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THE DYNAMIC MODEL OF RTTTR SERIAL ROBOT – ITERATIONS OUTWARDS AND INWARDS THE ROBOT'S MECHANICAL STRUCTURE

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Abstract: The paper presents the first part of the process of obtaining the dynamic model of RTTTR serial robot. After establishing the input data, the external forces and their moments are determined based on iterations outwards the robot's mechanical structure. The paper ends with iterations inwards the robot's mechanical structure, with the goal of determining the connection forces and their moments.

Key words: dynamic model, serial robot, symbolic computation, Newton-Euler's formulation.

1. INTRODUCTION

Unless the geometric and kinematic model of robots, where the operational kinematic parameters are determined based on the variation of the generalized coordinates, velocities and accelerations, the aim of the dynamic model is to determine the generalized driving forces, taking into account the dynamic requirements of the task, expressed by the useful payload (forces and moments) and the mass distribution parameters, such as: masses, mass centers, inertial tensors. The dynamic model is therefore more general and it takes into account the causes that produce motion and also the causes that restrict motion.

2. THE INPUT DATA

The dynamic model of RTTTR serial robot, whose structure is described in [1], is determined by Newton-Euler's method [2], implemented into the module *Robot_Dynamics* [3] from the generalized modeling application *Robot_Symbolic* [4].

The dynamic models of the previously analyzed robots – TRR [5, 6], TRTR [7], TRTTR [8], TRTTR1 [9] – were accomplished with some simplifying hypotheses, such as that a mass with a constant value was manipulated in the robot task, with no contact with the environment.

Unless the above mentioned robots, in the case of RTTTR robot, the mass center position vectors of links i with respect to the $\{i\}$ frame origin are generally expressed as:

$${}^1\bar{r}_{C_1} = \begin{bmatrix} {}^1r_{C_{1x}} \\ {}^1r_{C_{1y}} \\ {}^1r_{C_{1z}} \end{bmatrix}; \quad {}^2\bar{r}_{C_2} = \begin{bmatrix} {}^2r_{C_{2x}} \\ {}^2r_{C_{2y}} \\ {}^2r_{C_{2z}} \end{bmatrix}; \quad {}^3\bar{r}_{C_3} = \begin{bmatrix} {}^3r_{C_{3x}} \\ {}^3r_{C_{3y}} \\ {}^3r_{C_{3z}} \end{bmatrix}; \quad (1)$$

$${}^4\bar{r}_{C_4} = \begin{bmatrix} {}^4r_{C_{4x}} \\ {}^4r_{C_{4y}} \\ {}^4r_{C_{4z}} \end{bmatrix}; \quad {}^5\bar{r}_{C_5} = \begin{bmatrix} {}^5r_{C_{5x}} \\ {}^5r_{C_{5y}} \\ {}^5r_{C_{5z}} \end{bmatrix}; \quad (2)$$

and the generalized inertial tensors, corresponding to each link i , are:

$${}^1I_1^* = \begin{bmatrix} {}^1J_x^* & -{}^1J_{xy}^* & -{}^1J_{xz}^* \\ -{}^1J_{yx}^* & {}^1J_y^* & -{}^1J_{yz}^* \\ -{}^1J_{zx}^* & -{}^1J_{zy}^* & {}^1J_z^* \end{bmatrix}; \quad (3)$$

$${}^2I_2^* = \begin{bmatrix} {}^2J_x^* & -{}^2J_{xy}^* & -{}^2J_{xz}^* \\ -{}^2J_{yx}^* & {}^2J_y^* & -{}^2J_{yz}^* \\ -{}^2J_{zx}^* & -{}^2J_{zy}^* & {}^2J_z^* \end{bmatrix}; \quad (4)$$

$${}^3I_3^* = \begin{bmatrix} {}^3J_x^* & -{}^3J_{xy}^* & -{}^3J_{xz}^* \\ -{}^3J_{yx}^* & {}^3J_y^* & -{}^3J_{yz}^* \\ -{}^3J_{zx}^* & -{}^3J_{zy}^* & {}^3J_z^* \end{bmatrix}; \quad (5)$$

$${}^4I_4^* = \begin{bmatrix} {}^4J_x^* & -{}^4J_{xy}^* & -{}^4J_{xz}^* \\ -{}^4J_{yx}^* & {}^4J_y^* & -{}^4J_{yz}^* \\ -{}^4J_{zx}^* & -{}^4J_{zy}^* & {}^4J_z^* \end{bmatrix}; \quad (6)$$

$${}^5I_5^* = \begin{bmatrix} {}^5J_x^* & -{}^5J_{xy}^* & -{}^5J_{xz}^* \\ -{}^5J_{yx}^* & {}^5J_y^* & -{}^5J_{yz}^* \\ -{}^5J_{zx}^* & -{}^5J_{zy}^* & {}^5J_z^* \end{bmatrix}. \quad (7)$$

The useful payload is defined by the following force-moment vectors:

$${}^6\bar{f}_6 = \begin{bmatrix} {}^6f_x \\ {}^6f_y \\ {}^6f_z \end{bmatrix}; \quad {}^6\bar{n}_6 = \begin{bmatrix} {}^6n_x \\ {}^6n_y \\ {}^6n_z \end{bmatrix}. \quad (8)$$

Based on the results from the geometric and kinematic modeling [2], the mass center accelerations are computed as follows:

$${}^1\dot{\bar{v}}_{C_1} = \begin{bmatrix} -\ddot{q}_1^1 r_{C_{1y}} - \dot{q}_1^2 r_{C_{1x}} \\ \ddot{q}_1^1 r_{C_{1x}} - \dot{q}_1^2 r_{C_{1y}} \\ g \end{bmatrix}; \quad (9)$$

$${}^2\dot{\bar{v}}_{C_2} = \begin{bmatrix} -\ddot{q}_1^2 r_{C_{2y}} - \dot{q}_1^2 r_{C_{2x}} \\ \ddot{q}_1^2 r_{C_{2x}} - \dot{q}_1^2 r_{C_{2y}} \\ g + \ddot{q}_2 \end{bmatrix}; \quad (10)$$

$${}^3\dot{\bar{v}}_{C_3} = \begin{bmatrix} -\ddot{q}_1 \cdot q_3 - \ddot{q}_1 \cdot l_3 - \dot{q}_1^2 \cdot l_2 - 2 \cdot \dot{q}_3 \cdot \dot{q}_1 - \ddot{q}_1^3 r_{C_{3y}} - \dot{q}_1^2 \cdot \dot{q}_3 r_{C_{3x}} \\ \ddot{q}_1 \cdot l_2 - \dot{q}_1^2 \cdot q_3 - \dot{q}_1^2 \cdot l_3 + \ddot{q}_3 + \ddot{q}_1^3 r_{C_{3x}} - \dot{q}_1^2 \cdot \dot{q}_3 r_{C_{3y}} \\ g + \ddot{q}_2 \end{bmatrix}; \quad (11)$$

