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CONTRIBUTIONS TO THE BIOMECHANICS STUDY OF HUMAN CALF FOOT

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Abstract: *In the paper to make a comprehensive study on the biomechanical behavior of the human leg under the action of vibration. The request is applied through the sole of the foot and the vibrations are considered to act along the leg requested, up to the basin. The second leg is placed on a surface considered fixed, and does not act on his the exciting system. Mechanic System is analysed by integrating in Matlab Simulink of differential equations, which characterizes its dynamics. The results obtained are comparable to those existing in the literature. The work brings their contribution through the originality of the request and the manner of composing material system, forming the human lower leg.*
Key words: bio*mechanical system, human calf foot, vibrations****.***

1. MATHEMATICAL FOUNDATION OF BIOMECHANICAL SYSTEM

In the study of vibration of mechanical systems are made with different simplifying assumptions, which reduce the system to a real analytic model. The number of independent geometrical parameters, specifying the location of a system is the number of degrees of freedom. Even in the case of systems with more degrees of freedom, the movement is reduced to using two models: the model of mechanical translation and the model of rotation motion.

1.1. Biomechanical models of the human body undergoing vibrations

Theoretical models are made up of systems with finite number of freedom degrees, represented mathematically by means of differential equations. These equations describe the behaviour of physical models obtained by replacing system components with idealized elements.

Oscillating systems are made up of elements which accumulates potential energy (springs), elements which accumulates kinetic energy (or mass moments of inertia), and items where there is a continuous dissipation of energy [Tru 10].

2. BIOMECHANICAL MODEL OF THE LOWER LEG

It propuses a biomechanical model of the calf subjected to vibration given by a vibrant platform, as shown in Figure 1.

Fig. 1. The human leg subjected to vibration on a vibrating platform

It propuses a biomechanical model of the calf vibration products subjected to a vibrant platform, as shown in Figure a biomechanical model of the calf vibration products subjected to a vibrant platform, as shown in Figure

The force that a muscle develop in response to environmental conditions to carry out a specific work or to keep the balance of the body is the stimulus that leads to further his development of submission effort called "recovery time".

2.1. Biodynamic model of limb, subjected to vertical vibrations

Mathematical model proposed in this paper wants to do correlation between vibratory stimulus – induced energy in a certain portion of the human body and the force with which the muscles respond to environmental conditions in which the vibrator.

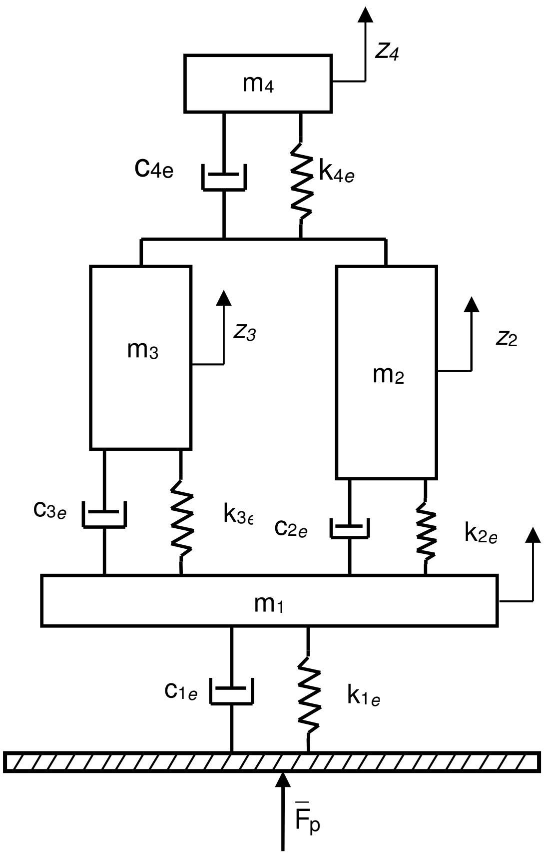

Fig. 2. Biodynamic model of limb, subjected to vertical vibrations

It is proposed the mechanical model (Fig. 2) with 4 degrees of freedom, of the lower limb, subjected to vertical vibration produced by a vibrant platform. This model consists of the following elements:

