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CONTRIBUTIONS TO THE TRANSMISSIBILITY STUDY OF ONE DEGREE OF FREEDOM MECHANICAL SYSTEM, WITH DIFFERENT ELASTIC AND VISCOUS ELEMENTS

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Abstract: *The transmissibility of one degree of freedom mechanical systems with oscillating support, performing harmonic movements, are studied. Different types of rheological models used as a link between support and the mass are considered. The differences between the transmissibility diagrams corresponding to the "classic" case, usually studied in the theory of vibration (oscillating support, Kelvin-Voigt model and mass) and the systems containing other rheological models are presented and discussed.*

Key words: *mechanical system, third order differential equation, transmissibility*

1. INTRODUCTION

Studying the dynamics of one degree of freedom mechanical systems, one important problem is to determine the amplitude of mass **m** displacements due to the oscillating support, that is linked with the mass via the Kelvin-Voigt model, [1], [5], [8], [9], [12], [15], [19] composed by a spring (**k** is the spring stiffness) and a damper (**c** is the damping coefficient), disposed in parallel. The transmissibility is defined as a ratio between the mass amplitude displacement and the oscillating support displacement amplitude [1], [5], [10], [15], [19],

$$T = \frac{x_{max}}{s_{max}} \qquad (1)$$

If the support displacements are harmonic, $s = s_{max} \cos \omega t = s_{max} \cos 2\pi f t$, the transmissibility expression is

$$T = \sqrt{\frac{1 + 4\varsigma^2 r^2}{(1 - r^2)^2 + 4\varsigma^2 r^2}} \qquad (2)$$

where $r = \frac{\omega}{\omega_0} = \frac{f}{f_0}$, $\omega_0 = \sqrt{\frac{k}{m}}$ circular eigen-frequency, $\varsigma = \frac{c}{c_0}$ damping ratio.

In figure 1 are represented the plots of the transmissibility for different values of the damping ratio, between 0.0 and 1.5

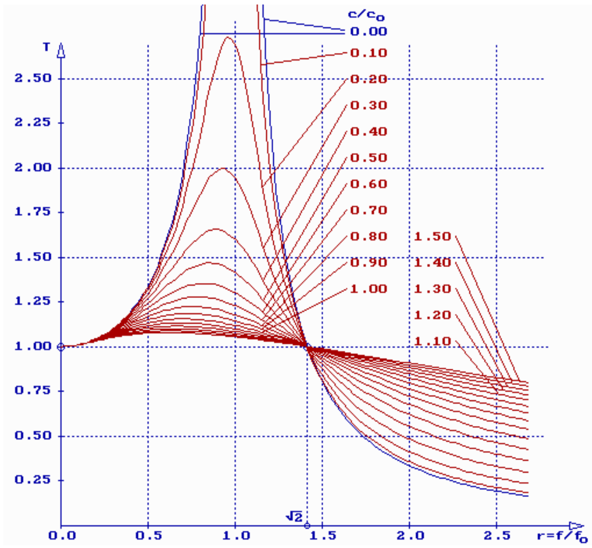

Fig. 1 The transmissibility diagrams [15]

If the mass displacements refer to a fixed, absolute frame, the transmissibility is named absolute transmissibility.

As it is known, the link between the mass and the oscillating support may be made with different elements, of Hooke's and Newton's type (figure 2),

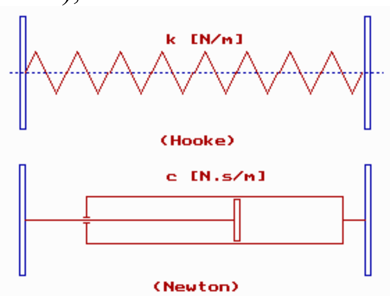

Fig. 2. The Hooke's and Newton's elements

in such cases the transmissibility and the other characteristics of the mechanical systems have to be studied, because many particularities may appear.

2. THEORETICAL CONSIDERATIONS

The transmissibility will be studied considering the mechanical systems represented in figures 3, 4 and 5.

We can notice that in the figure 3 the system has two Kelvin-Voigt models disposed in series, and in the figures 4 and 5 the Kelvin-Voigt model in series with Hooke's element.

