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CONTRIBUTIONS TO THE DYNAMICAL STUDY OF TRAVELLING LOAD ON AN ELASTIC BEAM

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Abstract: *In this paper there are studied two mechanical systems, containing an elastic element bar type, leaning to both ends, in the case of the first a mass is moving on the elastic beam with constant or variable speed and on the second case on the travelling mass there is a pendulum bound to it. In both cases it is determined the vertical movements of the mass that moves on the elastic beam according to different parameters. The obtained results can be used to improve the dynamic calculation which must be done in the construction of bridges, cranes and to the mobile elements of serial robots.*

Key words: *simultaneous nonlinear differential equations, Lagrange's equation, Runge-Kutta methods, traveling cranes, bridges, serial robots, programming in C.*

1. INTRODUCTION

On the 24th of May 1847 the bridge across the river Dee near Chester town–England collapsed when a passenger train was crossing on it and five people died due to this accident. The following investigation established that there were more reasons for that accident: the considered safety coefficient was too small, the metallic structure was made from cast steel (resistant to compressive but not to tension or variable stresses), the third cause was the different behavior of the metallic structure when a train is crossing over with speed or it stops.

This unfortunate accident was one of the factors that showed the necessity of the study of the dynamic mechanical system formed from an elastic beam, with negligible mass, and a load that moves on it, usually with a constant speed, the problem being during the time in the attention of many scientists.

Such a problem was imposed by practice, in the construction of bridges with a big lengths, in different type of cranes, the structure have to be verified both for static loads andthe dynamic ones, generated by the movement of some vehicle on it (the case of the bridges) or some weight that is moving (the case of the cranes and of the serial robots elastic elements).

Theoretical representation of some of these problems can be found in the monographic works [4], [6] and so on , in some chapters or paragraphs of some papers [5], [8], [9], [15] a. o. and the treatment of some particular cases is presented in many papers, e. g. [1], [2], [7], [11], [13], [14], [20] a. o.

In the present paper the authors perform a theoretical study followed by a series of numerical solving concerning the movement of a load on an elastic beam with negligible weight, with constant or variable speed. As variable elements there were considered: speed value (corresponding to the travelling portion with constant speed) and the time of linear variation of speed.

Also, the authors were solved another problem of great theoretical interest: the case in which a load moves on an elastic beam, linked with a pendulum which can make oscillations in the vertical plane containing the elastic beam. We obtained an extremely complex mathematic model, established using Lagrange’s equations [5], [15], [18], [19] and so on.

The numerical results are obtained with a C program, written by authors, for the numerical solving of the simultaneous differential equation being using the Runge-Kutta methods [12], [16], [18].

2. THEORETICAL BACKGROUND AND NUMERICAL RESULTS

Case 1. On an elastic beam, with d length, with negligible weight, supported to both ends, a load M+m moves with a negligible speed. The rigidity of the beam is EI. In this case, in a point of abscissa x where a force P acts (figure 1)the deformation of the beam in that point is calculated with the formula :

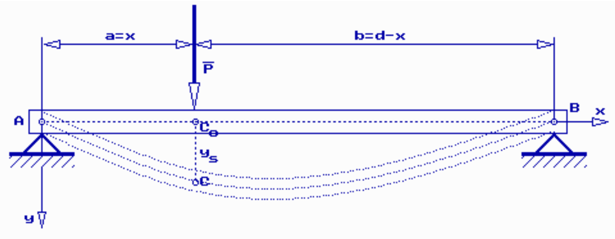

Fig. 1 The elastic beam and a force P

$$y=\frac{Px^2(d-x)^2}{3EId} \tag{1}$$

Because P is the weight of the mass, the formula becomes:

$$y=\frac{(M+m)gx^2(d-x)^2}{3\ EId} \tag{2}$$

Considering that the movement of the load M+m starts in the abscissa point $x_1$and it ends in the abscissa point x_4, we can calculate the covered trajectory, utilizing the formula (2).