$${}^4\dot{\bar{v}}_{C_4} = \begin{bmatrix} -cq_4 \cdot \ddot{q}_1 \cdot q_3 - cq_4 \cdot \ddot{q}_1 \cdot l_3 - cq_4 \cdot \dot{q}_1^2 \cdot l_2 - 2 \cdot cq_4 \cdot \dot{q}_3 \cdot \dot{q}_1 - cq_4 \cdot \ddot{q}_1 \cdot l_4 + sq_4 \cdot \ddot{q}_1 \cdot l_2 - \\ -sq_4 \cdot \dot{q}_1^2 \cdot q_3 - sq_4 \cdot \dot{q}_1^2 \cdot l_3 + sq_4 \cdot \ddot{q}_3 - sq_4 \cdot \dot{q}_1^2 \cdot l_4 - {}^4r_{C_{4y}} \cdot \ddot{q}_1 - {}^4r_{C_{4y}} \cdot \ddot{q}_4 - {}^4r_{C_{4x}} \cdot \dot{q}_1^2 - \\ -2 \cdot {}^4r_{C_{4x}} \cdot \dot{q}_1 \cdot \dot{q}_4 - {}^4r_{C_{4x}} \cdot \dot{q}_4^2 \\ sq_4 \cdot \ddot{q}_1 \cdot q_3 + sq_4 \cdot \ddot{q}_1 \cdot l_3 + sq_4 \cdot \dot{q}_1^2 \cdot l_2 + 2 \cdot sq_4 \cdot \dot{q}_3 \cdot \dot{q}_1 + sq_4 \cdot \ddot{q}_1 \cdot l_4 + cq_4 \cdot \ddot{q}_1 \cdot l_2 - \\ -cq_4 \cdot \dot{q}_1^2 \cdot q_3 - cq_4 \cdot \dot{q}_1^2 \cdot l_3 + cq_4 \cdot \ddot{q}_3 - cq_4 \cdot \dot{q}_1^2 \cdot l_4 + {}^4r_{C_{4x}} \cdot \ddot{q}_1 + {}^4r_{C_{4x}} \cdot \ddot{q}_4 - {}^4r_{C_{4y}} \cdot \dot{q}_1^2 - \\ -2 \cdot {}^4r_{C_{4y}} \cdot \dot{q}_1 \cdot \dot{q}_4 - {}^4r_{C_{4y}} \cdot \dot{q}_4^2 \\ g + \ddot{q}_2 \end{bmatrix}. \quad (12)$$

The vector ${}^5\dot{\bar{v}}_{C_5}$ is expressed by its Cartesian components, as:

$${}^5\dot{\bar{v}}_{C_{5x}} = -{}^5r_{C_{5y}} \cdot cq_5 \cdot \ddot{q}_4 - {}^5r_{C_{5y}} \cdot cq_5 \cdot \ddot{q}_1 + cq_5 \cdot sq_4 \cdot \ddot{q}_3 - cq_5 \cdot cq_4 \cdot \ddot{q}_1 \cdot q_3 - cq_5 \cdot cq_4 \cdot \ddot{q}_1 \cdot l_3 - \\ -{}^5r_{C_{5x}} \cdot \dot{q}_5^2 - {}^5r_{C_{5x}} \cdot c^2 q_5 \cdot \dot{q}_1^2 - 2cq_5 \cdot sq_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_1 \cdot \dot{q}_4 - sq_5 \cdot g - sq_5 \cdot \ddot{q}_2 - \\ -cq_5 \cdot cq_4 \cdot \dot{q}_1^2 \cdot l_2 - 2 \cdot cq_5 \cdot cq_4 \cdot \dot{q}_3 \cdot \dot{q}_1 - cq_5 \cdot cq_4 \cdot \ddot{q}_1 \cdot l_4 + cq_5 \cdot sq_4 \cdot \ddot{q}_1 \cdot l_2 - \\ -cq_5 \cdot sq_4 \cdot \dot{q}_1^2 \cdot q_3 - cq_5 \cdot sq_4 \cdot \dot{q}_1^2 \cdot l_3 - cq_5 \cdot sq_4 \cdot \dot{q}_1^2 \cdot l_4 - 2 \cdot {}^5r_{C_{5z}} \cdot c^2 q_5 \cdot \dot{q}_1 \cdot \dot{q}_4 - \\ -cq_5 \cdot sq_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_1^2 - cq_5 \cdot sq_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_4^2 + \ddot{q}_5 \cdot {}^5r_{C_{5z}} - {}^5r_{C_{5x}} \cdot c^2 q_5 \cdot \dot{q}_4^2 \quad (13)$$

$${}^5\dot{\bar{v}}_{C_{5y}} = -{}^5r_{C_{5y}} \cdot \dot{q}_4^2 - {}^5r_{C_{5y}} \cdot \dot{q}_1^2 - 2 \cdot {}^5r_{C_{5y}} \cdot \dot{q}_1 \cdot \dot{q}_4 - cq_4 \cdot \dot{q}_1^2 \cdot q_3 + cq_4 \cdot \ddot{q}_1 \cdot l_2 + sq_4 \cdot \ddot{q}_1 \cdot l_4 + \\ + 2sq_4 \cdot \dot{q}_1 \cdot \dot{q}_3 + sq_4 \cdot \dot{q}_1^2 \cdot l_2 + sq_4 \cdot \ddot{q}_1 \cdot l_3 + sq_4 \cdot \ddot{q}_1 \cdot q_3 - cq_4 \cdot \dot{q}_1^2 \cdot l_4 - cq_4 \cdot \dot{q}_1^2 \cdot l_3 + \\ + cq_4 \cdot \ddot{q}_3 - 2 \cdot {}^5r_{C_{5x}} \cdot \dot{q}_5 \cdot sq_5 \cdot \dot{q}_1 - 2 \cdot {}^5r_{C_{5x}} \cdot \dot{q}_5 \cdot sq_5 \cdot \dot{q}_4 + 2 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_5 \cdot cq_5 \cdot \dot{q}_1 + \\ + 2 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_5 \cdot cq_5 \cdot \dot{q}_4 + {}^5r_{C_{5z}} \cdot sq_5 \cdot \ddot{q}_4 + {}^5r_{C_{5x}} \cdot cq_5 \cdot \ddot{q}_1 + {}^5r_{C_{5x}} \cdot cq_5 \cdot \ddot{q}_4 + {}^5r_{C_{5z}} \cdot sq_5 \cdot \ddot{q}_1 \quad (14)$$

$${}^5\dot{\bar{v}}_{C_{5z}} = {}^5r_{C_{5z}} \cdot \dot{q}_1^2 \cdot c^2 q_5 - {}^5r_{C_{5z}} \cdot \dot{q}_5^2 + cq_5 \cdot g + cq_5 \cdot \ddot{q}_2 - sq_5 cq_4 \cdot \dot{q}_1^2 \cdot l_2 - sq_5 cq_4 \cdot \dot{q}_1^2 \cdot l_4 + \\ + sq_5 \cdot sq_4 \cdot \ddot{q}_1 \cdot l_2 - sq_5 sq_4 \cdot \dot{q}_1^2 \cdot q_3 - sq_5 sq_4 \cdot \dot{q}_1^2 \cdot l_3 - sq_5 sq_4 \cdot \dot{q}_1^2 \cdot l_4 - \\ -cq_5 sq_5 \cdot {}^5r_{C_{5x}} \cdot \dot{q}_1^2 - sq_5 \cdot cq_4 \cdot \ddot{q}_1 \cdot q_3 - sq_5 \cdot cq_4 \cdot \ddot{q}_1 \cdot l_3 - 2sq_5 \cdot cq_4 \cdot \dot{q}_3 \cdot \dot{q}_1 - \\ -2cq_5 \cdot sq_5 \cdot {}^5r_{C_{5x}} \cdot \dot{q}_1 \cdot \dot{q}_4 - cq_5 sq_5 \cdot {}^5r_{C_{5x}} \cdot \dot{q}_4^2 + sq_5 sq_4 \cdot \ddot{q}_3 - {}^5r_{C_{5y}} sq_5 \cdot \ddot{q}_1 - {}^5r_{C_{5y}} sq_5 \cdot \ddot{q}_4 + \\ + {}^5r_{C_{5z}} \cdot \dot{q}_4^2 \cdot c^2 q_5 - 2 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_1 \cdot \dot{q}_4 + 2 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_1 \cdot \dot{q}_4 \cdot c^2 q_5 - {}^5r_{C_{5z}} \cdot \dot{q}_1^2 - {}^5r_{C_{5z}} \cdot \dot{q}_4^2 - \ddot{q}_5 \cdot {}^5r_{C_{5x}} \quad (15)$$

The above described parameters are used as input data, in the process of computing the external forces and their moments, passing the mechanical structure by outwards iterations and in the process of computing the connection forces and their moments, by iterations inwards the robot's mechanical structure.