- m_1 – mass of foot [kg];
- m_2 – mass of the bone structure of the lower leg of the foot (tibia and fibula) [kg];
- m_3 – muscular structure mass of the lower leg of the foot [kg];
- m_4 – visible mass of the part body at the top of the knee (half of the femur mass) [kg];
- c_{1e} - equivalent damping constant of the foot [Ns/m];
- c_{2e} - equivalent damping constant of the bone structure of the lower leg foot (tibia and fibula) [Ns/m];
- c_{3e} - equivalent damping constant of the muscular structure of the lower leg foot [Ns/m];
- c_{4e} - equivalent damping constant of the femur [Ns/m];
- k_{1e} - equivalent elasticity constant of foot [N/m];
- k_{2e} - equivalent elasticity constant of the bone structure of the lower leg foot (tibia and fibula) [N/m];
- k_{3e} - equivalent elasticity constant of the muscular structure of the lower leg foot [N/m];
- k_{4e} - equivalent elasticity constant of the femur [N/m];
- $\bar{F}_p$ - force vector harmonic vibration of vibrating platform [N].

2.2. Initial conditions and the constants values

Harmonic vibration force of vibratory platform has the expression:

$$F = F_0 \cdot \sin(\omega t) \qquad (1)$$

Where:

- F – scalar value of harmonic vibration of vibrating platform [N]; - F_0 – amplitude harmonic vibration force of vibratory platform [N]; - ω – vibration pulsation [rad/s], ω = 2*pi*f=138,16 rad/s; - f – frequency 22Hz; - t – time [s].

Individual values of the masses, elastic constants, and damping constants are given in table 1.

Table 1.
Individual values of the masses, elastic constants, and damping constants [Abb 10]

No. Crt.	Element	m [kg]	k [N/m]	c [Ns/m]
1.	Leg	0.957	120000	850

2.	Bone (tibia+fibula)	0.51	500000	10000
3.	Calf muscles	2.559	80000	1000
4.	Half femur + knee	3.3	130000	2200

Since the elements of the model are connected in series, it is necessary to determine the elastic constants and the equivalent depreciation, between two successive elements. It should be borne in mind that the masses of m_2 and m_3, of the item are parallel linked [Abd,08].

Table 2.

Constants of elasticity and constants of damping corresponding to elements of biodynamic model

No. Crt	Element	k_e [N/m]	c_e [Ns/m]
1.	Leg	120000	850
2.	Bone (tibia + fibula)	320000	459.5
3.	Calf muscles	70000	459.5
4.	Half femur + knee	100000	1047

2.3. Dynamics differential equations for the biomechanical system

Equations of equilibrium for each element of the biomechanical model, as determined by the application of d'Alembert principle.

Mathematical system consisting of differential equations, obtained for each mass of the biodynamic, with 4 degrees of freedom, of lower limb represents the mathematical model corresponding to this model.

The system of differential equations for the biomechanical model, given in the figure 2 is the following:

The system of the dynamical differential equations is:

$$\begin{cases} m_1\ddot{z}_1 - c_{2e}(\dot{z}_2 - \dot{z}_1) - c_{3e}(\dot{z}_3 - \dot{z}_1) - k_{2e}(z_2 - z_1) - k_{3e}(z_3 - z_1) = F_p \\ m_2\ddot{z}_2 - c_{2e}(\dot{z}_2 - \dot{z}_1) - c_{4e}(\dot{z}_4 - \dot{z}_2) - k_{2e}(z_2 - z_1) - k_{4e}(z_4 - z_1) = 0 \\ m_3\ddot{z}_3 - c_{3e}(\dot{z}_3 - \dot{z}_1) - c_{4e}(\dot{z}_4 - \dot{z}_3) - k_{3e}(z_3 - z_1) - k_{4e}(z_4 - z_1) = 0 \\ m_4\ddot{z}_4 - c_{4e}(\dot{z}_4 - \dot{z}_3) - c_{4e}(\dot{z}_4 - \dot{z}_2) - k_{4e}(z_4 - z_3) - k_{4e}(z_4 - z_2) = 0 \end{cases} \quad (2)$$

Matrix form writing system has the expression:

$$[M]\cdot\{\ddot{Z}\} + [C]\cdot\{\dot{Z}\} + [K]\cdot\{Z\} = \{P\} \quad (3)$$

In the relation (3) the matrices are:

- ✓ Inertia matrix

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \quad (4)$$

- ✓ Accelerations vector, velocities vector, displacements vector and active force are:

$$\{\ddot{Z}\} = \begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \end{bmatrix} \quad \{\dot{Z}\} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \end{bmatrix} \quad \{Z\} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} \quad F_P = \begin{bmatrix} Z_p \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