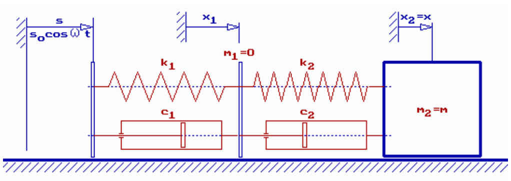

Fig. 3 Two Kelvin-Voigt models in series

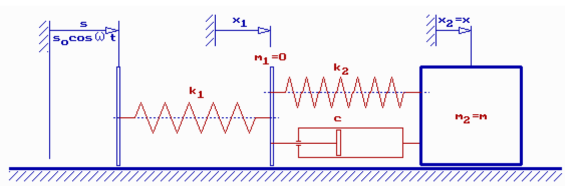

Fig. 4 Hooke and Kelvin-Voigt models in series

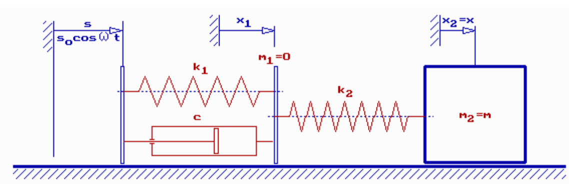

Fig. 5 Kelvin-Voigt and Hooke models in series

Based on the second Newton's law one may write the following differential equations, the first corresponding to the fictitious mass m_1 and the second to the existing mass m_2 [1], [6], [10], [12], [15], [19],

$$\begin{cases} m_1\ddot{x}_1 = k_1(s-x_1) - k_2(x_1-x_2) + \\ \qquad + c_1(\dot{s}-\dot{x}_1) - c_2(\dot{x}_1-\dot{x}_2) \\ m_2\ddot{x}_2 = k_2(x_1-x_2) + c_2(\dot{x}_1-\dot{x}_2) \end{cases} \quad (3)$$

These equations may be also written in the matrix form, as it is shown

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k_1 s_0\cos\omega t - c_1\omega s_0\sin\omega t \\ 0 \end{bmatrix}$$

Because $m_1 = 0$ and noting $m_2 = m$, $x_2 = x$, the system (1) become

$$\begin{cases} (k_1+k_2)x_1 + (c_1+c_2)\dot{x}_1 = k_2 x + c_2\dot{x} + \\ \qquad + k_1 s_0\cos\omega t - c_1\omega s_0\sin\omega t \\ k_2 x_1 + c_2\dot{x}_1 = m\ddot{x} + c_2\dot{x} + k_2 x \end{cases} \quad (4)$$

obtaining simultaneous linear equations, the unknowns being considered x_1 and $\dot{x}_1$.

Solving this system we obtain the unknowns x_1 and $\dot{x}_1$

$$x_1 = \frac{\begin{vmatrix} k_2 x + c_2\dot{x} + k_1 s_0\cos\omega t - c_1\omega s_0\sin\omega t & c_1+c_2 \\ m\ddot{x} + c_2\dot{x} + k_2 x & c_2 \end{vmatrix}}{\begin{vmatrix} k_1+k_2 & c_1+c_2 \\ k_2 & c_2 \end{vmatrix}}$$

$$\dot{x}_1 = \frac{\begin{vmatrix} k_1+k_2 & k_2 x + c_2\dot{x} + k_1 s_0\cos\omega t - c_1\omega s_0\sin\omega t \\ k_2 & m\ddot{x} + c_2\dot{x} + k_2 x \end{vmatrix}}{\begin{vmatrix} k_1+k_2 & c_1+c_2 \\ k_2 & c_2 \end{vmatrix}} \quad (5)$$

The derivative with respect of time of variable x_1 is

$$\dot{x}_1 = \frac{\begin{vmatrix} k_2 \dot{x} + c_2 \ddot{x} - k_1 \omega s_0 \sin \omega t - \\ - c_1 \omega^2 s_0 \cos \omega t & c_1 + c_2 \\ m \dddot{x} + c_2 \ddot{x} + k_2 \dot{x} & c_2 \end{vmatrix}}{\begin{vmatrix} k_1 + k_2 & c_1 + c_2 \\ k_2 & c_2 \end{vmatrix}} \qquad (6)$$

and equaling it with the previously obtained expression of unknown $\dot{x}_1$, we obtain

$$\begin{vmatrix} k_2 \dot{x} + c_2 \ddot{x} - k_1 \omega s_0 \sin \omega t - \\ - c_1 \omega^2 s_0 \cos \omega t & c_1 + c_2 \\ m \dddot{x} + c_2 \ddot{x} + k_2 \dot{x} & c_2 \end{vmatrix} =$$

$$= \begin{vmatrix} k_1 + k_2 & k_2 x + c_2 \dot{x} + k_1 s_0 \cos \omega t - \\ & - c_1 \omega s_0 \sin \omega t \\ k_2 & m \ddot{x} + c_2 \dot{x} + k_2 x \end{vmatrix}$$