3. ITERATIONS OUTWARDS THE ROBOT'S MECHANICAL STRUCTURE

The next step is the determination of the external forces, by iterations outwards the

robot's mechanical structure, starting from the fixed base:

$${}^1\bar{F}_1 = \begin{bmatrix} -M_1 \cdot (\ddot{q}_1^1 r_{c_{1y}} + \dot{q}_1^{2,1} r_{c_{1x}}) \\ -M_1 \cdot (-\ddot{q}_1^1 r_{c_{1x}} + \dot{q}_1^{2,1} r_{c_{1y}}) \\ M_1 g \end{bmatrix}; \quad (16)$$

$${}^2\bar{F}_2 = \begin{bmatrix} -M_2 \cdot (\ddot{q}_1^2 r_{c_{2y}} + \dot{q}_1^{2,2} r_{c_{2x}}) \\ -M_2 \cdot (-\ddot{q}_1^2 r_{c_{2x}} + \dot{q}_1^{2,2} r_{c_{2y}}) \\ M_2 (g + \ddot{q}_2) \end{bmatrix}; \quad (17)$$

$${}^3\bar{F}_3 = \begin{bmatrix} -M_3 \cdot (\ddot{q}_1 q_3 + \ddot{q}_1 l_3 + \dot{q}_1^2 \cdot l_2 + 2\dot{q}_3 \dot{q}_1 + \ddot{q}_1^3 r_{c_{3y}} + \dot{q}_1^{2,3} r_{c_{3x}}) \\ M_3 \cdot (\ddot{q}_1 l_2 - \dot{q}_1^2 \cdot q_3 - \dot{q}_1^2 \cdot l_3 + \ddot{q}_3 + \ddot{q}_1^3 r_{c_{3x}} - \dot{q}_1^{2,3} r_{c_{3y}}) \\ M_3 (g + \ddot{q}_2) \end{bmatrix}; \quad (18)$$

$${}^4\bar{F}_4 = \begin{bmatrix} -M_4 c q_4 \cdot \ddot{q}_1 q_3 - M_4 c q_4 \cdot \ddot{q}_1 l_3 - M_4 c q_4 \cdot \dot{q}_1^2 \cdot l_2 - 2M_4 c q_4 \cdot \dot{q}_3 \dot{q}_1 - M_4 c q_4 \cdot \ddot{q}_1 l_4 + \\ + M_4 s q_4 \cdot \ddot{q}_1 l_2 - M_4 s q_4 \cdot \dot{q}_1^2 \cdot q_3 - M_4 s q_4 \cdot \dot{q}_1^2 \cdot l_3 + M_4 s q_4 \cdot \ddot{q}_3 - M_4 s q_4 \cdot \dot{q}_1^2 \cdot l_4 - \\ - M_4^4 r_{c_{4y}} \ddot{q}_1 - M_4^4 r_{c_{4y}} \ddot{q}_4 - M_4^4 r_{c_{4x}} \dot{q}_1^2 - 2M_4^4 r_{c_{4x}} \dot{q}_1 \dot{q}_4 - M_4^4 r_{c_{4x}} \dot{q}_4^2 \\ - M_4 s q_4 \cdot \ddot{q}_1 q_3 + M_4 s q_4 \cdot \ddot{q}_1 l_3 + M_4 s q_4 \cdot \dot{q}_1^2 \cdot l_2 + 2M_4 s q_4 \cdot \dot{q}_3 \dot{q}_1 + M_4 s q_4 \cdot \ddot{q}_1 l_4 + \\ + M_4 c q_4 \cdot \ddot{q}_1 l_2 - M_4 c q_4 \cdot \dot{q}_1^2 q_3 - M_4 c q_4 \cdot \dot{q}_1^2 \cdot l_3 + M_4 c q_4 \cdot \ddot{q}_3 - M_4 c q_4 \cdot \dot{q}_1^2 \cdot l_4 + \\ + M_4^4 r_{c_{4x}} \ddot{q}_1 + M_4^4 r_{c_{4x}} \ddot{q}_4 - M_4^4 r_{c_{4y}} \dot{q}_1^2 - 2M_4^4 r_{c_{4y}} \dot{q}_1 \dot{q}_4 - M_4^4 r_{c_{4y}} \dot{q}_4^2 \\ M_4 (g + \ddot{q}_2) \end{bmatrix}. \quad (19)$$

The external force ${}^5\bar{F}_5$ is expressed by its Cartesian components, as:

$${}^5F_x = -M_5^5 r_{c_{5y}} c q_5 \cdot \ddot{q}_4 - M_5^5 r_{c_{5y}} c q_5 \cdot \ddot{q}_1 + M_5 c q_5 s q_4 \cdot \ddot{q}_3 - M_5 c q_5 c q_4 \cdot \ddot{q}_1 q_3 - \\ - M_5 c q_5 c q_4 \cdot \ddot{q}_1 l_3 - M_5^5 r_{c_{5x}} \dot{q}_5^2 - M_5^5 r_{c_{5x}} c^2 q_5 \cdot \dot{q}_1^2 - 2M_5 c q_5 s q_5^5 r_{c_{5z}} \cdot \dot{q}_1 \dot{q}_4 - \\ - M_5 s q_5 \cdot g - M_5 s q_5 \cdot \ddot{q}_2 - M_5 c q_5 c q_4 \cdot \dot{q}_1^2 \cdot l_2 - 2M_5 c q_5 c q_4 \cdot \dot{q}_3 \dot{q}_1 - M_5 c q_5 c q_4 \cdot \ddot{q}_1 l_4 + \\ + M_5 c q_5 s q_4 \cdot \ddot{q}_1 l_2 - M_5 c q_5 s q_4 \cdot \dot{q}_1^2 \cdot q_3 - M_5 c q_5 s q_4 \cdot \dot{q}_1^2 \cdot l_3 - M_5 c q_5 s q_4 \cdot \dot{q}_1^2 \cdot l_4 - \\ - 2M_5^5 r_{c_{5x}} c^2 q_5 \cdot \dot{q}_1 \dot{q}_4 - M_5 c q_5 s q_5^5 r_{c_{5z}} \cdot \dot{q}_1^2 - M_5 c q_5 s q_5^5 r_{c_{5z}} \cdot \dot{q}_4 + M_5 \ddot{q}_5^5 r_{c_{5z}} - \\ - M_5^5 r_{c_{5x}} c^2 q_5 \cdot \dot{q}_4^2 \quad (20)$$

$${}^5F_y = -M_5^5 r_{c_{5y}} \cdot \dot{q}_4^2 - M_5^5 r_{c_{5y}} \dot{q}_1^2 - 2M_5^5 r_{c_{5y}} \dot{q}_1 \dot{q}_4 - M_5 c q_4 \cdot \dot{q}_1^2 \cdot q_3 + M_5 c q_4 \cdot \ddot{q}_1 l_2 + \\ + M_5 s q_4 \cdot \ddot{q}_1 l_4 + 2M_5 s q_4 \cdot \dot{q}_3 \dot{q}_1 + M_5 s q_4 \cdot \dot{q}_1^2 \cdot l_2 + M_5 s q_4 \cdot \ddot{q}_1 l_3 + M_5 s q_4 \cdot \ddot{q}_1 q_3 - \\ - M_5 c q_4 \cdot \dot{q}_1^2 \cdot l_4 - M_5 c q_4 \cdot \dot{q}_1^2 \cdot l_3 + M_5 c q_4 \cdot \ddot{q}_3 - 2M_5^5 r_{c_{5x}} \cdot \dot{q}_5 \cdot s q_5 \cdot \dot{q}_1 - \\ - 2M_5^5 r_{c_{5x}} \dot{q}_5 s q_5 \cdot \dot{q}_4 + 2M_5^5 r_{c_{5z}} \cdot \dot{q}_5 \cdot c q_5 \cdot \dot{q}_1 + 2M_5^5 r_{c_{5z}} \dot{q}_5 c q_5 \cdot \dot{q}_4 + \\ + M_5^5 r_{c_{5z}} \cdot s q_5 \cdot \ddot{q}_4 + M_5^5 r_{c_{5x}} c q_5 \cdot \ddot{q}_1 + M_5^5 r_{c_{5x}} \cdot c q_5 \cdot \ddot{q}_4 + M_5^5 r_{c_{5z}} s q_5 \cdot \ddot{q}_1 \quad (21)$$