- ✓ Damping matrix:

$$[C] = \begin{bmatrix} -(C_{2e} + C_{3e}) & C_{2e} & C_{3e} & 0 \\ C_{2e} & -(C_{2e} + C_{4e}) & 0 & C_{4e} \\ C_{3e} & 0 & -(C_{3e} + C_{4e}) & C_{4e} \\ 0 & C_{4e} & C_{4e} & -2C_{4e} \end{bmatrix} \quad (6)$$

- ✓ Rigidity matrix:

$$[K] = \begin{bmatrix} -(K_{2e} + K_{3e}) & K_{2e} & K_{3e} & 0 \\ K_{2e} & -(K_{2e} + K_{4e}) & 0 & K_{4e} \\ K_{3e} & 0 & -(K_{3e} + K_{4e}) & K_{4e} \\ 0 & K_{4e} & K_{4e} & -2K_{4e} \end{bmatrix} \quad (7)$$

3. THE SOLUTION OF DIFFERENTIAL EQUATIONS SYSTEM

For numerical solving of differential equations system, determine the analytical expressions of masses accelerations of the 4 freedom degrees, through the successive integration with Runge-Kutta method of 4.5 order - using Math Lab Simulink programming environment, it may cause movements, accelerations and velocities vibrations masses of biodynamic model.

The input signal is determined by the F_p, and it has two sinusoidal components, namely:

- Elastique component having the expression $z_p k_{1e} \sin\omega t = 10^{-3} * 1.2 * 10^5 \sin\omega t = 120 \sin\omega t$

- Damping Component with expression $z_p c_{1e} \omega$ cosωt $=10^{-3}$* 850*138.16 cosωt=117 sinωt

The system (2) becomes successively:

$$\begin{cases} \ddot{z}_1 = \frac{1}{m_1}\left(F_p - (c_{2e} + c_{3e})\dot{z}_1 + c_{2e}\dot{z}_2 + c_{3e}\dot{z}_3 - (k_{2e} + k_{3e})z_1 + k_{2e}z_2 + k_{3e}z_3\right) \\ \ddot{z}_2 = \frac{1}{m_2}(c_{2e}\dot{z}_1 - (c_{2e} + c_{4e})\dot{z}_2 + c_{4e}\dot{z}_4 + k_{2e}z_1 - (k_{2e} + k_{4e})z_2 + k_{4e}z_4) \\ \ddot{z}_3 = \frac{1}{m_3}(c_{3e}\dot{z}_1 - (c_{3e} + c_{4e})\dot{z}_3 + c_{4e}\dot{z}_4 + k_{3e}z_1 - (k_{3e} + k_{4e})z_3 + k_{4e}z_4) \\ \ddot{z}_4 = \frac{1}{m_4}(c_{4e}\dot{z}_2 + c_{4e}\dot{z}_3 - 2c_{4e}\dot{z}_4 + k_{4e}z_2 + k_{4e}z_3 - 2k_{4e}z_4) \end{cases} \quad (8)$$

$$\begin{cases} \ddot{z}_1 = 1.05\left(F_p - 1.31\dot{z}_1 + 0.46\dot{z}_2 - 440z_1 + 320z_2 + 70z_3\right) \\ \ddot{z}_2 = 1.96(0.46\dot{z}_1 - 1.5\dot{z}_2 + 1.047\dot{z}_4 + 320z_1 - 420z_2 + 100z_4) \\ \ddot{z}_3 = 0.39(0.46\dot{z}_1 - 1.5\dot{z}_3 + 1.047\dot{z}_4 + 70z_1 - 170z_3 + 100z_4) \\ \ddot{z}_4 = 0.30(1.047\dot{z}_2 + 1.047\dot{z}_3 - 2.094\dot{z}_4 + 100z_2 + 100z_3 - 200z_4) \end{cases} \quad (9)$$