The following third order nonhomogeneous differential equation will result:

$$\begin{aligned} m(c_1 + c_2)\dddot{x} &+ [m(k_1 + k_2) + c_1 c_2]\ddot{x} + \\ &+ (k_1 c_2 + k_2 c_1)\dot{x} + k_1 k_2 x = \\ &= (k_1 k_2 - \omega^2 c_1 c_2) s_0 \cos \omega t - \\ &- (k_1 c_2 + k_2 c_1) \omega s_0 \sin \omega t \end{aligned} \qquad (7)$$

The second term of this differential equation may be written in a simplified form:

$$\begin{aligned} &(k_1 k_2 - \omega^2 c_1 c_2) s_0 \cos \omega t - \\ &- (k_1 c_2 + k_2 c_1) \omega s_0 \sin \omega t = A \cos(\omega t + \varphi) \end{aligned}$$

the amplitude A and the phase angle **φ** having the expressions:

$$A = s_0 \sqrt{\begin{aligned}&(k_1 k_2 - \omega^2 c_1 c_2)^2 + \\ &+ \omega^2 (k_1 c_2 + k_2 c_1)^2\end{aligned}} \qquad (8)$$

$$\varphi = a \tan \left[\frac{\omega (k_1 c_2 + k_2 c_1)}{k_1 k_2 - \omega^2 c_1 c_2} \right] \qquad (9)$$

As a consequence, the differential equation becomes

$$\begin{aligned} &m(c_1 + c_2)\dddot{x} + [m(k_1 + k_2) + c_1 c_2]\ddot{x} + \\ &+ (k_1 c_2 + k_2 c_1)\dot{x} + k_1 k_2 x = A \cos(\omega t + \varphi) \end{aligned}$$

and its particular solution will look like the second term, therefore this solution will be a harmonic function.

To obtain this solution will be used the method of transforming the differential equation in complex one, replacing [15],

$$A \cos(\omega t + \varphi) \rightarrow A e^{j(\omega t + \varphi)}$$

$$x \rightarrow Z = x_{max} e^{j(\omega t + \varphi + \psi)}$$

and will result the complex differential equation

$$\begin{aligned} &m(c_1 + c_2)\dddot{Z} + [m(k_1 + k_2) + c_1 c_2]\ddot{Z} + \\ &+ (k_1 c_2 + k_2 c_1)\dot{Z} + k_1 k_2 Z = A e^{j(\omega t + \varphi)} \end{aligned} \qquad (10)$$

After replacing the derivatives of complex variable Z,

$$\dot{Z} = j\omega Z, \quad \ddot{Z} = -\omega^2 Z, \quad \dddot{Z} = -j\omega^3 Z$$

on obtains

$$\begin{aligned} &\{-[m(k_1 + k_2) + c_1 c_2]\omega^2 + k_1 k_2 + \\ &+ j\omega[-m(c_1 + c_2)\omega^2 + k_1 c_2 + k_2 c_1]\} Z = A e^{j(\omega t + \varphi)} \end{aligned}$$

The complex expression Z of the mass m displacement has the form,

$$Z = \frac{A}{\begin{aligned}&k_1 k_2 - \omega^2 [m(k_1 + k_2) + c_1 c_2] + \\ &+ j\omega[k_1 c_2 + k_2 c_1 - m\omega^2 (c_1 + c_2)]\end{aligned}} e^{j(\omega t + \varphi)}$$

and using the formula (8) of the amplitude A will result

$$Z = \frac{s_0 \sqrt{\begin{aligned}&(k_1 k_2 - \omega^2 c_1 c_2)^2 + \\ &+ \omega^2 (k_1 c_2 + k_2 c_1)^2\end{aligned}}}{\begin{aligned}&k_1 k_2 - \omega^2 [m(k_1 + k_2) + c_1 c_2] + \\ &+ j\omega[k_1 c_2 + k_2 c_1 - m\omega^2 (c_1 + c_2)]\end{aligned}} e^{j(\omega t + \varphi)} \qquad (11)$$