$${}^5F_z = -M_5^5 r_{c_{5z}} \dot{q}_5^2 - M_5^5 r_{c_{5z}} \dot{q}_1^2 - M_5^5 r_{c_{5z}} \dot{q}_4^2 - M_5 \ddot{q}_5^5 r_{c_{5x}} + M_5 c q_5 \cdot \ddot{q}_2 + M_5 c q_5 \cdot g - \\ - M_5 c q_5 s q_5^5 r_{c_{5x}} \dot{q}_1^2 - M_5 s q_5 c q_4 \cdot \dot{q}_1^2 \cdot l_2 - M_5 s q_5 c q_4 \cdot \ddot{q}_1 \cdot l_4 - M_5 s q_5 s q_4 \cdot \dot{q}_1^2 \cdot q_3 + \\ + M_5^5 r_{c_{5z}} \dot{q}_1^2 \cdot c^2 q_5 - M_5^5 r_{c_{5y}} s q_5 \cdot \ddot{q}_4 - M_5^5 r_{c_{5y}} s q_5 \cdot \ddot{q}_1 + M_5 s q_5 s q_4 \cdot \ddot{q}_3 - \\ - M_5 s q_5 s q_4 \cdot \dot{q}_1^2 \cdot l_3 - M_5 s q_5 s q_4 \cdot \dot{q}_1^2 \cdot l_4 - M_5 s q_5 c q_4 \cdot \ddot{q}_1 q_3 - M_5 s q_5 c q_4 \cdot \ddot{q}_1 l_3 - \\ - M_5 c q_5 s q_5^5 r_{c_{5x}} \dot{q}_4^2 - 2M_5 c q_5 s q_5^5 r_{c_{5x}} \dot{q}_1 \dot{q}_4 + M_5 s q_5 s q_4 \cdot \ddot{q}_1 l_2 - 2M_5 s q_5 c q_4 \cdot \dot{q}_3 \dot{q}_1 + \\ + M_5^5 r_{c_{5z}} \dot{q}_4^2 c^2 q_5 + 2M_5^5 r_{c_{5z}} \dot{q}_1 \dot{q}_4 \cdot c^2 q_5 - 2M_5^5 r_{c_{5z}} \dot{q}_1 \dot{q}_4 \quad (22)$$

By iterations outwards the robot's mechanical structure, the moments of the external forces are determined as:

$${}^1\bar{N}_1 = \begin{bmatrix} -{}^1J_{xz}^* \cdot \ddot{q}_1 + \dot{q}_1^2 \cdot {}^1J_{yz}^* \\ -{}^1J_{yz}^* \cdot \ddot{q}_1 - \dot{q}_1^2 \cdot {}^1J_{xz}^* \\ {}^1J_z^* \cdot \ddot{q}_1 \end{bmatrix}; \quad (23)$$

$${}^2\bar{N}_2 = \begin{bmatrix} -{}^2J_{xz}^* \cdot \ddot{q}_1 + \dot{q}_1^2 \cdot {}^2J_{yz}^* \\ -{}^2J_{yz}^* \cdot \ddot{q}_1 - \dot{q}_1^2 \cdot {}^2J_{xz}^* \\ {}^2J_z^* \cdot \ddot{q}_1 \end{bmatrix}; \quad (24)$$

$${}^3\bar{N}_3 = \begin{bmatrix} -{}^3J_{xz}^* \cdot \ddot{q}_1 + \dot{q}_1^2 \cdot {}^3J_{yz}^* \\ -{}^3J_{yz}^* \cdot \ddot{q}_1 - \dot{q}_1^2 \cdot {}^3J_{xz}^* \\ {}^3J_z^* \cdot \ddot{q}_1 \end{bmatrix}; \quad (25)$$

$${}^4\bar{N}_4 = \begin{bmatrix} -{}^4J_{xz}^* \cdot \ddot{q}_1 - {}^4J_{xz}^* \cdot \ddot{q}_4 + {}^4J_{yz}^* \cdot \dot{q}_1^2 + 2 \cdot {}^4J_{yz}^* \cdot \dot{q}_1 \dot{q}_4 + {}^4J_{yz}^* \cdot \dot{q}_4^2 \\ -{}^4J_{yz}^* \cdot \ddot{q}_1 - {}^4J_{yz}^* \cdot \ddot{q}_4 - {}^4J_{xz}^* \cdot \dot{q}_1^2 - 2 \cdot {}^4J_{xz}^* \cdot \dot{q}_1 \dot{q}_4 - {}^4J_{xz}^* \cdot \dot{q}_4^2 \\ {}^4J_z^* (\ddot{q}_1 + \ddot{q}_4) \end{bmatrix}; \quad (26)$$

$${}^5\bar{N}_5 = [{}^5N_x \ {}^5N_y \ {}^5N_z]^T, \quad (27)$$

where the Cartesian components of ${}^5\bar{N}_5$ are given by:

$$\begin{aligned} {}^5N_x = & -{}^5J_{xz}^* s q_5 \cdot \ddot{q}_1 - {}^5J_{xz}^* s q_5 \cdot \ddot{q}_4 - {}^5J_{xz}^* \dot{q}_5 c q_5 \cdot \dot{q}_1 - {}^5J_{xz}^* \dot{q}_5 c q_5 \cdot \dot{q}_4 - {}^5J_{xy}^* \ddot{q}_5 - {}^5J_{xz}^* c q_5 \cdot \ddot{q}_1 - {}^5J_{xz}^* c q_5 \cdot \ddot{q}_4 + \\ & + {}^5J_{xz}^* \dot{q}_5 s q_5 \cdot \dot{q}_1 + {}^5J_{xz}^* \dot{q}_5 s q_5 \cdot \dot{q}_4 - s q_5 c q_5 \cdot {}^5J_{yx}^* \dot{q}_1^2 - 2 s q_5 c q_5 \cdot {}^5J_{yx}^* \dot{q}_1 \dot{q}_4 - s q_5 c q_5 \cdot {}^5J_{yx}^* \dot{q}_4^2 + \\ & + s q_5 \cdot q_5 \cdot {}^5J_{zx}^* \dot{q}_1 + s q_5 \dot{q}_5 \cdot {}^5J_{zx}^* \dot{q}_4 - \dot{q}_5 c q_5 \cdot {}^5J_y^* \dot{q}_1 - \dot{q}_5 c q_5 \cdot {}^5J_y^* \dot{q}_4 - \dot{q}_5^2 \cdot {}^5J_{zy}^* + {}^5J_{yz}^* c^2 q_5 \cdot \dot{q}_1^2 + \\ & + 2 \cdot {}^5J_{yz}^* c^2 q_5 \cdot \dot{q}_1 \dot{q}_4 + {}^5J_{yz}^* c^2 q_5 \cdot \dot{q}_4^2 + c q_5 \cdot \dot{q}_5 \cdot {}^5J_z^* \dot{q}_1 + c q_5 \cdot \dot{q}_5 \cdot {}^5J_z^* \dot{q}_4 \end{aligned} \quad (28)$$