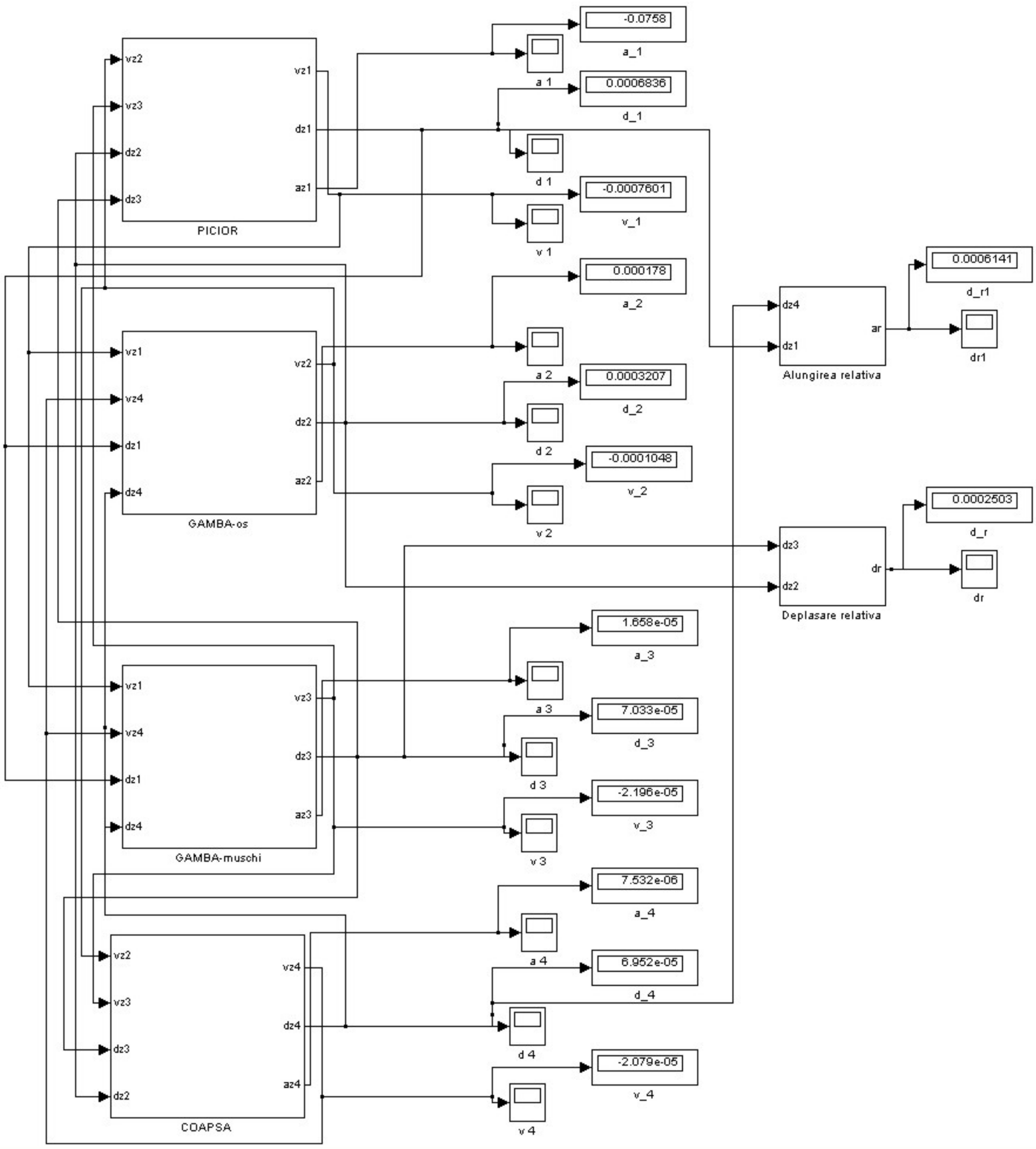

Fig. 3. The system of differential equations in Matlab Simulink

4. RESULTS

It shows in succession for each mass of the biomechanical system, the graphic representations of the displacement, velocity and acceleration.