The modulus of the complex displacement Z has the following expression

$$|Z| = \frac{s_0 \sqrt{\begin{aligned}&(k_1 k_2 - \omega^2 c_1 c_2)^2 + \\ &+ \omega^2 (k_1 c_2 + k_2 c_1)^2\end{aligned}}}{\sqrt{\begin{aligned}&\{k_1 k_2 - \omega^2 [m(k_1 + k_2) + c_1 c_2]\}^2 + \\ &+ \omega^2 [k_1 c_2 + k_2 c_1 - m\omega^2 (c_1 + c_2)]^2\end{aligned}}}$$

According to the definition, the absolute transmissibility of displacements is computed as $T=\frac{|Z|}{s_0}$, obtaining :

$$T=\frac{\sqrt{\begin{array}{l}(k_1 k_2-\omega^2 c_1 c_2)^2+\\ \omega^2 (k_1 c_2+k_2 c_1)^2\end{array}}}{\sqrt{\begin{array}{l}\{k_1 k_2-\omega^2[m(k_1+k_2)+c_1 c_2]\}^2+\\ +\omega^2[k_1 c_2+k_2 c_1-m\omega^2(c_1+c_2)]^2\end{array}}} \tag{12}$$

The expression of the mass displacement is,

$$x(t)=s_0 T\cos(\omega t+\varphi+\psi)$$

where the angle ψ is the argument of complex number, its algebraic form being

$$\frac{A}{\begin{array}{l}k_1 k_2-\omega^2[m(k_1+k_2)+c_1 c_2]+\\ j\omega[k_1 c_2+k_2 c_1-m\omega^2(c_1+c_2)]\end{array}}=$$

$$=\frac{A\{k_1 k_2-\omega^2[m(k_1+k_2)+c_1 c_2]\}}{\begin{array}{l}\{k_1 k_2-\omega^2[m(k_1+k_2)+c_1 c_2]\}^2+\\ \omega^2[k_1 c_2+k_2 c_1-m\omega^2(c_1+c_2)]^2\end{array}}+$$

$$+j\frac{-A\omega[k_1 c_2+k_2 c_1-m\omega^2(c_1+c_2)]}{\begin{array}{l}\{k_1 k_2-\omega^2[m(k_1+k_2)+c_1 c_2]\}^2+\\ \omega^2[k_1 c_2+k_2 c_1-m\omega^2(c_1+c_2)]^2\end{array}}$$

therefore

$$\psi=a\tan\left\{\frac{\omega[m(c_1+c_2)\omega^2-k_1 c_2-k_2 c_1]}{k_1 k_2-[m(k_1+k_2)+c_1 c_2]\omega^2}\right\} \tag{13}$$

The final expression of the mass m displacement are the following:

$$x(t)=\frac{s_0\sqrt{\begin{array}{l}(k_1 k_2-\omega^2 c_1 c_2)^2+\\ +\omega^2(k_1 c_2+k_2 c_1)^2\end{array}}}{\sqrt{\begin{array}{l}\{k_1 k_2-\omega^2[m(k_1+k_2)+c_1 c_2]\}^2+\\ +\omega^2[k_1 c_2+k_2 c_1-m\omega^2(c_1+c_2)]^2\end{array}}}\cdot$$

$$.\cos\left\{\omega t+a\tan\left[\frac{\omega(k_1 c_2+k_2 c_1)}{k_1 k_2-\omega^2 c_1 c_2}\right]+\right.$$

$$\left.+a\tan\left[\frac{\omega[m(c_1+c_2)\omega^2-k_1 c_2-k_2 c_1]}{k_1 k_2-[m(k_1+k_2)+c_1 c_2]\omega^2}\right]\right\} \tag{14}$$