$$\begin{aligned} {}^5N_y = & {}^5J_{yx}^* s q_5 \cdot \ddot{q}_1 + {}^5J_y^* \cdot \ddot{q}_5 + {}^5J_{yx}^* s q_5 \cdot \ddot{q}_4 - 2 s q_5 c q_5 \cdot {}^5J_{xz}^* \dot{q}_1 \dot{q}_4 + 2 c q_5 s q_5 \cdot {}^5J_z^* \dot{q}_1 \dot{q}_4 + s q_5 c q_5 \cdot {}^5J_z^* \dot{q}_4^2 + \\ & + {}^5J_{yx}^* \dot{q}_5 c q_5 \cdot \dot{q}_1 + {}^5J_{yx}^* \dot{q}_5 c q_5 \cdot \dot{q}_4 + {}^5J_{yz}^* \dot{q}_5 s q_5 \cdot \dot{q}_1 + {}^5J_{yx}^* \dot{q}_5 s q_5 \cdot \dot{q}_4 - s q_5 c q_5 \cdot {}^5J_x^* \dot{q}_1^2 - \\ & - s q_5 c q_5 \cdot {}^5J_x^* \dot{q}_4^2 - \dot{q}_5 c q_5 \cdot {}^5J_{xy}^* \dot{q}_1 - \dot{q}_5 c q_5 \cdot {}^5J_{xy}^* \dot{q}_4 - \dot{q}_5 s q_5 \cdot {}^5J_{zy}^* \dot{q}_1 - \dot{q}_5 s q_5 \cdot {}^5J_{zy}^* \dot{q}_4 - \\ & - 2 c^2 q_5 \cdot {}^5J_{xz}^* \dot{q}_1 \dot{q}_4 + c q_5 s q_5 \cdot {}^5J_z^* \dot{q}_1^2 - {}^5J_{yz}^* c q_5 \cdot \ddot{q}_1 - {}^5J_{yz}^* c q_5 \cdot \ddot{q}_4 - c^2 q_5 \cdot {}^5J_{xz}^* \dot{q}_1^2 - \\ & - c^2 q_5 \cdot {}^5J_{xz}^* \dot{q}_4^2 + {}^5J_{zx}^* \dot{q}_1^2 - {}^5J_{zx}^* \dot{q}_4^2 \cdot c^2 q_5 + {}^5J_{zx}^* \dot{q}_4^2 \cdot c^2 q_5 + 2 \cdot {}^5J_{zx}^* \dot{q}_1 \dot{q}_4 - 2 \cdot {}^5J_{zx}^* \dot{q}_1 \dot{q}_4 c^2 q_5 \end{aligned} \quad (29)$$

$$\begin{aligned} {}^5N_z = & -{}^5J_{zy}^* \cdot \ddot{q}_5 + {}^5J_{yx}^* \cdot \dot{q}_1^2 c^2 q_5 + \dot{q}_5^2 \cdot {}^5J_{xy}^* - 2 \cdot {}^5J_{yx}^* \dot{q}_1 \dot{q}_4 + 2 \cdot {}^5J_{yx}^* \dot{q}_1 \dot{q}_4 c^2 q_5 - {}^5J_{yx}^* \dot{q}_1^2 - {}^5J_{yx}^* \dot{q}_4^2 + \\ & + {}^5J_{yx}^* \dot{q}_4^2 c^2 q_5 + 2 c q_5 s q_5 \cdot {}^5J_{yz}^* \dot{q}_1 \dot{q}_4 + {}^5J_{zx}^* s q_5 \cdot \ddot{q}_1 + {}^5J_{zx}^* s q_5 \cdot \ddot{q}_4 + {}^5J_z^* c q_5 \cdot \ddot{q}_1 + {}^5J_z^* c q_5 \cdot \ddot{q}_4 + \\ & + {}^5J_{zx}^* \cdot \dot{q}_5 c q_5 \cdot \dot{q}_1 + {}^5J_{zx}^* \cdot \dot{q}_5 c q_5 \cdot \dot{q}_4 - {}^5J_z^* \cdot \dot{q}_5 s q_5 \cdot \dot{q}_1 - {}^5J_z^* \cdot \dot{q}_5 s q_5 \cdot \dot{q}_4 + s q_5 \dot{q}_5 \cdot {}^5J_x^* \cdot \dot{q}_1 + \\ & + s q_5 \dot{q}_5 \cdot {}^5J_x^* \cdot \dot{q}_4 - \dot{q}_5 s q_5 \cdot {}^5J_y^* \cdot \dot{q}_1 - \dot{q}_5 s q_5 \cdot {}^5J_y^* \cdot \dot{q}_4 + c q_5 \dot{q}_5 \cdot {}^5J_{xz}^* \cdot \dot{q}_1 + c q_5 \dot{q}_5 \cdot {}^5J_{xz}^* \cdot \dot{q}_4 + \\ & + c q_5 s q_5 \cdot {}^5J_{yz}^* \cdot \dot{q}_1^2 + c q_5 s q_5 \cdot {}^5J_{yz}^* \cdot \dot{q}_4^2 \end{aligned} \quad (30)$$

4. ITERATIONS INWARDS THE ROBOT'S MECHANICAL STRUCTURE

The second part of Newton-Euler's formulation is dedicated to passing the robot's mechanical structure by inwards iterations. The connection forces, as resulted from Newton-Euler's algorithm, are presented in the equations below (31-43).

Given the complexity of the vectors ${}^i\bar{n}_i, i = \overline{5,1}$, representing the moments of connection forces, they cannot be shown extensively in this paper, but the reader is invited to find them, as resulted from the modeling application, at the following URL: http://www.ism.utcluj.ro/documents/ovidiu.detsan/2016_RTTRR_ATN.pdf

$$\begin{aligned}
{}^5\bar{f}_5 = & \left[\begin{aligned}
& {}^6f_x - M_5 \cdot {}^5r_{C_{5x}} \cdot c^2 q_5 \cdot \dot{q}_4^2 + M_5 c q_5 \cdot s q_4 \cdot \ddot{q}_3 - M_5 \cdot {}^5r_{C_{5y}} c q_5 \cdot \ddot{q}_4 - M_5 c q_5 \cdot c q_4 \cdot \ddot{q}_1 q_3 - \\
& - M_5 c q_5 \cdot c q_4 \cdot \ddot{q}_1 l_3 - M_5 c q_5 \cdot c q_4 \cdot \dot{q}_1^2 \cdot l_2 - 2 M_5 c q_5 \cdot c q_4 \cdot \dot{q}_3 \dot{q}_1 - M_5 c q_5 \cdot c q_4 \cdot \ddot{q}_1 l_4 + \\
& + M_5 c q_5 \cdot s q_4 \cdot \ddot{q}_1 l_2 - M_5 c q_5 \cdot s q_4 \cdot \dot{q}_1^2 \cdot q_3 - M_5 c q_5 \cdot s q_4 \cdot \dot{q}_1^2 \cdot l_3 - M_5 c q_5 \cdot s q_4 \cdot \dot{q}_1^2 \cdot l_4 - \\
& - 2 M_5 \cdot {}^5r_{C_{5x}} c^2 q_5 \cdot \dot{q}_1 \dot{q}_4 - M_5 c q_5 s q_5 \cdot {}^5r_{C_{5z}} \dot{q}_1^2 - M_5 c q_5 s q_5 \cdot {}^5r_{C_{5z}} \dot{q}_4^2 - M_5 s q_5 \cdot g - \\
& - M_5 s q_5 \cdot \ddot{q}_2 - M_5 \cdot {}^5r_{C_{5x}} \dot{q}_5^2 - 2 M_5 c q_5 s q_5 \cdot {}^5r_{C_{5x}} \dot{q}_1 \dot{q}_4 - M_5 \cdot {}^5r_{C_{5y}} c q_5 \ddot{q}_1 - M_5 \cdot {}^5r_{C_{5x}} c^2 q_5 \dot{q}_1^2 + \\
& + M_5 \ddot{q}_5 \cdot {}^5r_{C_{5z}} \\
& \hline
{}^6f_y - M_5 \cdot {}^5r_{C_{5y}} \cdot \dot{q}_4^2 - M_5 \cdot {}^5r_{C_{5y}} \cdot \dot{q}_1^2 - 2 M_5 \cdot {}^5r_{C_{5y}} \dot{q}_1 \cdot \dot{q}_4 - M_5 c q_4 \cdot \dot{q}_1^2 \cdot l_4 - M_5 c q_4 \cdot \dot{q}_1^2 l_3 - \\
& - M_5 c q_4 \cdot \dot{q}_1^2 q_3 + M_5 c q_4 \cdot \ddot{q}_1 l_2 + M_5 s q_4 \cdot \ddot{q}_1 l_4 + 2 M_5 s q_4 \cdot \dot{q}_3 \cdot \dot{q}_1 + \\
& + M_5 s q_4 \cdot \dot{q}_1^2 l_2 + M_5 s q_4 \cdot \ddot{q}_1 l_3 + M_5 s q_4 \cdot \ddot{q}_1 q_3 + M_5 c q_4 \cdot \ddot{q}_3 - 2 M_5 \cdot {}^5r_{C_{5x}} \cdot \dot{q}_5 s q_5 \cdot \dot{q}_1 - \\
& - 2 M_5 \cdot {}^5r_{C_{5x}} \cdot \dot{q}_5 s q_5 \cdot \dot{q}_4 + 2 M_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_5 c q_5 \cdot \dot{q}_1 + 2 M_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_5 c q_5 \cdot \dot{q}_4 + \\
& + M_5 \cdot {}^5r_{C_{5x}} \cdot c q_5 \cdot \ddot{q}_1 + M_5 \cdot {}^5r_{C_{5x}} \cdot c q_5 \cdot \ddot{q}_4 + M_5 \cdot {}^5r_{C_{5z}} \cdot s q_5 \cdot \ddot{q}_1 + M_5 \cdot {}^5r_{C_{5z}} \cdot s q_5 \cdot \ddot{q}_4 \\
& \hline
M_5 c q_5 \cdot g - M_5 \ddot{q}_5 \cdot {}^5r_{C_{5x}} + M_5 c q_5 \cdot \ddot{q}_2 - M_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_5^2 - M_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_4^2 - M_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_1^2 - \\
& - M_5 s q_5 c q_5 \cdot \ddot{q}_1 q_3 - 2 M_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_1 \dot{q}_4 + {}^6f_z + M_5 s q_5 s q_4 \cdot \ddot{q}_3 + M_5 \cdot {}^5r_{C_{5z}} \cdot \dot{q}_1^2 c^2 q_5 - \\
& - M_5 \cdot {}^5r_{C_{5y}} s q_5 \cdot \ddot{q}_4 - M_5 \cdot {}^5r_{C_{5y}} s q_5 \cdot \ddot{q}_1 + M_5 \cdot {}^5r_{C_{5z}} \dot{q}_4^2 c^2 q_5 - M_5 s q_5 c q_4 \cdot \ddot{q}_1 l_3 - \\
& - M_5 s q_5 s q_4 \cdot \dot{q}_1^2 l_4 - M_5 s q_5 c q_4 \cdot \ddot{q}_1 \cdot l_4 - M_5 s q_5 s q_4 \cdot \dot{q}_1^2 \cdot q_3 - M_5 s q_5 s q_4 \cdot \dot{q}_1^2 \cdot l_3 - \\
& - M_5 s q_5 c q_4 \cdot \dot{q}_1^2 \cdot l_2 - M_5 c q_5 s q_5 \cdot {}^5r_{C_{5x}} \dot{q}_1^2 - M_5 c q_5 s q_5 \cdot {}^5r_{C_{5x}} \dot{q}_4^2 - 2 M_5 c q_5 s q_5 \cdot {}^5r_{C_{5x}} \dot{q}_1 \cdot \dot{q}_4 + \\
& + 2 M_5 \cdot {}^5r_{C_{5z}} \dot{q}_1 \dot{q}_4 c^2 q_5 - 2 M_5 s q_5 c q_4 \cdot \dot{q}_3 \cdot \dot{q}_1 + M_5 s q_5 s q_4 \cdot \ddot{q}_1 \cdot l_2
\end{aligned} \right] \quad (31)
\end{aligned}$$