4.1. The m_1 mass of foot

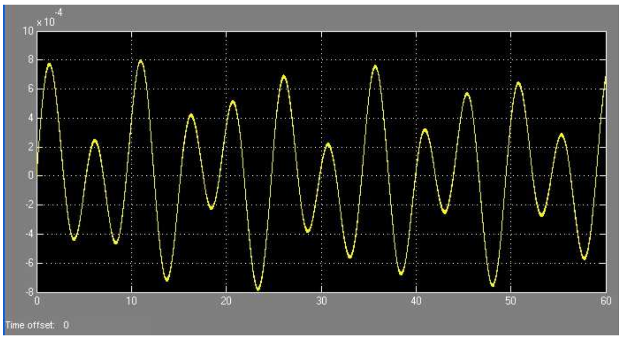

Fig. 4. The displacement of the m_1 mass

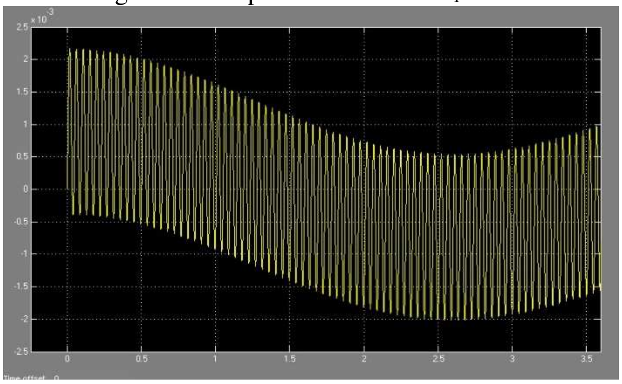

Fig. 5. The velocity of the m_1 mass

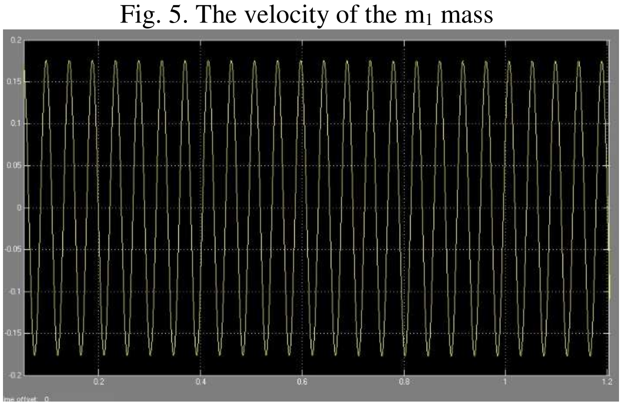

Fig. 6. The acceleration of the m_1 mass

The m_1 mass has a motion given by the vibrating platform. It is characterized by a displacement with maximum $0.8*10^{-4}$ m, velocity variable with pulsation magnitude of $2.5*10^{-3}$m/s, and 0.2 m/s acceleration in harmonic variation. This part of the leg is linked with the vibrating platform, and has a small damping.

4.2. The m_2 mass of the bone structure of the lower leg (tibia and fibula)

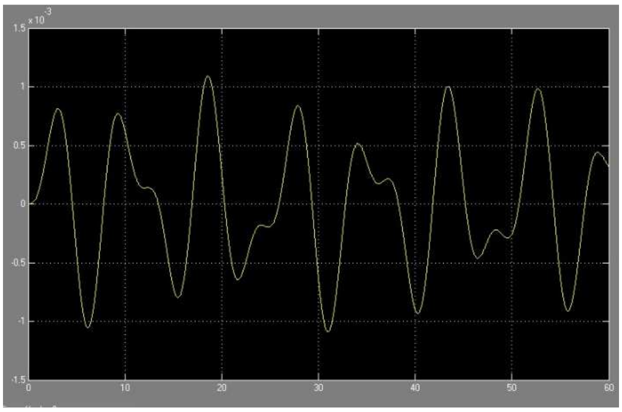

Fig. 7. The displacement of the m_2 mass

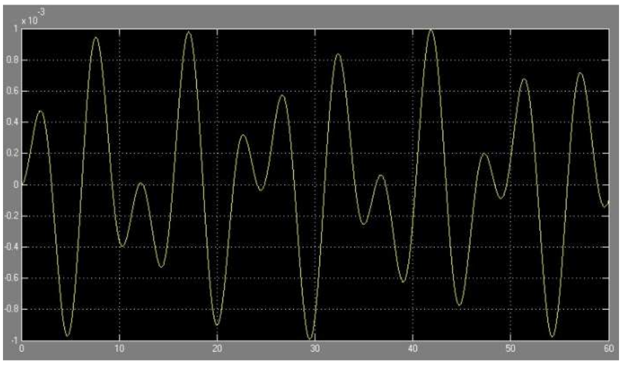

Fig. 8. The velocity of the m_2 mass

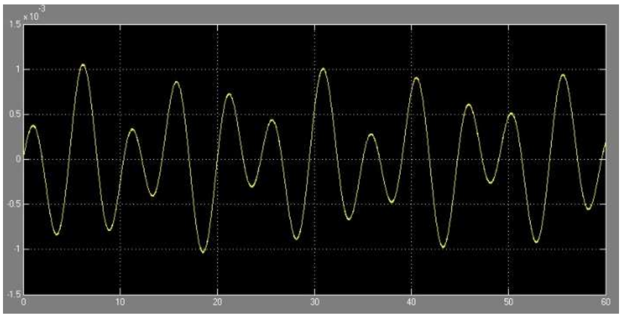

Fig. 9. The acceleration of the m_2 mass

In the figures 7, 8, and 9 there are the kinematics characteristics of the m_2 mass of the bones (tibia and fibula) of the lower leg. They give the time variation laws for the displacement (maximum at$1.1*10^{-3}$m), velocity (maximum at $1*10^{-3}$m/s), and acceleration ($1.1*10^{-3}$m/s^2).

