operatorname{atan2}\left(\frac{1-c(2q_5)}{(2-2\cdot c(2q_5))^{\frac{1}{2}}},cq_5\right) \quad (15)$$

$$\gamma_z=\operatorname{atan2}\left(\frac{-2s(q_5)}{(2-2\cdot c(2q_5))^{\frac{1}{2}}},0\right). \quad (16)$$

By the simplification of results, the vector of rotation parameters can be written as:

$$\Psi=\begin{bmatrix}\alpha_z\\ \beta_x\\ \gamma_z\end{bmatrix}=\begin{bmatrix}\pi/2+q_1+q_4\\ q_5\\ -\pi/2\end{bmatrix}. \quad (17)$$

As a conclusion, the geometric model of the small-sized modular robot RTTRR can be expressed by one of the equation sets (5) and (13) or (17) and (13), respectively. The obtained symbolic data is saved into the file *RTTRR_geom.mat* and it can be used in order to get the kinematic model of the analyzed robot.

4. THE KINEMATIC MODEL OF RTTRR SMALL-SIZED MODULAR ROBOT

The kinematic model of RTTRR robot is accomplished using the iterative method [8] implemented into the *Robot_Kinematics* [4] generalized module from the *Robot_Symbolic*

[1] modeling application. After entering the robot's name (RTTRR), the module loads the data obtained in the step of geometric modeling and computes the inverses of the rotation matrices between the consecutive frames:

$$ {}_{0}^{1}[R]=\begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\quad {}_{1}^{2}[R]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};(18) $$

$$ {}_{2}^{3}[R]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\quad {}_{3}^{4}[R]=\begin{bmatrix} cq_4 & sq_4 & 0 \\ -sq_4 & cq_4 & 0 \\ 0 & 0 & 1 \end{bmatrix};(19) $$

$$ {}_{4}^{5}[R]=\begin{bmatrix} cq_5 & 0 & -sq_5 \\ 0 & 1 & 0 \\ sq_5 & 0 & cq_5 \end{bmatrix};\quad {}_{5}^{6}[R]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.(20) $$

The kinematic parameters of the robot's frame are given by the equations:

$$ {}^{0}\overline{\omega}_0=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{0}\overline{v}_0=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{0}\dot{\overline{\omega}}_0=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{0}\dot{\overline{v}}_0=\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}.(21) $$

After performing the necessary computation, the operational angular velocities of the links (i), $i=\overline{1,6}$, are expressed into the frames $\{i\}$ as follows:

$$ {}^{1}\overline{\omega}_1=\begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix};\quad {}^{2}\overline{\omega}_2=\begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix};\qquad (22) $$

$$ {}^{3}\overline{\omega}_3=\begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix};\quad {}^{4}\overline{\omega}_4=\begin{bmatrix} 0 \\ 0 \\ \dot{q}_1+\dot{q}_4 \end{bmatrix}\qquad (23) $$

$$ {}^{5}\overline{\omega}_5=\begin{bmatrix} -sq_5\cdot(\dot{q}_1+\dot{q}_4) \\ \dot{q}_5 \\ cq_5\cdot(\dot{q}_1+\dot{q}_4) \end{bmatrix};\quad {}^{6}\overline{\omega}_6=\begin{bmatrix} -sq_5\cdot(\dot{q}_1+\dot{q}_4) \\ \dot{q}_5 \\ cq_5\cdot(\dot{q}_1+\dot{q}_4) \end{bmatrix}.(24) $$

The operational linear velocities corresponding to the links (i), $i=\overline{1,6}$, expressed into the frames $\{i\}$ as computed by the application, are the following:

$$ {}^{1}\overline{v}_1=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\quad {}^{2}\overline{v}_2=\begin{bmatrix} 0 \\ 0 \\ \dot{q}_2 \end{bmatrix};\quad {}^{3}\overline{v}_3=\begin{bmatrix} (-q_3-l_3)\cdot\dot{q}_1 \\ \dot{q}_1\cdot l_2+\dot{q}_3 \\ \dot{q}_2 \end{bmatrix};\ (25) $$

$$ {}^{4}\overline{v}_4=\begin{bmatrix} (cq_4(-q_3-l_3-l_4)+sq_4\cdot l_2)\cdot\dot{q}_1+sq_4\cdot\dot{q}_3 \\ (-sq_4(-q_3-l_3-l_4)+cq_4\cdot l_2)\cdot\dot{q}_1+cq_4\cdot\dot{q}_3 \\ \dot{q}_2 \end{bmatrix};(26) $$