We must notice the fact that the reaction N, between the load and the elastic beam, is equal in this case, permanently, with the weight force (M+m)g.

The equations of the deformed beam are the following ([10] pg. 224-226).

$$y=\frac{Pb}{6EId}\left[-x^2+a(a+2b)\right]x\ \ ,\ x\in[0;a]$$

(x is measured from the beam left end)

$$y=\frac{Pa}{6EId}\left[-x^2+b(2a+b)\right]x\ \ ,\ x\in[0;b]$$

(x is measured from the beam right end)

In the figure 2 are shown (with dotted line) some diagrams of deformed beam corresponding to the situations when the weight position P=(M+m)g, determined by the abscissa is 0.1d, 0.3d, 0.5d, 0.7d and 0.9d, as well as the position of the application point of the weight force P when these weight moves along the elastic beam, with a negligible speed (the symmetric diagram with thick line).

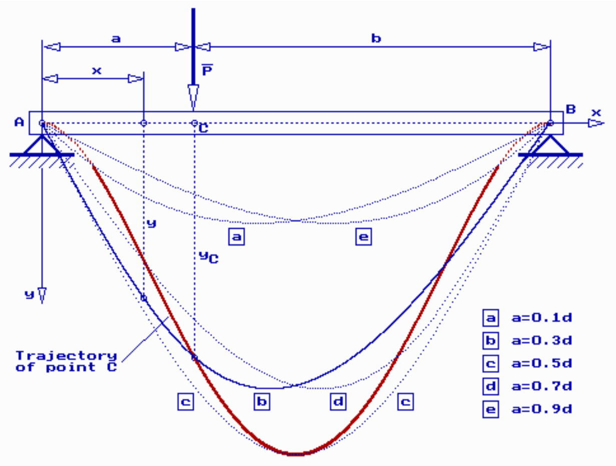

Fig. 2 The different shapes of the elastic beam, depending on the force positions

The numerical data for the diagram were: d=a+b=10.0 [m], P=(M+m)g=800x9.81 [kg·m·s^{-2}], EI=1.0x107 [kg·m^4·s^{-2}]. The maximal deformation of the beam, when P acts in the middle of it is 0.01635 [m].

To obtain the drown diagrams from the figure, highlighting the bending deformations there have been used different scales to represent the horizontal and vertical lengths, their report being 1:400.

Next, knowing the travelled trajectory of the application point of the weight which moves on the elastic beam, presents an interest in the determination of the description of the same point in the case when the weight moves on the elastic beam with a certain speed.

Case 2. The movement on the elastic beam of the load M+m is made with the speed v(t) = x(t), constant or variable (figure 3).

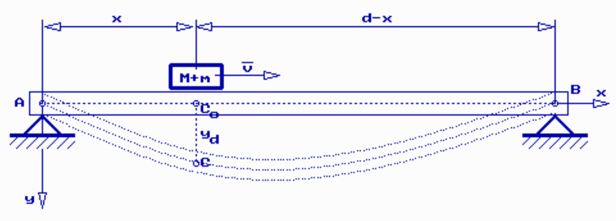

Fig. 3 The elastic beam and a moving load

In this case the reaction N is variable, depending on time. The value of reaction is computed as follows

$$N=\frac{3EIdy}{x^2(d-x)^2} \tag{3}$$

where x is calculated with the formula

$$x=x_1+\begin{cases} v_0\,t & (\text{constant speed}) \\ \int_0^t v(\xi)\,d\xi & (\text{variable speed}) \end{cases} \tag{4}$$

The differential equation of the movement of the load M+m on the vertical direction can be written based on Newton's law,

$$(M+m)\ddot{y}=(M+m)g-N$$

and considering the relations (3) and (4), the differential equation will be:

$$(M+m)\ddot{y}=(M+m)g-\frac{3EIdy}{(x_1+x)^2(d-x)^2}$$

or

$$\ddot{y}+\frac{3EId}{(M+m)\,x^2(d-x)^2}\,y=g \tag{5}$$

obtaining a linear differential equation, with variable coefficient, which can be solved only with a numerical method.