The vector ${}^4\bar{f}_4$ is expressed by its Cartesian components, as:

$$\begin{aligned}
{}^4f_x = & M_5 s q_4 \cdot \ddot{q}_3 + s q_5 \cdot {}^6f_z + c q_5 \cdot {}^6f_x - 2 s q_5 M_5 \cdot {}^5r_{C_{5z}} \dot{q}_1 \dot{q}_4 - s q_5 M_5 \ddot{q}_5 \cdot {}^5r_{C_{5x}} - s q_5 M_5 \cdot {}^5r_{C_{5z}} \dot{q}_5^2 - \\
& - s q_5 M_5 \cdot {}^5r_{C_{5z}} \dot{q}_4^2 - s q_5 M_5 \cdot {}^5r_{C_{5z}} \dot{q}_1^2 - c q_5 M_5 \cdot {}^5r_{C_{5x}} \dot{q}_5^2 + c q_5 M_5 \ddot{q}_5 \cdot {}^5r_{C_{5z}} - M_5 s q_4 \dot{q}_1^2 l_3 - \\
& - M_5 s q_4 \dot{q}_1^2 \cdot l_4 - M_5 c q_4 \ddot{q}_1 l_4 - M_5 s q_4 \dot{q}_1^2 q_3 - M_5 \cdot {}^5r_{C_{5y}} \ddot{q}_4 - M_5 \cdot {}^5r_{C_{5y}} \ddot{q}_1 - \\
& - 2 M_5 c q_5 \cdot {}^5r_{C_{5x}} \dot{q}_1 \dot{q}_4 - 2 M_5 c q_4 \cdot \dot{q}_3 \dot{q}_1 - M_5 c q_4 \ddot{q}_1 q_3 + M_5 s q_4 \cdot \ddot{q}_1 l_2 - M_5 c q_4 \dot{q}_1^2 \cdot l_2 - \\
& - M_5 c q_5 \cdot {}^5r_{C_{5x}} \dot{q}_1^2 - M_5 c q_5 \cdot {}^5r_{C_{5x}} \dot{q}_4^2 - M_5 c q_4 \cdot \ddot{q}_1 l_3 + M_4 s q_4 \cdot \ddot{q}_3 - M_4 \cdot {}^4r_{C_{4y}} \ddot{q}_1 - \\
& - M_4 \cdot {}^4r_{C_{4y}} \ddot{q}_4 - M_4 \cdot {}^4r_{C_{4x}} \dot{q}_1^2 - M_4 \cdot {}^4r_{C_{4x}} \dot{q}_4^2 - M_4 c q_4 \cdot \ddot{q}_1 q_3 - M_4 c q_4 \cdot \ddot{q}_1 l_3 - \\
& - M_4 c q_4 \cdot \dot{q}_1^2 l_2 - 2 M_4 c q_4 \cdot \dot{q}_3 \dot{q}_1 - M_4 c q_4 \cdot \ddot{q}_1 l_4 + M_4 s q_4 \cdot \ddot{q}_1 l_2 - M_4 s q_4 \cdot \dot{q}_1^2 q_3 - \\
& - M_4 s q_4 \cdot \dot{q}_1^2 l_3 - M_4 s q_4 \cdot \dot{q}_1^2 l_4 - 2 M_4 \cdot {}^4r_{C_{4x}} \dot{q}_1 \dot{q}_4 \quad (32)
\end{aligned}$$