$$ {}^{5}\overline{v}_5=\begin{bmatrix} cq_5\cdot[(cq_4\cdot(-q_3-l_3-l_4)+sq_4\cdot l_2)\cdot\dot{q}_1+ \\ +sq_4\cdot\dot{q}_3]-sq_5\cdot\dot{q}_2 \\ \hline [-sq_4(-q_3-l_3-l_4)+cq_4\cdot l_2]\cdot\dot{q}_1+cq_4\cdot\dot{q}_3 \\ \hline sq_5\cdot[(cq_4\cdot(-q_3-l_3-l_4)+sq_4\cdot l_2)\cdot\dot{q}_1+ \\ +sq_4\cdot\dot{q}_3]+cq_5\cdot\dot{q}_2 \end{bmatrix}; \qquad (27) $$

$$ {}^{6}\overline{v}_6=\begin{bmatrix} cq_5\cdot[(cq_4\cdot(-q_3-l_3-l_4)+sq_4\cdot l_2)\cdot\dot{q}_1+ \\ +sq_4\cdot\dot{q}_3]-sq_5\cdot\dot{q}_2-cq_5\cdot(\dot{q}_1+\dot{q}_4)\cdot l_6 \\ \hline [-sq_4(-q_3-l_3-l_4)+cq_4\cdot l_2]\cdot\dot{q}_1+cq_4\cdot\dot{q}_3 \\ \hline sq_5\cdot[(cq_4\cdot(-q_3-l_3-l_4)+sq_4\cdot l_2)\cdot\dot{q}_1+ \\ +sq_4\cdot\dot{q}_3]+cq_5\cdot\dot{q}_2-sq_5\cdot(\dot{q}_1+\dot{q}_4)\cdot l_6 \end{bmatrix}.(28) $$

The operational angular accelerations are:

$$ {}^{1}\dot{\overline{\omega}}_1=\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix};\ {}^{2}\dot{\overline{\omega}}_2=\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix};\qquad (29) $$

$$ {}^{3}\dot{\overline{\omega}}_3=\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix};\ {}^{4}\dot{\overline{\omega}}_4=\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1+\ddot{q}_4 \end{bmatrix};\qquad (30) $$

$$ {}^{5}\dot{\overline{\omega}}_5=\begin{bmatrix} -sq_5\cdot(\ddot{q}_1+\ddot{q}_4)+\dot{q}_5\cdot cq_5\cdot(-\dot{q}_1-\dot{q}_4) \\ \ddot{q}_5 \\ cq_5\cdot(\ddot{q}_1+\ddot{q}_4)+\dot{q}_5\cdot sq_5\cdot(-\dot{q}_1-\dot{q}_4) \end{bmatrix};(31) $$

$$ {}^{6}\dot{\overline{\omega}}_6=\begin{bmatrix} -sq_5\cdot(\ddot{q}_1+\ddot{q}_4)+\dot{q}_5\cdot cq_5\cdot(-\dot{q}_1-\dot{q}_4) \\ \ddot{q}_5 \\ cq_5\cdot(\ddot{q}_1+\ddot{q}_4)+\dot{q}_5\cdot sq_5\cdot(-\dot{q}_1-\dot{q}_4) \end{bmatrix};(32) $$

and the operational linear accelerations are expressed as:

$$ {}^{1}\dot{\overline{v}}_1=\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix};\quad {}^{2}\dot{\overline{v}}_2=\begin{bmatrix} 0 \\ 0 \\ g+\ddot{q}_2 \end{bmatrix};\qquad (33) $$

$$ {}^{3}\dot{\bar{v}}_3 = \begin{bmatrix} (-q_3 - l_3)\cdot \ddot{q}_1 - \dot{q}_1^2 \cdot l_2 - 2\cdot \dot{q}_3 \cdot \dot{q}_1 \\ \ddot{q}_1 \cdot l_2 - \dot{q}_1^2 \cdot (q_3 + l_3) + \ddot{q}_3 \\ g + \ddot{q}_2 \end{bmatrix}; \qquad (34) $$

$$ {}^{4}\dot{\bar{v}}_4 = \begin{bmatrix} cq_4[(-q_3 - l_3)\cdot \ddot{q}_1 - \dot{q}_1^2 \cdot l_2 - 2\cdot \dot{q}_3 \cdot \dot{q}_1 - \ddot{q}_1 \cdot l_4] + \\ + sq_4 \cdot [\ddot{q}_1 \cdot l_2 - \dot{q}_1^2 \cdot (q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 \cdot l_4] \\ \hline - sq_4[(-q_3 - l_3)\cdot \ddot{q}_1 - \dot{q}_1^2 \cdot l_2 - 2\cdot \dot{q}_3 \cdot \dot{q}_1 - \ddot{q}_1 \cdot l_4] + \\ + cq_4 \cdot [\ddot{q}_1 \cdot l_2 - \dot{q}_1^2 \cdot (q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 \cdot l_4] \\ \hline g + \ddot{q}_2 \end{bmatrix}; \qquad (35) $$