For the differential equation (5) solving it must be added the initial conditions,

$$t=0,\quad \begin{cases} y=\dfrac{(M+m)g\,x_1^2(d-x_1)^2}{3EId} \\ \dot{y}=0 \end{cases} \tag{6}$$

In the case of numerical solving of the differential equations (6), of order two, it must be written in form of a system of two differential equations of first order, to which we can get after the variable changes

$$y=z_1,\quad \dot{y}=z_2 \tag{7}$$

resulting:

$$\begin{cases} \dot{z}_1=z_2 \\ \dot{z}_2=g-\dfrac{3EId}{(M+m)\,x^2(d-x)^2}\,z_1 \end{cases} \tag{8}$$

with the initial conditions

$$t=0,\quad \begin{cases} z_1=\dfrac{(M+m)g\,x_1^2(d-x_1)^2}{3EId} \\ z_2=0 \end{cases} \tag{9}$$

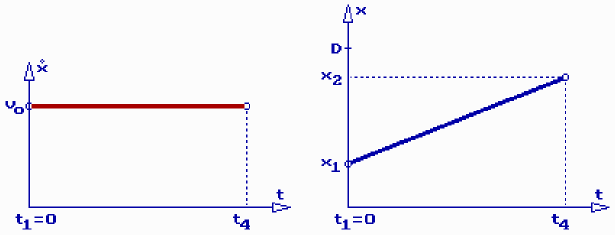

Fig. 4 Diagrams of constant speed and corresponding displacement

In the case when the speed is constant (figure 4) the traveled space is calculated with the formula $x=x_1+v_0t$. The load starts moving from the determined position by $x=x_1>0$ and it ends in the position $x=x_4<d$, so it must to be fulfilled the condition

$$x_4=x_1+v_0t_4<d,\qquad v_0t_4<d-x_1 \tag{10}$$

The system of differential equations (8) with the initial conditions (9) was solved numerically, determining the deformations y_d of the beam produced as an effect of the moving of the load on it and the bending deformation of the beam under the action of the load, fixed, were noted with y_s ([16], [18]). In the figures 5~9 were represented the differences y_d-y_s of these deformations, considering different moving speeds of the load on the elastic beam.

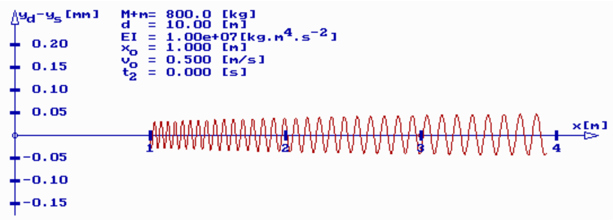

Fig. 5 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0=0.50 [m/s]

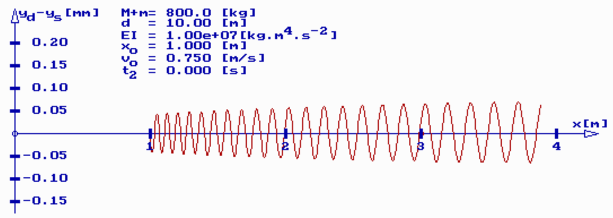

Fig. 6 Diagram of the deflections $y_d - y_s$when the constant mass speed is v_0=0.75 [m/s]

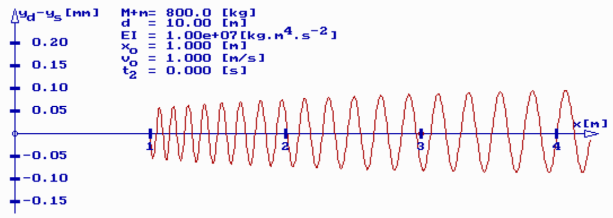

Fig. 7 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0=1.25 [m/s]