4.3. The m_3 mass of the muscular structure of the lower leg

The muscular structure of the lower leg of the foot is presented in time variation law for the displacement, velocity, and acceleration.

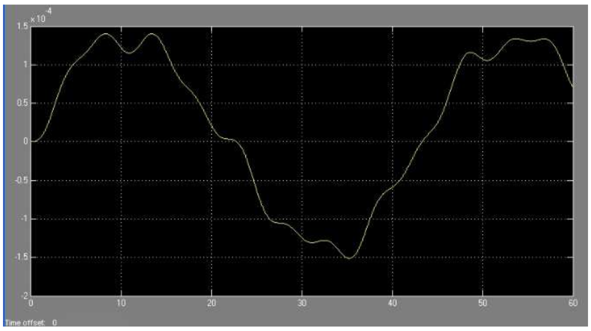

Fig. 10. The displacement of the m_3 mass

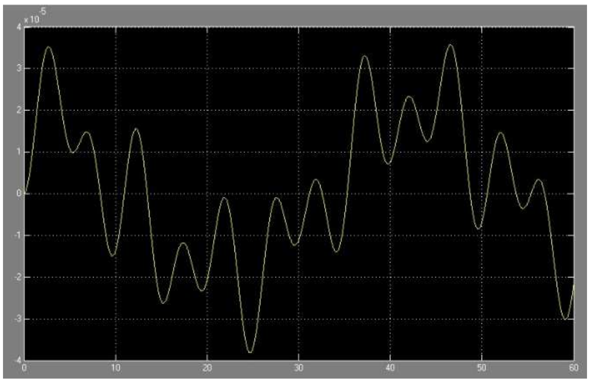

Fig. 11. The velocity of the m_3 mass

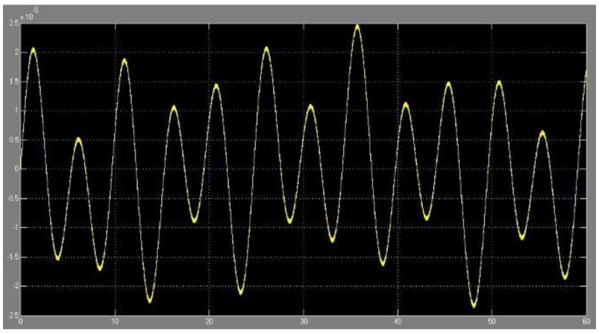

Fig. 12. The acceleration of the m_3 mass

The representations in the figures 10, 11, and 12 give us a diminish motion of the m_3 mass as the m_2 mass. That means the muscular structure has a greater dumping as the bone structure.