$$ {}^{5}\dot{\bar{v}}_5 = \begin{bmatrix} cq_5\{cq_4[(-q_3 - l_3)\ddot{q}_1 - \dot{q}_1^2 l_2 - 2\dot{q}_3\dot{q}_1 - \ddot{q}_1 l_4] + \\ + sq_4[\ddot{q}_1 l_2 - \dot{q}_1^2(q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 l_4]\} - sq_5(g + \ddot{q}_2) \\ \hline - sq_4[(-q_3 - l_3)\ddot{q}_1 - \dot{q}_1^2 l_2 - 2\cdot \dot{q}_3\dot{q}_1 - \ddot{q}_1 l_4] + cq_4[\ddot{q}_1 l_2 - \dot{q}_1^2(q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 l_4] \\ \hline sq_5\{cq_4[(-q_3 - l_3)\ddot{q}_1 - \dot{q}_1^2 l_2 - 2\dot{q}_3\dot{q}_1 - \ddot{q}_1 l_4] + \\ + sq_4[\ddot{q}_1 l_2 - \dot{q}_1^2(q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 l_4]\} + cq_5(g + \ddot{q}_2) \end{bmatrix}; \qquad (36) $$

$$ {}^{6}\dot{\bar{v}}_6 = \begin{bmatrix} cq_5\{cq_4[(-q_3 - l_3)\ddot{q}_1 - \dot{q}_1^2 l_2 - 2\dot{q}_3\dot{q}_1 - \ddot{q}_1 l_4] + sq_4[\ddot{q}_1 l_2 - \dot{q}_1^2(q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 l_4]\} - \\ - sq_5(g + \ddot{q}_2) + [-cq_5(\ddot{q}_1 + \ddot{q}_4) - \dot{q}_5 sq_5(-\dot{q}_1 - \dot{q}_4)]l_6 - \dot{q}_5 sq_5(\dot{q}_1 + \dot{q}_4)l_6 \\ \hline - sq_4[(-q_3 - l_3)\ddot{q}_1 - \dot{q}_1^2 l_2 - 2\dot{q}_3\dot{q}_1 - \ddot{q}_1 l_4] + cq_4[\ddot{q}_1 l_2 - \dot{q}_1^2(q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 l_4] + \\ + [-c^2 q_5(\dot{q}_1 + \dot{q}_4)^2 - s^2 q_5 \cdot (\dot{q}_1 + \dot{q}_4)^2]l_6 \\ \hline sq_5\{cq_4[(-q_3 - l_3)\ddot{q}_1 - \dot{q}_1^2 l_2 - 2\dot{q}_3\dot{q}_1 - \ddot{q}_1 l_4] + sq_4[\ddot{q}_1 l_2 - \dot{q}_1^2(q_3 + l_3) + \ddot{q}_3 - \dot{q}_1^2 l_4]\} + \\ + cq_5(g + \ddot{q}_2) + [-sq_5(\ddot{q}_1 + \ddot{q}_4) + \dot{q}_5 cq_5(-\dot{q}_1 - \dot{q}_4)]l_6 + cq_5(\dot{q}_1 + \dot{q}_4)\dot{q}_5 l_6 \end{bmatrix}. \qquad (37) $$

After the transformation of the frames, the operational kinematic parameters (operational velocities and accelerations, linear and angular) can be written with respect to the fixed frame {0}, as:

$$ {}^{0}\bar{v}_6 = \begin{bmatrix} - sq_1 \cdot \dot{q}_3 - sq_1 \cdot \dot{q}_1 \cdot l_2 - c(q_1 + q_4)l_6 \cdot \dot{q}_4 - cq_1 \cdot \dot{q}_1 \cdot l_3 - \\ - cq_1 \cdot \dot{q}_1 \cdot q_3 - cq_1 \cdot \dot{q}_1 \cdot l_4 - c(q_1 + q_4) \cdot l_6 \cdot \dot{q}_1 \\ \hline - sq_1 \cdot \dot{q}_3 - sq_1 \cdot \dot{q}_1 \cdot l_2 - c(q_1 + q_4) \cdot l_6 \cdot \dot{q}_4 - cq_1 \cdot \dot{q}_1 \cdot l_3 - \\ - cq_1 \cdot \dot{q}_1 \cdot q_3 - cq_1 \cdot \dot{q}_1 \cdot l_4 - c(q_1 + q_4) \cdot l_6 \cdot \dot{q}_1 \\ \hline \dot{q}_2 \end{bmatrix}; \quad {}^{0}\bar{\omega}_6 = \begin{bmatrix} - s(q_1 + q_4) \cdot \dot{q}_5 \\ c(q_1 + q_4) \cdot \dot{q}_5 \\ \dot{q}_1 + \dot{q}_4 \end{bmatrix}; \qquad (38) $$