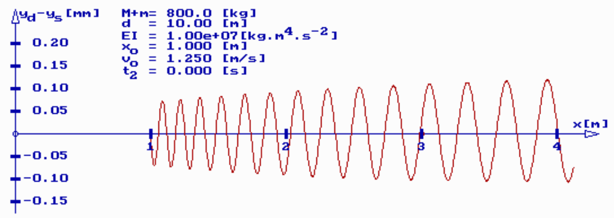

Fig. 8 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0= 1.00 [m/s]

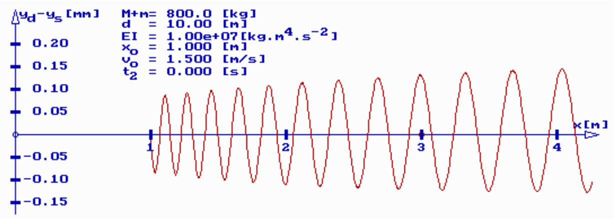

Fig. 9 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0= 1.5 [m/s]

The trajectory described by the mass during the movement on the elastic beam with constant speed (v_0=1.5 [m/s]) and the shape of static deflection are presented in figure 10.

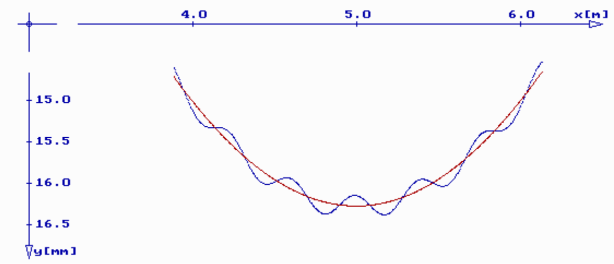

Fig. 10 The trajectory of the mass during thedisplacement on the elastic beam and the static deflections (the displacement speed is v_0 =1.5 [m/s])

The last case, closer to the reality, is the one in which the speed has a variation according to a trapezoidal law (figure 11), the computing of the speeds being made according to the relations

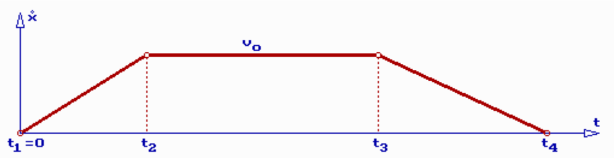

Fig. 11 The speed diagram corresponding to the trapezoidal law

$$\dot{x}(t) = \begin{cases} \dfrac{v_0}{t_2} t & t_1 = 0 \le t \le t_2 \\ v_0 & t_2 < t < t_3 \\ \dfrac{v_0}{t_4 - t_3}(t_4 - t) & t_3 \le t \le t_4 \\ 0 & t > t_4 \end{cases} \qquad (11)$$

As it can be seen in the above image the speed has a linear variation in the [t_1=0; t_2] and [t_3, t_4] intervals and it is constant in the [t_2, t_3] interval. It is interesting to make a study to see how the adopted value for the $t_2$time influences the vertical movementsof the mass of the elastic beam.

The obtained numerical results are presented in the figures 12~15, considering the same travelling speed v_0 =1.5 [m] but different values for time t_2 (0.025, 0.050, 0.075, 0.1 [s]).

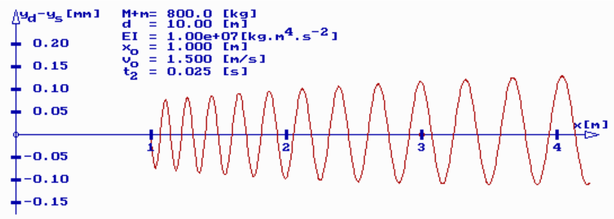

Fig. 12 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0=1.50 [m/s] and t_2 =0.025 [s]