Considering the particular cases showed in figures 4 ($c_1=c\neq0$, $c_2=0$) and 5 ($c_1=0$, $c_2=c\neq0$) on obtain some simplified expressions for the transmissibility computing:

$$T_{(c_1=0,c_2\neq0)}=\frac{k_1\sqrt{k_2^2+\omega^2 c_2^2}}{\sqrt{\begin{array}{l}[k_1 k_2-\omega^2 m(k_1+k_2)]^2+\\ +\omega^2 c_2^2(k_1-m\omega^2)^2\end{array}}} \tag{15}$$

$$T_{(c_1\neq0,c_2=0)}=\frac{k_2\sqrt{k_1^2+\omega^2 c_1^2}}{\sqrt{\begin{array}{l}[k_1 k_2-\omega^2 m(k_1+k_2)]^2+\\ +\omega^2 c_1^2(k_2-m\omega^2)^2\end{array}}} \tag{16}$$

The deduced formulas (12), (15) and (16) are proper to compute the transmissibility when the rheological model is composed by two Kelvin-Voigt model in series or a Kelvin-Voigt model in series with Hooke's model.

In present days were identified different materials which characteristics may be modeled using other rheological model – the Zener model, such model being a component of the mechanical system represented in figure 6.

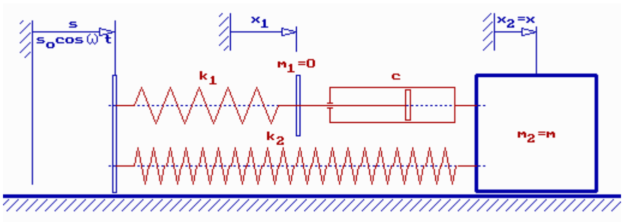

Fig. 6. The mechanical system containing the Zener's rheological model

As it is shown the Zener model is composed from a Maxwell model (Hooke and Newton models in series) and in parallel with the Hooke model.

Using the similar method on may deduce the transmissibility expression for the system presented above.

Considering the two masses m_1 and m_2 the following differential equations may be written:

$$\begin{cases} m_1\ddot{x}_1=k_1(s_0\cos\omega t-x_1)-c(\dot{x}_1-\dot{x}_2)\\ m_2\ddot{x}_2=k_2(s_0\cos\omega t-x_2)+c(\dot{x}_1-\dot{x}_2)\end{cases} \tag{17}$$

or in matrix form

$$\begin{bmatrix} m_1 & 0\\ 0 & m_2\end{bmatrix}\begin{bmatrix}\ddot{x}_1\\ \ddot{x}_2\end{bmatrix}+\begin{bmatrix} c & -c\\ -c & c\end{bmatrix}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}+$$

$$+\begin{bmatrix} k_1 & 0\\ 0 & k_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}k_1 s_0\cos\omega t\\ k_2 s_0\cos\omega t\end{bmatrix}$$

Because $m_1=0$ and $m_2=m$ will result

$$\begin{cases} k_1\,x_1 + c\,\dot{x}_1 = c\,\dot{x}_2 + k_1\,s_0\,\cos\omega\,t \\ c\,\dot{x}_1 = m\,\ddot{x}_2 + k_2\,x_2 + c\,\dot{x}_2 - k_2\,s_0\,\cos\omega\,t \end{cases} \quad (18)$$

Removing the unknown x_1, after some calculi is obtained the third order differential equation, nonhomogeneous,

$$\begin{aligned} & m\,c\,\dddot{x} + m\;k_1\,\ddot{x} + (k_1 + k_2)\,c\,\dot{x} + k_1\,k_2\,x = \\ & = s_0\,[k_1\,k_2\,\cos\omega\,t - c\,(k_1 + k_2)\,\omega\sin\omega\,t] \end{aligned} \quad (19)$$

The second term may be written as follows

$$\begin{aligned} & s_0\,[k_1\,k_2\,s_0\cos\omega\,t - c\,(k_1 + k_2)\,\omega\sin\omega\,t] = \\ & \qquad = B\cos(\omega\,t + \xi) \end{aligned}$$

where

$$B = s_0\,\sqrt{k_1^2\,k_2^2 + c^2\,\omega^2\,(k_1 + k_2)^2}$$

$$\xi = a\tan\left[\frac{c\,\omega\,(k_1 + k_2)}{k_1\,k_2}\right]$$

The following differential equation will be considered in the future

$$\begin{aligned} & m\,c\,\dddot{x} + m\;k_1\,\ddot{x} + (k_1 + k_2)\,c\,\dot{x} + k_1\,k_2\,x = \\ & \qquad = B\cos(\omega\,t + \xi) \end{aligned} \quad (20)$$

