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A DESIGN METHOD OF THE CYLINDRICAL WORM GEARS

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Abstract: *The technical literature doesn't give a unique method for choosing the module and the diameter factor in case of cylindrical worm gears. This paper gives a method of choosing the worm diameter factor with the help of a Matlab application, while the module is computed automatically in order to keep all necessary fulfillments related to the strength and the rigidity. Results are given so that the designer can select the diameter factor that gives the best efficiency of the worm gear.*

Key words: *cylindrical worm gears, module, diameter factor, worm threads, addendum modification.*

1. INTRODUCTION

When designing cylindrical worm gears an important parameter is the worm diameter factor (q). This determines the worm stiffness, efficiency of the teeth meshing, auto-breaking condition, consumption of materials and others. Worm gears with high efficiency can be used in designing parallel robots ([1], [2]) this paper gives a solution that meets these demands.

2. EFFICIENCY OF THE WORM GEAR

To obtain a good efficiency (η_z) the diameter factor must have a low value. The module m, must be determined so that the worm has a good stiffness and the right contact pressure in operation. Considering the number of the worm threads (z_1), the diameter factor, the friction coefficient between the teeth flanks (μ_z), the teeth efficiency is given by:

$$\eta_z = \frac{z_1(q - z_1\mu_z)}{q(z_1 + g\mu_z)} \quad (1)$$

3. DETERMINATION OF THE MODULE FOR THE WORM GEAR

The condition for the worm shaft not to bend in operation is given, in the technical literature [3], by the following expression:

$$\delta_m = 2\cdot 10^{-6} l_1^3 F_{tm2} \frac{\sqrt{\tan^2(\alpha_x) + \tan^2(\gamma_m + \rho_z)}}{d_{m1}^4} \le \delta_{\lim} \quad (2)$$

where:

l_1 is the distance between the supporting bearing of the worm shaft considered to be arranged symmetrically to the middle and given in [mm];

F_{tm2} is the tangential force on the worm wheel in [N];

α_x is the spiral profile angle in axial section;

γ_m is the worm spiral slope angle on the division cylinder;

ρ_z is the angle of friction;

d_{m1} is the worm division diameter in [mm]

δ_{lim} is the admissible bending deformation ($\delta_{lim} \approx 0.004\cdot m$ at hardened worm and $\delta_{lim} \approx 0.01\cdot m$ at improved worm);

Expression (2) can be computed using the following [4]:

$$l_1 \approx 1.5\cdot a = 1.5\frac{d_{m1} + d_{m2}}{2} = 1.5\frac{m_1(z_2 + q + 2x)}{2} \quad (3)$$

$$F_{tm2} = \frac{2000\cdot T_2}{m(z_2 + 2x)} \quad (4)$$

$$d_{m1} = mq \qquad (5)$$

$$\tan(\gamma_m) = \frac{z_1}{q} \qquad (6)$$

$$\tan(\alpha_x) = \frac{\tan(\alpha_n)}{\cos(\gamma_m)} = \tan(20^0)\frac{\sqrt{q^2 + z_1^2}}{q} \qquad (7)$$

$$\tan(\rho_z) = \mu_z \qquad (8)$$

where:
a is the distance between the axes in [mm];
m is the worm module in [mm] measured at axial section;
T_2 is the torque on the axle of the worm wheel in [Nm].

Considering the relations from (3) to (7), for the hardened worm, from (2), the axial module can be determined using the following relation:

$$m \geq (z_2 + q + 2x) \times \left[\frac{T_2^2}{5.62(z_2 + 2x)^2 q^8}\right]^{1/6} \times \left[\frac{q^2 + z_1^2}{q^2}\tan^2(20^0) + \frac{(z_1 + q\mu_z)^2}{(q - z_1\mu_z)^2}\right]^{1/6} \qquad (9)$$

4. THE UNDERCUTTING CHECK

To avoid the undercutting phenomena the number of the teeth for the worm wheel must satisfy the following condition:

$$z_2 \geq \frac{2(h_a^* + c^* - x)[q^2 + (q^2 + z_1^2)\tan(20^0)]}{(q^2 + z_1^2)\tan(20^0)} \qquad (10)$$

where:
h_a^* is the tooth head height coefficient;
$c^* = 0.25$ is the radial clearance coefficient;
x is the addendum modification.