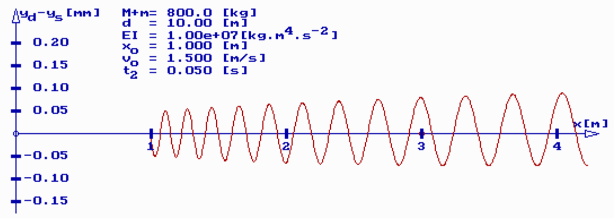

Fig. 13 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0=1.50 [m/s] and t_2 = 0.050 [s]

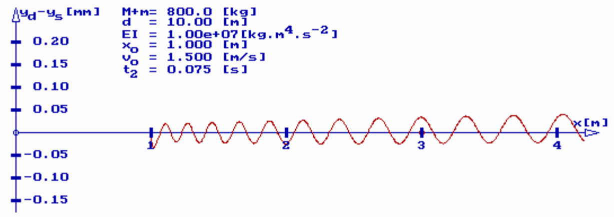

Fig. 14 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0=1.50 [m/s] and t_2 = 0.075 [s]

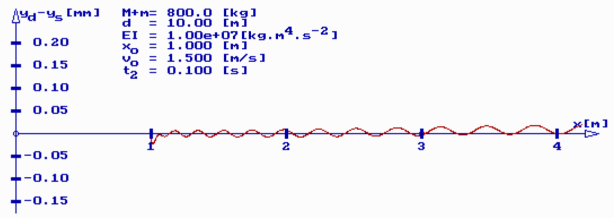

Fig. 15 Diagram of the deflections $y_d - y_s$ when the constant mass speed is v_0=1.50 [m/s] and t_2 = 0.100 [s]

It can be noticed that increasing the time value $t_2$we will obtain a movement in the dynamic regime more closely to the static deformation, the difference y_d– y_s diminishing.

Case 3. On an elastic beam with a negligible weight travels a load M, with a pendulumlinked to it, with the length ℓ and a weight m at its end, the system starting from a resting position (figure 16).

The mechanical system has two degrees of freedom, the generalized coordinates being y (the vertical movement of the load M) and φ (the pendulum angle of oscillation).It is considered that the traveling speed of the M load is variable, being noted with $\dot{x}$.

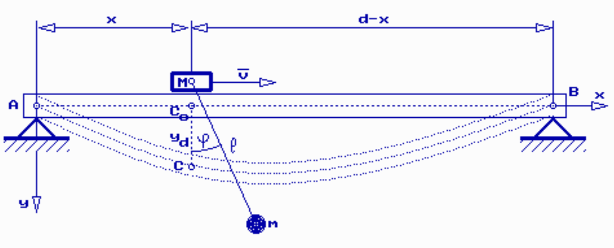

Fig. 16 The elastic beam, moving load and a pendulum

The determination of the differential equation isperformed using Lagrange's equation, these being:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)-\frac{\partial L}{\partial y}=0\ ,\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right)-\frac{\partial L}{\partial \varphi}=0 \qquad (12)$$

with the initial conditions

$$t=0,\ y=\frac{(M+m)g x_1^2 (d-x_1)^2}{3EId},\ \dot{y}=0,\ \ \varphi=0,\ \dot{\varphi}=0 \qquad (13)$$

Lagrange's function has the expression

$$L=E_c-E_p \qquad (14)$$

The kinetic energy E_c is determined by summing the kinetic energies of the masses M and m.

The load M speed components are $\dot{x}$ (on horizontal) and $\dot{y}$ (on vertical) and for determining the load m absolute speed components it must be summed (on the horizontal and vertical) the components of transport and relative speeds.