$$\begin{aligned}
{}^4f_y = & -M_5 \cdot {}^5r_{C_{5y}} \dot{q}_1^2 - M_5 \cdot {}^5r_{C_{5y}} \dot{q}_4^2 + M_5 c q_4 \cdot \ddot{q}_3 - 2 M_5 \cdot {}^5r_{C_{5y}} \dot{q}_1 \dot{q}_4 - M_5 c q_4 \cdot \dot{q}_1^2 l_3 - \\
& - M_5 c q_4 \cdot \dot{q}_1^2 l_4 + M_5 \cdot {}^5r_{C_{5z}} s q_5 \ddot{q}_4 + M_5 \cdot {}^5r_{C_{5z}} s q_5 \ddot{q}_1 + M_5 \cdot {}^5r_{C_{5x}} c q_5 \ddot{q}_4 + M_5 \cdot {}^5r_{C_{5x}} c q_5 \ddot{q}_1 + \\
& + M_5 s q_4 \cdot \ddot{q}_1 q_3 + M_5 s q_4 \cdot \ddot{q}_1 l_3 + M_5 s q_4 \cdot \dot{q}_1^2 l_2 + 2 M_5 \cdot s q_4 \cdot \dot{q}_3 \dot{q}_1 + M_5 s q_4 \cdot \ddot{q}_1 l_4 + \\
& + M_5 c q_4 \cdot \ddot{q}_1 l_2 - M_5 c q_4 \cdot \dot{q}_1^2 q_3 + {}^6f_y - 2 M_5 \cdot {}^5r_{C_{5x}} \dot{q}_5 s q_5 \cdot \dot{q}_4 - 2 M_5 \cdot {}^5r_{C_{5x}} \dot{q}_5 s q_5 \cdot \dot{q}_1 + \\
& + 2 M_5 \cdot {}^5r_{C_{5z}} \dot{q}_5 c q_5 \cdot \dot{q}_4 + 2 M_5 \cdot {}^5r_{C_{5z}} \dot{q}_5 c q_5 \cdot \dot{q}_1 + M_4 c q_4 \cdot \ddot{q}_3 + M_4 \cdot {}^4r_{C_{4x}} \ddot{q}_1 + M_4 \cdot {}^4r_{C_{4x}} \ddot{q}_4 - \\
& - M_4 \cdot {}^4r_{C_{4y}} \dot{q}_1^2 - M_4 \cdot {}^4r_{C_{4y}} \dot{q}_4^2 + M_4 s q_4 \cdot \ddot{q}_1 q_3 + M_4 s q_4 \cdot \ddot{q}_1 l_3 + M_4 s q_4 \cdot \dot{q}_1^2 l_2 + \\
& + 2 M_4 s q_4 \cdot \dot{q}_3 \dot{q}_1 + M_4 s q_4 \cdot \ddot{q}_1 l_4 + M_4 c q_4 \cdot \ddot{q}_1 l_2 - M_4 c q_4 \cdot \dot{q}_1^2 q_3 - M_4 c q_4 \cdot \dot{q}_1^2 l_3 - \\
& - M_4 c q_4 \cdot \dot{q}_1^2 l_4 - 2 M_4 \cdot {}^4r_{C_{4y}} \dot{q}_1 \dot{q}_4 \quad (33)
\end{aligned}$$

$$\begin{aligned}
{}^4f_z = & M_5 g - s q_5 \cdot {}^6f_x + M_4 \ddot{q}_2 + c q_5 \cdot {}^6f_z + M_4 g + s q_5 M_5 \cdot {}^5r_{C_{5x}} \dot{q}_5^2 - s q_5 \cdot M_5 \ddot{q}_5 \cdot {}^5r_{C_{5z}} - \\
& - c q_5 \cdot M_5 \ddot{q}_5 \cdot {}^5r_{C_{5x}} - c q_5 \cdot M_5 \cdot {}^5r_{C_{5z}} \dot{q}_5^2 + M_5 \ddot{q}_2 \quad (34)
\end{aligned}$$

The vector ${}^3\bar{f}_3$ has the following Cartesian components:

$$\begin{aligned}
{}^3f_x = & -M_5\ddot{q}_1l_3 - M_4\ddot{q}_1q_3 - M_5\dot{q}_1^2 \cdot l_2 - M_5\ddot{q}_1l_4 - 2M_5\dot{q}_3\dot{q}_1 - cq_4sq_5M_5\ddot{q}_5{}^5r_{C_{5x}} - \\
& -cq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_5^2 - cq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_4^2 - cq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_1^2 - cq_4cq_5M_5{}^5r_{C_{5x}}\dot{q}_5^2 + \\
& + cq_4cq_5M_5\ddot{q}_5{}^5r_{C_{5z}} - 2cq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_1\dot{q}_4 - 2cq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_1\dot{q}_4 - cq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_1^2 - \\
& - cq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_4^2 - 2cq_4M_4{}^4r_{C_{4x}}\dot{q}_1\dot{q}_4 + 2sq_4M_5{}^5r_{C_{5y}}\dot{q}_1\dot{q}_4 - 2sq_4M_5{}^5r_{C_{5z}}\dot{q}_5cq_5 \cdot \dot{q}_1 - \\
& - sq_4M_5{}^5r_{C_{5z}}sq_5\ddot{q}_4 - sq_4M_5{}^5r_{C_{5z}}sq_5\ddot{q}_1 - sq_4M_5{}^5r_{C_{5x}}cq_5\ddot{q}_4 - sq_4M_5{}^5r_{C_{5x}}cq_5\ddot{q}_1 + \\
& + 2sq_4 \cdot M_5{}^5r_{C_{5x}}\dot{q}_5sq_5 \cdot \dot{q}_1 - 2sq_4 \cdot M_5{}^5r_{C_{5z}}\dot{q}_5cq_5 \cdot \dot{q}_4 + 2sq_4 \cdot M_5{}^5r_{C_{5x}}\dot{q}_5sq_5 \cdot \dot{q}_4 + \\
& + 2sq_4 \cdot M_4{}^4r_{C_{4y}}\dot{q}_1\dot{q}_4 - sq_4{}^6f_y - M_3\ddot{q}_1q_3 - M_3\ddot{q}_1l_3 - M_3\dot{q}_1^2 \cdot l_2 - 2M_3\dot{q}_3\dot{q}_1 - \\
& - M_3\ddot{q}_1{}^3r_{C_{3y}} - M_3\dot{q}_1^2{}^3r_{C_{3x}} + cq_4sq_5{}^6f_z + cq_4cq_5{}^6f_x - cq_4M_5{}^5r_{C_{5y}}\ddot{q}_4 - cq_4M_5{}^5r_{C_{5y}}\ddot{q}_1 - \\
& - cq_4M_4{}^4r_{C_{4y}}\ddot{q}_1 - cq_4M_4{}^4r_{C_{4y}}\ddot{q}_4 - cq_4M_4{}^4r_{C_{4x}}\dot{q}_1^2 - cq_4M_4{}^4r_{C_{4x}}\dot{q}_4^2 + sq_4M_5{}^5r_{C_{5y}}\dot{q}_1^2 + \\
& + sq_4 \cdot M_5{}^5r_{C_{5y}}\dot{q}_4^2 - sq_4M_4{}^4r_{C_{4x}}\ddot{q}_1 - sq_4M_4{}^4r_{C_{4x}}\ddot{q}_4 + sq_4M_4{}^4r_{C_{4y}}\dot{q}_1^2 + \\
& + sq_4M_4{}^4r_{C_{4y}}\dot{q}_4^2 - M_4\ddot{q}_1l_3 - M_4\dot{q}_1^2l_2 - M_4\ddot{q}_1l_4 - M_5\ddot{q}_1q_3
\end{aligned} \tag{35}$$