This equation will be rewritten in complex form, considering

$$B\cos(\omega\,t + \xi) \;\rightarrow\; B\,e^{j(\omega t + \xi)}$$

$$x \;\rightarrow\; Z = x_{max}\,e^{j(\omega t + \xi + \varsigma)}$$

and the derivative expressions

$$\dot{Z} = j\,\omega\,Z\,, \quad \ddot{Z} = -\,\omega^2\,Z\,, \quad \dddot{Z} = -\,j\,\omega^3\,Z$$

obtaining:

$$\begin{aligned} & \left\{k_1\,k_2 - m\,k_1\,\omega^2 + j\,c\,\omega\,[k_1 + k_2 - m\,\omega^2]\right\} Z = \\ & \qquad = B\,e^{j(\omega t + \xi)} \end{aligned}$$

The complex mass m displacement is

$$Z = \frac{B\,e^{j(\omega t + \xi)}}{k_1\,(k_2 - m\,\omega^2) + j\,c\,\omega\,[k_1 + k_2 - m\,\omega^2]}$$

and it modulus

$$|Z| = x_{max} = s_0\,\sqrt{\frac{k_1^2\,k_2^2 + c^2\,\omega^2\,(k_1 + k_2)^2}{k_1^2\,(k_2 - m\,\omega^2)^2 + c^2\,\omega^2\,(k_1 + k_2 - m\,\omega^2)^2}}$$

According to the transmissibility definition will result the expressions of the module and of the phase angle

$$T = \frac{|Z|}{s_0} = \sqrt{\frac{k_1^2\,k_2^2 + c^2\,\omega^2\,(k_1 + k_2)^2}{k_1^2\,(k_2 - m\,\omega^2)^2 + c^2\,\omega^2\,(k_1 + k_2 - m\,\omega^2)^2}} \quad (21)$$

$$\varsigma = a\tan\left[\frac{-\,c\,\omega\,(k_1 + k_2 - m\,\omega^2)}{k_1\,(k_2 - m\,\omega^2)}\right] \quad (22)$$

3. NUMERICAL RESULTS

There is considered the mechanical system as in figure 3. The oscillating support amplitude is 0.005 [m], and variable frequency, other characteristics being: m=50 [kg], k_1=20000 [N/m], k_2=25000 [N/m], c_1=1000 [N.s/m], c_2=1500 [N.s/m]. In the figure 7 are presented the two plots corresponding to the amplitude and phase angle of the absolute transmissibility of this mechanical system.

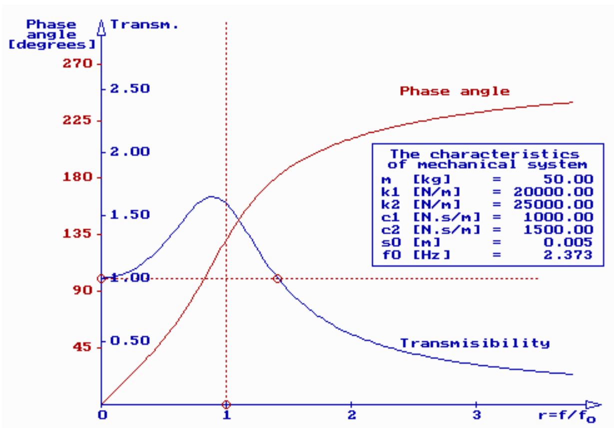

Fig. 7 Diagrams of transmissibility and phase angle for specified numerical values

The same types of diagrams are represented in the figures 8 and 9 the differences consist in the values of damping constants c_1 and c_2 .

The above presented diagrams correspond to the mechanical systems from the figures 4 and 5. We can notice that maximal value of the transmissibility amplitude depends on the damping coefficients and also the abscissa of this point is greater than unity when one of damping constants has zero value.

In the next three figures are shown families of diagrams obtained in the special case when the equivalent stiffness is the same (8000

[N/m]) and the ratio of k_2 and k_1 has values between 0.75 and 1.5 . In figures 10, 11 and 12 are considered different values for the damping coefficient c_1 (1000, 2000 and 3000) the second being c_2=0.