It is obtained the following expression of the kinetic energy of the mechanical system

$$E_c=\frac{1}{2}M(\dot{x}^2+\dot{y}^2)+\frac{1}{2}m(\dot{x}+\ell\dot{\varphi}\cos\varphi)^2+$$
$$+\frac{1}{2}m(\dot{y}-\ell\dot{\varphi}\sin\varphi)^2$$

For computing the potential energy E_p it is considered the elastic deformation of the beam and the movement on the vertical of the load M and m, having the following expression:

$$E_p=\frac{1}{2}\frac{3EId}{x^2(d-x)^2}y^2-(M+m)gy+$$
$$+mg\ell(1-\cos\varphi)$$

The Lagrange's function is:

$$\begin{aligned}L=&\frac{1}{2}(M+m)(\dot{x}^2+\dot{y}^2)+\frac{1}{2}m\ell^2\dot{\varphi}^2+\\&+2m\ell\,\dot{\varphi}\dot{x}\cos\varphi-2m\ell\,\dot{\varphi}\dot{y}\sin\varphi-\\&-\frac{1}{2}\frac{3EId}{x^2(d-x)^2}y^2+(M+m)gy-\\&-mg\ell(1-\cos\varphi)\end{aligned} \qquad (15)$$

After the computing the partial derivatives of Lagrange's function, the differential equations (12) are:

$$\begin{aligned}&(M+m)\ddot{y}-m\ell\ddot{\varphi}\sin\phi-m\ell\dot{\varphi}^2\cos\varphi+\\&+\frac{3EId}{x^2(d-x)^2}y-(M+m)g=0\end{aligned} \qquad (16)$$

$$\begin{aligned}&-m\ell\ddot{y}\sin\varphi+m\ell^2\ddot{\varphi}+\\&+m\ell(\ddot{x}\cos\varphi-\dot{x}\dot{\varphi}\sin\varphi-\dot{y}\dot{\varphi}\cos\varphi)+\\&+m\ell\dot{\varphi}(\dot{x}\sin\varphi+\dot{y}\cos\varphi)+mg\ell\sin\varphi=0\end{aligned} \qquad (17)$$

The equations (16) and (17) may be written in a matrix form:

$$\begin{bmatrix} M+m & -m\ell\sin\varphi \\ -\sin\varphi & \ell \end{bmatrix}\begin{bmatrix}\ddot{y}\\ \ddot{\varphi}\end{bmatrix}=\begin{bmatrix} m\ell\dot{\varphi}^2\cos\varphi-\dfrac{3EId}{x^2(d-x)^2}y+(M+m)g \\ -(\ddot{x}\cos\varphi-\dot{x}\dot{\varphi}\sin\varphi-\dot{y}\dot{\varphi}\cos\varphi)- \\ -\dot{\varphi}(\dot{x}\sin\varphi+\dot{y}\cos\varphi)-g\sin\varphi \end{bmatrix} \qquad (18)$$

If there are made the notations $y=z_1$, $\dot{y}=z_2$, $\varphi=z_3$, $\dot{\varphi}=z_4$, the matrix differential equation (18), can be written under another form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & M+m & 0 & -m\ell\sin z_3 \\ 0 & 0 & 1 & 0 \\ 0 & -\sin z_3 & 0 & \ell \end{bmatrix}\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\\ \dot{z}_4\end{bmatrix}=\begin{bmatrix} z_2 \\ m\ell z_4^2\cos z_3-\dfrac{3EId}{x^2(d-x)^2}z_1+(M+m)g \\ z_4 \\ -(\ddot{x}\cos z_3-\dot{x}z_4\sin z_3-z_2z_4\cos z_3)- \\ -z_4(\dot{x}\sin z_3+z_2\cos z_3)-g\sin z_3 \end{bmatrix}$$

and it can be determinate the first order derivatives,

$$\begin{bmatrix} \dot z_1 \\ \dot z_2 \\ \dot z_3 \\ \dot z_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & M+m & 0 & -m\ell\sin z_3 \\ 0 & 0 & 1 & 0 \\ 0 & -\sin z_3 & 0 & \ell \end{bmatrix}^{-1} \begin{bmatrix} z_2 \\ m\ell z_4^2\cos z_3 - \dfrac{3EId}{x^2(d-x)^2} z_1 + (M+m)g \\ z_4 \\ -\ (\ddot x\cos z_3 - \dot x z_4\sin z_3 - z_2 z_4\cos z_3) - \\ -\ z_4(\dot x\sin z_3 + z_2\cos z_3) - g\sin z_3 \end{bmatrix} \qquad (19)$$