4.4. The m_4 mass of the half femur + knee

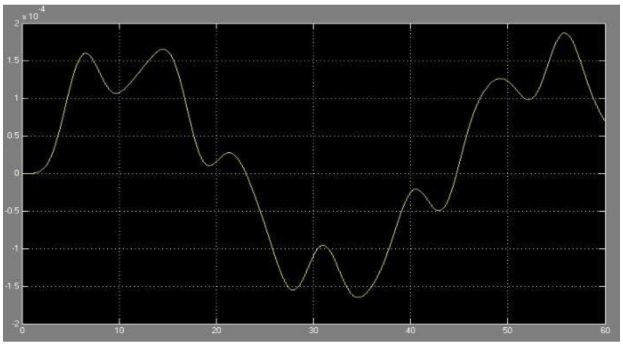

Fig. 13. The displacement of the m_4 mass

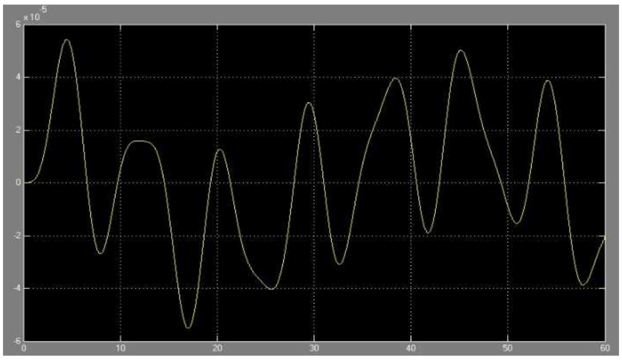

Fig. 14. The velocity of the m_4 mass

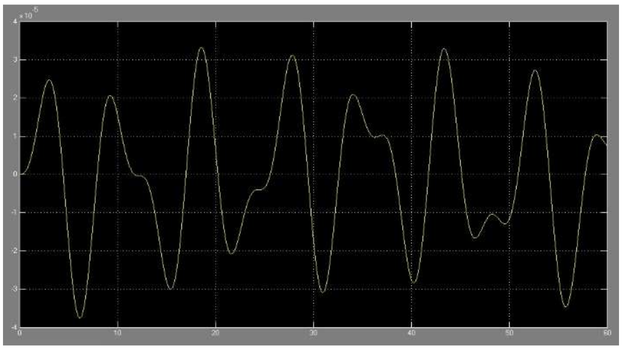

Fig. 15. The acceleration of the m_4 mass

The m_4 mass of the half thigh and knee is the sum of m_2 and m_3 masses motions, but its kinematics characteristics are lower there summed.

4.5. Relative displacement between the bone and muscle of the lower leg

The study makes an evaluation of the relative displacement between the bone and the muscle of the lower leg, and it is present in following figure.

Fig. 16. The relative displacement between the bone and the muscle of the lower leg

It can be observed that, the displacement between the components of the human lower leg is in order of $(-1.2+1.05)*10^{-3}$m, and it can not disturb the linkage between them.

4.6. Relative elongation of the calf muscle

For the calf muscle is important to know, what is the relative elongation regarding the bone of the lower leg. It is presented in the figure 17 the relative elongation of the calf muscle under the action of the vibrating motion produced by a vibrating platform.

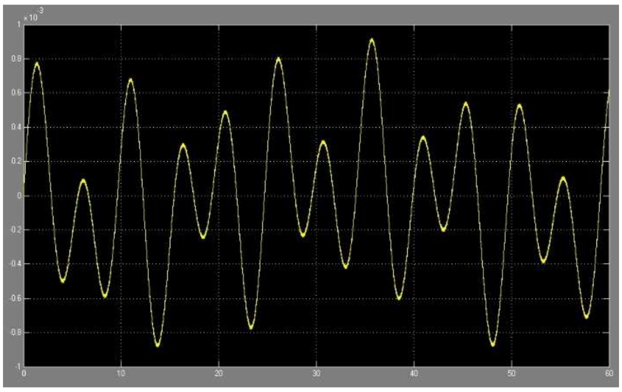

Fig. 17. The relative elongation of the calf muscle

The relative elongation under the vibration action is lower 1 mm, and our study demonstrated that it is not harmful to the human body.

5. CONCLUSIONS

At the base of theoretical grounding of mechanical vibration on the human body stays prior knowledge of morphological and functional structure of the human body. Mathematical modeling of biomechanical model of human body takes into account the reference planes and axes. With the help of the indications contained in the literature regarding the density of various segments of the human body, they can determine the masses of biomechanical model body segments, and the dimensions of the various parts of the human body, can be calculated by applying a constant of proportionality, knowing the height of the human subject.

The existing research literature offers basic principles concerning mechanical and mathematical modeling of the human body, subjected to mechanical vibration.

With the help of the program Matlab Simulink were determined the scale values of displacements, velocities and accelerations in vertical direction vibration and resulted parameters.

From the simulation results it is observed that the leg movements are relatively equal level with vibrant plate movement, which is normal because it is at the level of transmissibility of the lower leg.

Muscle fiber elongation calculated as the difference between the Centre of mass m_1 and m_4 has two components:

- a major component that corresponds to a large displacements in order of 10^{-3}, in our case, the parameters given maximum value reaches 0.9mm, but varies from a low frequency, approximately 1.88 rad/sec;
- a minor component corresponding to a small order of 10^{-6}, in the present case the elongation of peak to peak maximum is 20 µm, but have a major impact on muscle response to mechanical vibration. The perception of time in case of sudden muscular elongation of the muscle fiber is 5-15 µm. It should be noted that time is muscle and an acceleration sensor, being more sensitive in case of sudden elongation of the muscle fiber (Fig. 18).

A hypothesis what was born as a result of the results is that the force developed by the muscle response to vibrating stimulus varies over time in proportion to the major component of artificial muscle fiber.