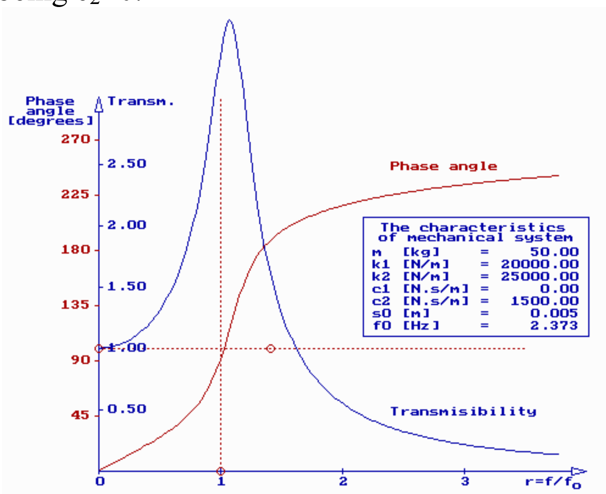

Fig. 8 Diagrams of transmissibility and phase angle for specified numerical values

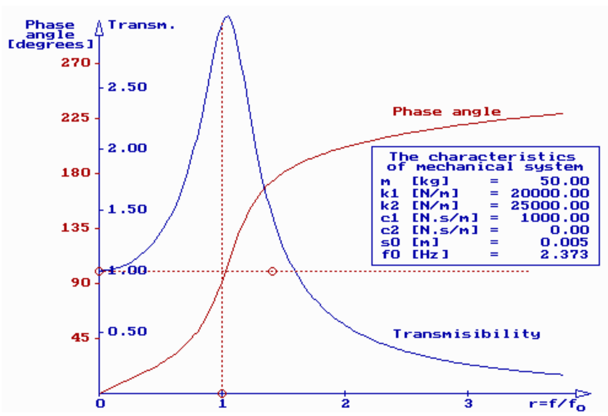

Fig. 9 Diagrams of transmissibility and phase angle for specified numerical values

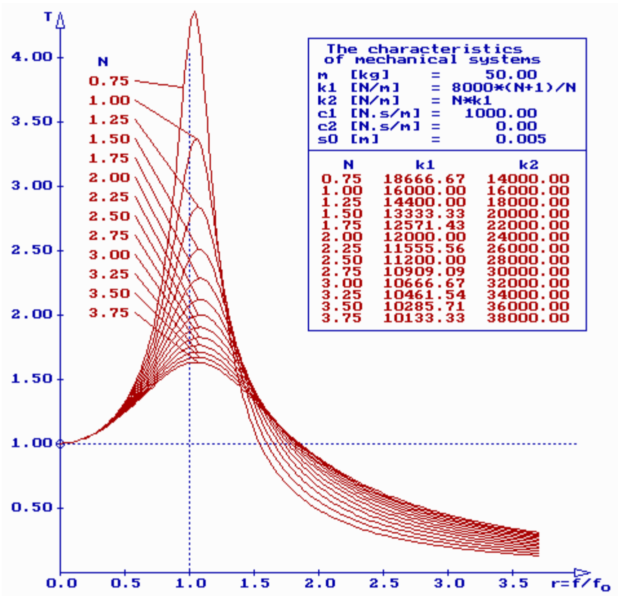

Fig.10 The transmissibility diagrams (c_1 =1000)

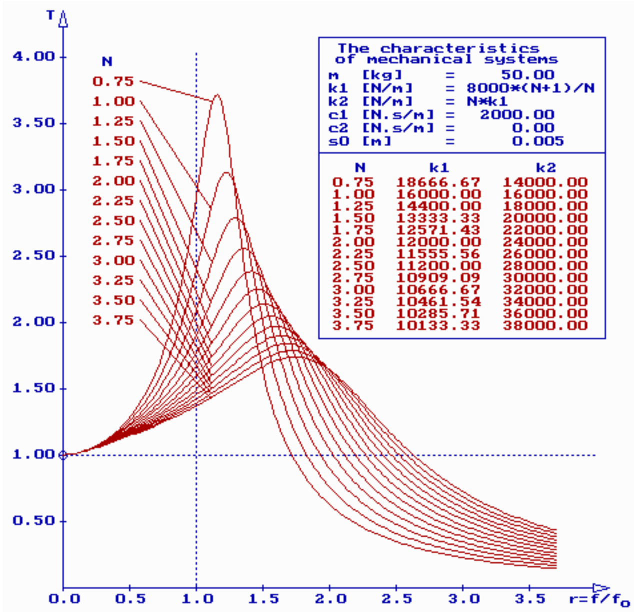

Fig. 11 The transmissibility diagrams (c_1 =2000)