5. THE CONTACT PRESSURE CHECK

The obtained parameters of the gear must also be checked against the contact pressure condition [3] so:

$$S_H = \frac{\sigma_{H\lim} Z_h Z_n \sqrt{m^3 (z_2 + q + 2x)^3}}{Z_E Z_\rho \sqrt{8000 T_2 K_A}} \geq 1.1 \cdots 1.5 \qquad (11)$$

where:
$\sigma_{H\lim}$ is the stress limit at contact pressure;
Z_h is the lifetime factor given by

$$Z_h = \left(\frac{2500}{L_h}\right)^{1/6} < 1.6 \qquad (12),$$

with L_h, the operating time in hours;
Z_n is the load substitution factor given by

$$Z_n = \left(\frac{1}{\frac{n_2}{8} + 1}\right)^{1/8} \qquad (13)$$

with n_2, the rotational speed of the worm, in [min^{-1}];
Z_E is the material factor given by

$$Z_E = \sqrt{\frac{1}{\pi\left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right)}} \qquad (14)$$

with ν_1 and ν_2 the Poisson coefficients for the material of the worm and of the wheel (ν_1=0.30 for the steel worm and ν_2 = 0.35 for the bronze worm); E_1 and E_2 are the elastic moduli of the worm and the wheel ($E_1 = 2.1 \times 10^5$N/mm^2 for steel worm and $E_2 = 0.883 \times 10^5$N/mm^2 for the bronze CuSn12 wheel);
Z_ρ is the contact factor;
K_a is the operating conditions factor.

6. CODE AND NUMERICAL RESULTS

Using the Matlab computing environment the basic geometrical dimensions of the worm gears are computed with the following input data: z_1={1, 2, 3, 4}, z_2=50, n_1=1500 min^{-1}, μ_z=0.08, T_2=430 Nm, $\sigma_{H\lim}$=256 N/mm^2, Z_E=145 (N/mm^2)$^{1/2}$, Z_h=1.6, K_a=1, Z_ρ=3. The z_1 variation is between 1 and 4 with step 1, the q variation is between 4 and 20 with step 1 and the *x* addendum modification variation is between -1 and 1.

The Matlab code is using two *for* cycles, one with the z_1 counter variable and the second, inside the first *for*, with the *q* counter variable. The code from the second *for* is testing the (10) and (11) expressions eliminating the *q* values that are not satisfying the test expressions. For the computed value of *m* from (9), if the test expressions are satisfied, the η_z and the *q* values are stored in vectors - as the interior *for* cycle, controlled by *q*, varies faster that the exterior one, controlled by z_1 - and plotted as part of a single curve. For each execution of the exterior cycle we obtain a curve that is plotted, with the help of the *hold* Matlab function, on the same graph with the previous plots. The pseudocode of the program is given below. Lines beginning with two consecutive slashed are comments. Where it was possible the Matlab programming language syntax and function names were kept.

```
//initialize constants

//the step for z1 is 1
for z1=1:4
 //c counts the valid points
 c=0;

 //qp and ep vectors are defined to store the
 //valid results
 //initialize vectors
 qp=zeros(1,2);
 ep=zeros(1,2);

 //the step for q is 1
 for q=4:20
  //the results of the test are stored in
  //two variables: z2test and shtest
  m = from (9)
  z2test = test expression (10)
  shtest = test expression (11)
  compute eta (1)

  if (z2test is True) and (shtest is True)
   //see Table 1 and 2 for results samples
   print q, m, eta, z2test, shtest
   //store the valid point
   qp(c) = q
   ep(c) = eta
   //prepare to store the next valid point
   //by incrementing the c counter
   c=c+1
   end //end of if

  end //end of q for

  //prepare the labels for the axes
  xlabel('q')           //q
  ylabel(texlabel('eta'))//η
  //plot the rezults
  plot(qp',ep')
  //hold from Matlab is used as all the data
  //should be drawn on a single plot, as
  //distinct curves in order to compare
  //the results
  hold on
end //end of z1 for
```

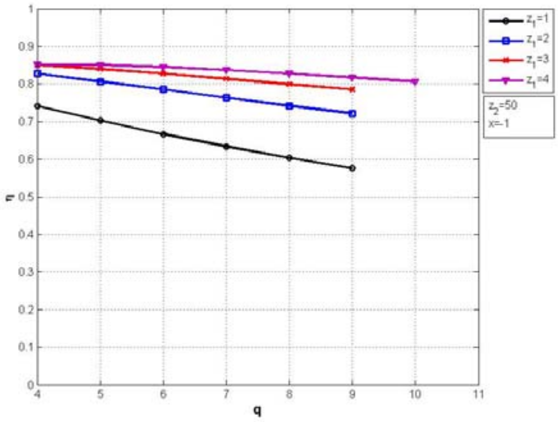

Fig. 1. Variation of *η* with respect of *q* for different z_1 values (x=-1).