6. BIBLIOGRAPHY

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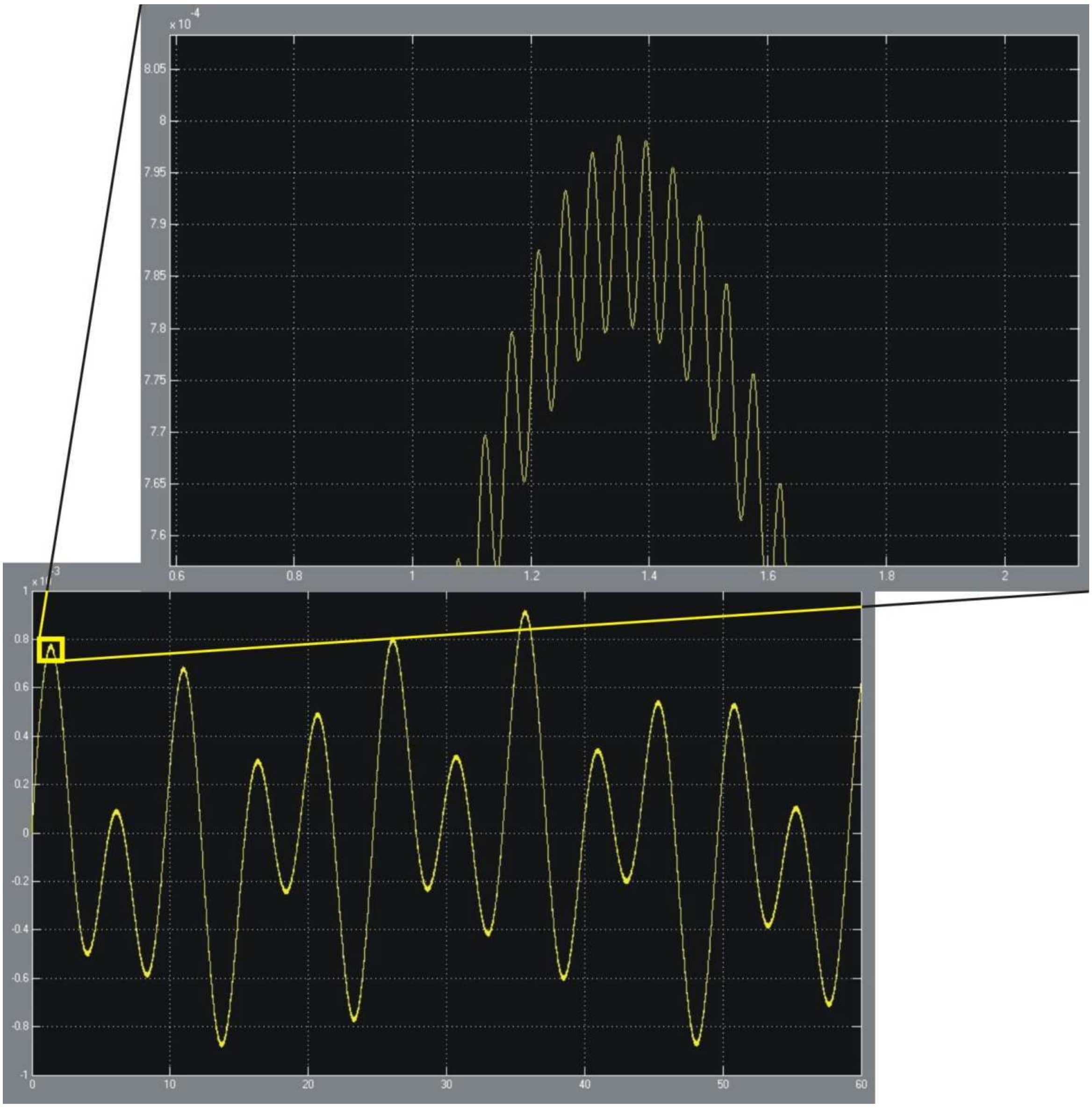

Fig. 18. Major and minor components of the displacement between m_1 and m_4 masses

Contribuţii la studiul biomecanic al gambei piciorului uman

Rezumat: *In lucrare se face un studiu complex, biomecanic asupra comportarii gambei piciorului uman sub actiunea vibratiilor. Solicitarea se aplica prin talpa piciorului si se considera ca actioneaza de-a lungul piciorului solicitat, pana la bazin. Cel de al doilea picior se considera asezat pe o suprafata fixa, iar asupra lui nu actioneaza sistemul excitant. Sistemul mecanic considerat este analizat prin integrarea in MatLab Simulinc a sistemului de ecuatii diferentiale, ce caracterizeaza dinamica acestuia. Rezultatele obtinute sunt comparabile cu cele existente in literatura de specialitate. Lucrarea isi aduce contributia prin originalitatea solicitarii si a modului de compunere a sistemului material, ce formeaza gamba piciorului uman.*

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