$$ {}^{0}\dot{\bar{v}}_6 = \begin{bmatrix} - cq_1 \cdot \ddot{q}_1 \cdot q_3 - c(q_1 + q_4) \cdot l_6 \cdot \ddot{q}_1 - c(q_1 + q_4) \cdot l_6 \cdot \ddot{q}_4 - sq_1 \cdot \ddot{q}_1 \cdot l_2 + \\ + 2 \cdot s(q_1 + q_4) \cdot l_6 \cdot \dot{q}_1 \cdot \dot{q}_4 + s(q_1 + q_4) \cdot l_6 \cdot \dot{q}_1^2 + s(q_1 + q_4) \cdot l_6 \cdot \dot{q}_4^2 + \\ + sq_1 \cdot \dot{q}_1^2 \cdot l_3 - cq_1 \cdot \ddot{q}_1 \cdot l_3 - cq_1 \cdot \dot{q}_1^2 \cdot l_2 + sq_1 \cdot \dot{q}_1^2 \cdot l_4 + \\ + sq_1 \cdot \dot{q}_1^2 \cdot q_3 - cq_1 \cdot \ddot{q}_1 \cdot l_4 - 2 \cdot cq_1 \cdot \dot{q}_3 \cdot \dot{q}_1 \text{-} sq_1 \cdot \ddot{q}_3 \\ \hline - cq_1 \cdot \dot{q}_1^2 \cdot l_4 - sq_1 \cdot \ddot{q}_1 \cdot l_4 - sq_1 \cdot \dot{q}_1^2 \cdot l_2 + cq_1 \cdot \ddot{q}_3 - cq_1 \cdot \dot{q}_1^2 \cdot l_3 + cq_1 \cdot \ddot{q}_1 \cdot l_2 - \\ - sq_1 \cdot \dot{q}_1 \cdot l_3 - cq_1 \cdot \dot{q}_1^2 \cdot q_3 - sq_1 \cdot \dot{q}_1 \cdot q_3 - 2sq_1 \cdot \dot{q}_3 \cdot \dot{q}_1 - s(q_1 + q_4) \cdot l_6 \cdot \dot{q}_4 - \\ - s(q_1 + q_4) \cdot l_6 \cdot \ddot{q}_1 - 2c(q_1 + q_4) \cdot l_6 \cdot \dot{q}_1 \cdot \dot{q}_4 - \\ - s(q_1 + q_4) \cdot l_6 \cdot \dot{q}_1^2 - c(q_1 + q_4) \cdot l_6 \cdot \dot{q}_4^2 \\ \hline g + \ddot{q}_2 \end{bmatrix}; \qquad (39) $$

$$ {}^{0}\dot{\bar{\omega}}_6 = \begin{bmatrix} -c(q_1+q_4)\cdot\dot{q}_5\cdot\dot{q}_1 - c(q_1+q_4)\cdot\dot{q}_5\cdot\dot{q}_4 - s(q_1+q_4)\cdot\ddot{q}_5 \\ -s(q_1+q_4)\cdot\dot{q}_5\cdot\dot{q}_1 - s(q_1+q_4)\cdot\dot{q}_5\cdot\dot{q}_4 + c(q_1+q_4)\cdot\ddot{q}_5 \\ \ddot{q}_1+\ddot{q}_4 \end{bmatrix}. \quad (40) $$

The kinematic model of RTTRR small-sized modular robot can be expressed by one of the following relation sets: (24), (28), (32) and (37), with respect to the frame {6}, or (38)-(40), with respect to the fixed frame {0}. The obtained symbolic data is saved into the file *RTTRR_kin.mat* and it will be afterwards used as an input data at the dynamic modeling of this robot [6].

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Modelarea geometrică şi cinematică a minirobotului modular de tip RTTRR

Rezumat: Prima parte a lucrării descrie structura mecanică a minirobotului modular RTTRR. A doua parte este rezervată determinării modelului geometric al robotului, folosind metoda matricelor de rotaţie [2], iar partea a treia prezintă modelul cinematic, determinat prin utilizarea metodei iterative de modelare cinematică [2]. Ecuaţiile modelului geometric şi cinematic sunt determinate folosind primele trei module ale aplicaţiei generalizate de modelare *Robot_Symbolic* [1] şi acestea pot fi utilizate la simularea compotamentului geometric şi cinematic al robotului analizat.

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