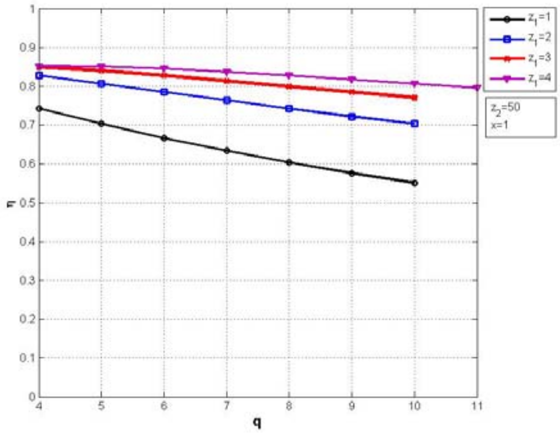

Fig. 2. Variation of *η* with respect of *q* for different z_1 values (x=1).

Table 1

The *q* and *m* results given by the program (*x*=1, z_1=1, z_2=50).

q	*m*	η_z	z2test	shtest
4	10.64	0.74242	4.92604	6.29540
5	7.84	0.70286	5.00289	4.09240
6	6.16	0.66667	5.04605	2.92291
7	5.04	0.63370	5.07257	2.22355
8	4.26	0.60366	5.08999	1.76944
9	3.68	0.57623	5.10203	1.45629
10	3.23	0.55111	5.11069	1.23022
11	2.88	0.52805	5.11712	1.06104
12	2.60	0.50680	5.12203	0.93070
13	2.37	0.48718	5.12585	0.82786
14	2.17	0.46900	5.12890	0.74509
15	2.01	0.45212	5.13135	0.67733
16	1.87	0.43640	5.13337	0.62105
17	1.74	0.42173	5.13504	0.57371
18	1.64	0.40801	5.13644	0.53347
19	1.54	0.39515	5.13763	0.49891
20	1.46	0.38308	5.13864	0.46898

Table 2

The q and m results given by the program (x=1, z_1=4, z_2=50).

q	m	η_z	z2test	shtest
4	14.52	0.85185	3.26090	8.62454
5	10.27	0.85091	3.67515	5.26660
6	7.79	0.84524	3.98673	3.57228
7	6.20	0.83709	4.21899	2.60362
8	5.11	0.82759	4.39319	1.99916
9	4.33	0.81733	4.52549	1.59672
10	3.74	0.80667	4.62746	1.31507
11	3.29	0.79583	4.70726	1.11000
12	2.93	0.78495	4.77062	0.95581
13	2.64	0.77411	4.82163	0.83675
14	2.40	0.76339	4.86320	0.74275
15	2.20	0.75282	4.89748	0.66711
16	2.03	0.74242	4.92604	0.60525
17	1.89	0.73222	4.95006	0.55394
18	1.76	0.72222	4.97044	0.51085
19	1.65	0.71243	4.98787	0.47426
20	1.56	0.70286	5.00289	0.44290

As shown in the plots from Fig. 1 and Fig. 2, the effect of x addendum modification variation is that of limitation of the solutions that are passing the test conditions. As the value of x goes higher more solutions are found.

An example of the m and q values for the worm gear computed by the program is given in Table1 and Table 2. As z_2=50 all the values from the '**z2test**' column are passing the test from (10).

However, the shaded row results from Table 1 and Table 2 are not passing the test given at (11). That is the values from the '**shtest**' column are under then 1.1, the lowest admitted value, so the contact pressure conditions are not checked and these rows will be eliminated from any further computations.

7. CONCLUSIONS

The design method given in the paper concerning the worm gears main parameters gives a method of computing the m module based on a chosen value of the q diameter factor. The technical literature doesn't give a method for choosing these parameters and in most cases arbitrary values are used. The q value can be selected so that the efficiency of the worm gear is highest, while the strength conditions are satisfied and the worm gear work conditions are appropriate. The specific addendum modification must have positive values in order to obtain a lager domain of geometrical dimensions.

The numerical results from Table 1 and Table 2 are proving that a low value of q and a high value of m is required to have high efficiency worm gears. These favorable conditions are however badly influenced by the standardization used over the q values.

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O metoda de calcul pentru angrenajele cilindrice melcate

Rezumat: În literatura de specialitate nu apare o modalitate unică de alegere a modului şi a coeficientului diametral În cazul angrenajelor melcate cilindrice. Lucrarea dă o metodă de alegere a coeficientului diametral a angrenajului melcat cilindric modulul rezultând ca urmare a satisfacerii condiţiilor În funcţionare, legate de rigiditate şi de rezistenţă la presiune de contact.

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