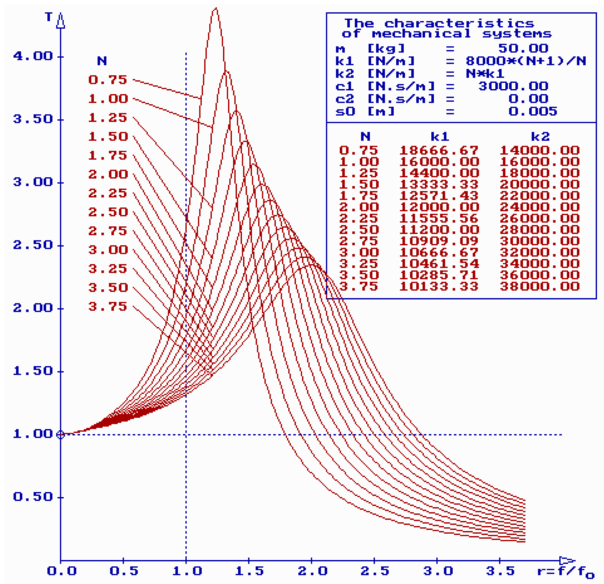

Fig. 12 The transmissibility diagrams (c_1 =3000)

Observing the plotted diagrams we can see that the abscissas of the maximal points are moving to the right when the damping coefficient increases.

Considering the expressions (21) and (22) of the transmissibility and phase angle in the case of mechanical system having Zener's type springs and dampers we obtain the diagrams presented in the figure 13.

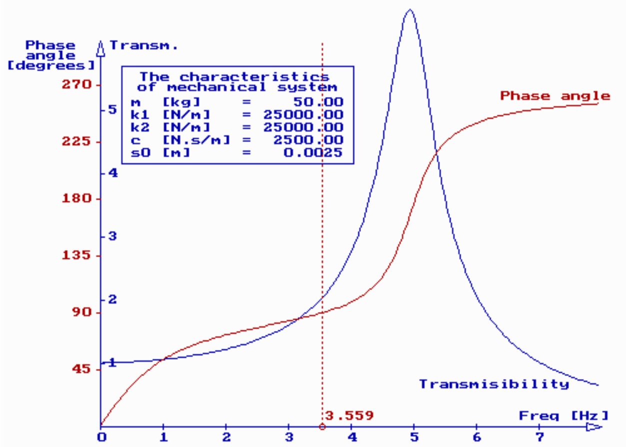

Fig. 13 Transmissibility and phase angle (Zener model)

4. CONCLUSIONS

The study of the mechanical system transmissibility arise in many areas: the vibration isolation [4], [13], the movement of a whole human body subjected to vibrations, in standing or sitting positions [3], [6], [7], the determination of the optimal parameters of a drivers auto seats [14], [20], [21], a. o.

The recently used isolation materials have elastic and damping properties that can not be modeled with rheological model of Kelvin-Voigt type.

Many researches are focused on the study of other rheological models, more complicated than the well known Kelvin-Voigt model, the proposed paper belongs to the same category.

Using the deduced formulas and a C program [16], was possible to show a lot of diagrams of the transmissibility and to compare its with the diagrams corresponding to the system endowed with Kelvin-Voigt rheological models.

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CONTRIBUŢII LA STUDIUL TRANSMISIBILITĂŢII SISTEMELOR MECANICE CU UN GRAD DE LIBERTATE, CU SUPORT OSCILANT, CU DIFERITE ELEMENTE ELASTICE ŞI VISCOASE

Se efectuează un studiu al transmisibilităţii sistemelor mecanice cu un grad de libertate, mişcarea cărora este datorată unui suport oscilant. Sunt considerate diferite tipuri de modele reologice care fac legătura dintre suportul oscilant şi masa sistemului. Se arată diferenţele care apar în ceea ce priveşte diagramele de transmisibilitate intâlnite în cazul "clasic" (suport oscilant, model reologic de tip Kelvin-Voigt, masă) şi sistemele care conţin alte modele reologice de legătură dintre suport şi masă.

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