Calculating the inverse of the matrix, it is obtained:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & M+m & 0 & -m\ell\sin z_3 \\ 0 & 0 & 1 & 0 \\ 0 & -\sin z_3 & 0 & \ell \end{bmatrix}^{-1} =$$

$$= \frac{1}{\ell(M+m\cos^2 z_3)} \begin{bmatrix} \ell(M+m\cos^2 z_3) & 0 & 0 & 0 \\ 0 & \ell & 0 & \sin z_3 \\ 0 & 0 & \ell(M+m\cos^2 z_3) & 0 \\ 0 & \sin z_3 & 0 & M+m \end{bmatrix}$$

and after introducing it (19) it can be written:

$$\begin{bmatrix} \dot z_1 \\ \dot z_2 \\ \dot z_3 \\ \dot z_4 \end{bmatrix} = \frac{1}{m\ell^2(M+m\cos^2 z_3)} \begin{bmatrix} m\ell^2(M+m\cos^2 z_3) & 0 & 0 & 0 \\ 0 & m\ell^2 & 0 & m\ell\sin z_3 \\ 0 & 0 & m\ell^2(M+m\cos^2 z_3) & 0 \\ 0 & m\ell\sin z_3 & 0 & M+m \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} z_2 \\ m\ell z_4^2\cos z_3 - \dfrac{3EId}{x^2(d-x)^2} z_1 + (M+m)g \\ z_4 \\ -\ m\ell(\ddot x\cos z_3 - \dot x z_4\sin z_3 - z_2 z_4\cos z_3) - \\ -m\ell z_4(\dot x\sin z_3 + z_2\cos z_3) - mg\ell\sin z_3 \end{bmatrix}$$

After calculating (the multiplying of the inverse matrix with the column matrix) it will result the final form of the first order system of differential equations,

$$\begin{bmatrix} \dot z_1 \\ \dot z_2 \\ \dot z_3 \\ \dot z_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ \dfrac{1}{m\ell^2(M+m\cos^2 z_3)} \left\{ \begin{matrix} m\ell^2\left[m\ell z_4^2 - \dfrac{3EId}{x^2(d-x)^2} z_1 + (M+m)g \right] - \\ -m\ell\sin z_3\,[\,m\ell(\ddot x\cos z_3 - \dot x z_4\sin z_3 - z_2 z_4\cos z_3) + \\ +m\ell z_4(\dot x\sin z_3 + z_2\cos z_3) + mg\ell\sin z_3\,] \end{matrix} \right\} \\ z_4 \\ \dfrac{1}{m\ell^2(M+m\cos^2 z_3)} \left\{ \begin{matrix} m\ell\sin z_3\left[m\ell z_4^2 - \dfrac{3EId}{x^2(d-x)^2} z_1 + (M+m)g \right] - \\ -(M+m)[\,m\ell(\ddot x\cos z_3 - \dot x z_4\sin z_3 - z_2 z_4\cos z_3) + \\ +m\ell z_4(\dot x\sin z_3 + z_2\cos z_3) + mg\ell\sin z_3\,] \end{matrix} \right\} \end{bmatrix} . \qquad (20)$$

to which are added the four initial conditions:

$$t=0, \quad z_1 = \frac{(M+m)g x_1^2(d-x_1)^2}{3EId}, \quad z_2 = 0, \quad z_3 = 0, \quad z_4 = 0 \qquad (21)$$

The numerical solving of this system of differential equations was performed, determining the trajectory described by the fixed point of the pendulum and also its deviation from the static deformed beam.

In the figures 17, 18 and 19 are presented the trajectories described by the fixed point of the pendulum in case of some different values of the two moving masses M (which moves on the beam) and m (which oscillates), considering three sets of values: M=600, m=200 (first), M=m=400 (second) respectively M=200, m=600 (third), the travelling speed being 1.5 [m/s].

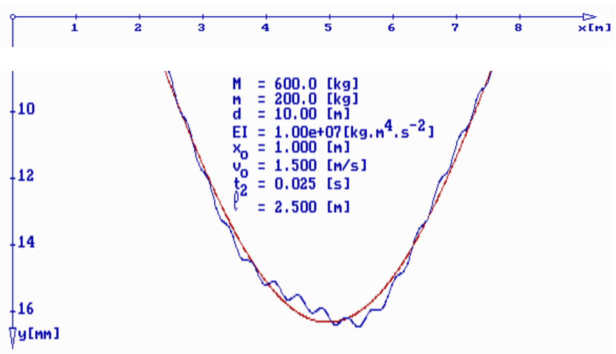

Fig. 17 The trajectory of fixed point of the pendulum (the mass values are M= 600 [kg], m= 200 [kg])

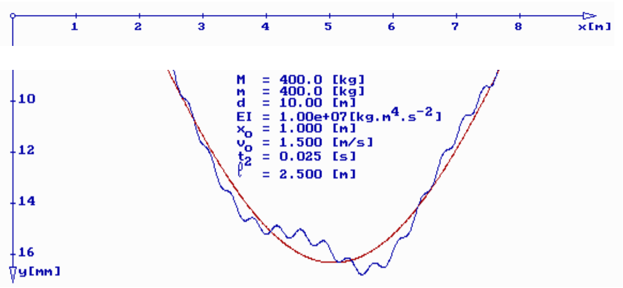

Fig. 18 The trajectory of fixed point of the pendulum (the mass values are M=400 [kg], m=400 [kg])

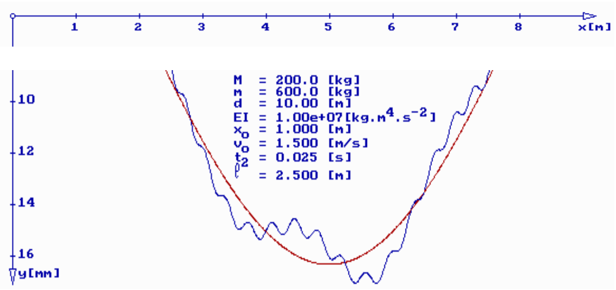

Fig. 19 The trajectory of fixed point of the pendulum (the mass values are M= 200 [kg], m= 600 [kg])

Examining the above presented diagram we can notice the influence of the ratio M/m on the shape of the mass M trajectory.

3. CONCLUSIONS

Using the C programs for solving numerically the dynamical problems of influence on the elastic structure of moving loads, based on the elaborated mathematical models, in this paper it has been performed a detailed study of the dynamical behavior of some mechanical systems considering the influence of different parameters..

There are considered varied numerical values of some parameters (traveling speed, acceleration and deceleration time, the ratio M/m of the two loads (their sum remaining the same) in the studies of elements displacements and oscillations.

The obtained mathematical models were simultaneous nonlinear differential equations of second order, numerically solved with Runge-Kutta methods of high accuracy.

A part of the obtained results were presented in this paper.

In the future the authors plan to extend the studies in the domain of serial robots with flexible elements that perform relative planar and spatial movements.

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Contribuţii la studiul dinamic al unei mase care se deplasează pe o bară elastică

Rezumat: *În lucrare sunt studiate două sisteme mecanice, conţinând un element elastic tip bară, rezemată la ambele capete, în cazul primului deplasându-se pe bara elastică o masă cu viteză constantă sau variabilă iar în cazul celui de-al doilea de masa care se deplasează fiind legat un pendul. În ambele cazuri se determină deplasările pe verticală ale masei care se mişcă pe bara elastică în funcţie de diferiţi parametri. Rezultatele obţinute pot fi folosite la o îmbunătăţire a calculului dinamic ce trtebuie efectuat la proiectarea podurilor, a macaralelor şi a elementelor mobile ale roboţilor seriali.*

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