$$\begin{aligned}
{}^3f_y = & M_4\ddot{q}_3 + sq_4sq_5{}^6f_z + sq_4cq_5{}^6f_x - M_3\dot{q}_1^2{}^3r_{C_{3y}} + M_3\ddot{q}_1{}^3r_{C_{3x}} - M_3\dot{q}_1^2 \cdot l_3 - M_3\dot{q}_1^2q_3 + \\
& + M_3\dot{q}_3 + cq_4M_5{}^5r_{C_{5z}}sq_5 \cdot \ddot{q}_4 + cq_4M_5{}^5r_{C_{5z}}sq_5 \cdot \ddot{q}_1 + cq_4M_5{}^5r_{C_{5x}}cq_5 \cdot \ddot{q}_4 - \\
& - 2cq_4M_5{}^5r_{C_{5y}}\dot{q}_4 \cdot \dot{q}_4 - sq_4M_4{}^4r_{C_{4y}}\ddot{q}_4 + cq_4{}^6f_y - sq_4M_5{}^5r_{C_{5y}}\ddot{q}_4 - sq_4M_5{}^5r_{C_{5y}}\ddot{q}_1 - \\
& - sq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_1^2 - sq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_4^2 - 2sq_4M_4{}^4r_{C_{4x}}\dot{q}_1 \cdot \dot{q}_4 - sq_4sq_5M_5\ddot{q}_5{}^5r_{C_{5x}} - \\
& - 2sq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_1 \cdot \dot{q}_4 - sq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_5^2 - sq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_4^2 - sq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_1^2 - \\
& - sq_4cq_5M_5{}^5r_{C_{5x}}\dot{q}_5^2 + sq_4cq_5M_5\ddot{q}_5{}^5r_{C_{5z}} - 2sq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_1 \cdot \dot{q}_4 + cq_4M_5{}^5r_{C_{5x}}cq_5 \cdot \ddot{q}_1 + \\
& + 2cq_4M_5{}^5r_{C_{5z}}\dot{q}_5cq_5 \cdot \dot{q}_4 + 2cq_4M_5{}^5r_{C_{5z}}\dot{q}_5cq_5 \cdot \dot{q}_1 - 2cq_4M_5{}^5r_{C_{5x}}\dot{q}_5sq_5 \cdot \dot{q}_4 - \\
& - 2cq_4M_4{}^4r_{C_{4z}}\dot{q}_1\dot{q}_4 - 2cq_4M_5{}^5r_{C_{5x}}\dot{q}_5sq_5 \cdot \dot{q}_1 - cq_4M_4{}^4r_{C_{4y}}\dot{q}_4^2 - cq_4M_4{}^4r_{C_{4y}}\dot{q}_1^2 - \\
& - cq_4M_5{}^5r_{C_{5y}}\dot{q}_4^2 + cq_4M_4{}^4r_{C_{4x}}\ddot{q}_1 + cq_4M_4{}^4r_{C_{4x}}\ddot{q}_4 - cq_4M_5{}^5r_{C_{5y}}\dot{q}_1^2 - sq_4M_4{}^4r_{C_{4y}}\ddot{q}_1 - \\
& - sq_4M_4{}^4r_{C_{4x}}\dot{q}_1^2 - sq_4M_4{}^4r_{C_{4x}}\dot{q}_4^2 + M_3\ddot{q}_1l_2 + M_5\dot{q}_1^2 \cdot l_3 - M_4\dot{q}_1^2 \cdot l_4 + M_4\ddot{q}_1l_2 - \\
& - M_5\dot{q}_1^2 \cdot l_4 + M_5\ddot{q}_1l_2 - M_5\dot{q}_1^2 \cdot q_3 + M_5\ddot{q}_3 + M_4\dot{q}_1^2 \cdot l_3 - M_4\dot{q}_1^2 \cdot q_3
\end{aligned} \tag{36}$$

$$\begin{aligned}
{}^3f_z = & M_5g - sq_5{}^6f_x + M_4\ddot{q}_2 + cq_5{}^6f_z + M_4g + sq_5M_5{}^5r_{C_{5x}}\dot{q}_5^2 - sq_5M_5\ddot{q}_5{}^5r_{C_{5z}} - \\
& - cq_5M_5\ddot{q}_5{}^5r_{C_{5x}} - cq_5M_5{}^5r_{C_{5z}}\dot{q}_5^2 + M_5\ddot{q}_2 + M_3g + M_3\ddot{q}_2
\end{aligned} \tag{37}$$

and the vector ${}^2\bar{f}_2$ is characterized by:

$$\begin{aligned}
{}^2f_x = & -M_2\dot{q}_1^2{}^2r_{C_{2x}} - M_2\ddot{q}_1{}^2r_{C_{2y}} - M_2\ddot{q}_1l_3 - M_4\ddot{q}_1q_3 - M_5\dot{q}_1^2 \cdot l_2 - M_5\ddot{q}_1l_4 - 2M_5\dot{q}_3\dot{q}_1 - \\
& - 2M_4\dot{q}_3\dot{q}_1 - cq_4sq_5M_5\ddot{q}_5{}^5r_{C_{5x}} - cq_4sq_5M_5{}^5r_{C_{5x}}\dot{q}_5^2 - cq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_4^2 - \\
& - cq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_1^2 - cq_4cq_5M_5{}^5r_{C_{5x}}\dot{q}_5^2 + cq_4cq_5M_5\ddot{q}_5{}^5r_{C_{5z}} - 2cq_4sq_5M_5{}^5r_{C_{5z}}\dot{q}_1\dot{q}_4 - \\
& - 2cq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_1\dot{q}_4 - cq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_1^2 - cq_4M_5cq_5{}^5r_{C_{5x}}\dot{q}_4^2 - 2cq_4M_4{}^4r_{C_{4x}}\dot{q}_1\dot{q}_4 + \\
& + 2sq_4M_5{}^5r_{C_{5y}}\dot{q}_1\dot{q}_4 - 2sq_4M_5{}^5r_{C_{5z}}\dot{q}_5cq_5 \cdot \dot{q}_1 - sq_4M_5{}^5r_{C_{5z}}sq_5\ddot{q}_4 - sq_4M_5{}^5r_{C_{5z}}sq_5\ddot{q}_1 - \\
& - sq_4M_5{}^5r_{C_{5x}}cq_5\ddot{q}_4 - sq_4M_5{}^5r_{C_{5x}}cq_5\ddot{q}_1 + 2sq_4M_5{}^5r_{C_{5x}}\dot{q}_5sq_5 \cdot \dot{q}_1 - \\
& - 2sq_4M_5{}^5r_{C_{5z}}\dot{q}_5cq_5 \cdot \dot{q}_4 + 2sq_4M_5{}^5r_{C_{5x}}\dot{q}_5sq_5 \cdot \dot{q}_4 + 2sq_4M_4{}^4r_{C_{4y}}\dot{q}_1\dot{q}_4 - sq_4{}^6f_y - \\
& - M_3\ddot{q}_1q_3 - M_3\ddot{q}_1l_3 - M_3\dot{q}_1^2 \cdot l_2 - 2M_3\dot{q}_3\dot{q}_1 - M_3\ddot{q}_1{}^3r_{C_{3y}} - M_3\dot{q}_1^2{}^3r_{C_{3x}} + cq_4sq_5{}^6f_z + \\
& + cq_4cq_5{}^6f_x - cq_4M_5{}^5r_{C_{5y}}\ddot{q}_4 - cq_4M_5{}^5r_{C_{5y}}\ddot{q}_1 - cq_4M_4{}^4r_{C_{4y}}\ddot{q}_1 - cq_4M_4{}^4r_{C_{4y}}\ddot{q}_4 - \\
& - cq_4M_4{}^4r_{C_{4x}}\dot{q}_1^2 - cq_4M_4{}^4r_{C_{4x}}\dot{q}_4^2 + sq_4M_5{}^5r_{C_{5y}}\dot{q}_1^2 + sq_4M_5{}^5r_{C_{5y}}\dot{q}_4^2 - sq_4M_4{}^4r_{C_{4x}}\ddot{q}_1 - \\
& - sq_4M_4{}^4r_{C_{4x}}\ddot{q}_4 + sq_4M_4{}^4r_{C_{4y}}\dot{q}_1^2 + sq_4M_4{}^4r_{C_{4y}}\dot{q}_4^2 - M_4\ddot{q}_1l_3 - M_4\dot{q}_1^2 \cdot l_2 - M_4\ddot{q}_1l_4 - M_5\ddot{q}_1q_3
\end{aligned} \tag{38}$$

$$\begin{aligned}
{}^2f_y = & M_4\ddot{q}_3 + sq_4sq_5\cdot{}^6f_z + sq_4cq_5\cdot{}^6f_x - M_3\dot{q}_1^2\cdot{}^3r_{C_{3y}} + M_3\ddot{q}_1\cdot{}^3r_{C_{3x}} - M_3\dot{q}_1^2\cdot l_3 - M_3\dot{q}_1^2\cdot q_3 + \\
& + M_3\ddot{q}_3 + cq_4M_5\cdot{}^5r_{C_{5z}}sq_5\ddot{q}_4 + cq_4M_5\cdot{}^5r_{C_{5x}}sq_5\ddot{q}_1 + cq_4M_5\cdot{}^5r_{C_{5x}}cq_5\ddot{q}_4 - \\
& - 2cq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_1\dot{q}_4 - sq_4M_4\cdot{}^4r_{C_{4y}}\ddot{q}_4 + cq_4\cdot{}^6f_y - sq_4M_5\cdot{}^5r_{C_{5y}}\ddot{q}_4 - sq_4M_5\cdot{}^5r_{C_{5y}}\ddot{q}_1 - \\
& - sq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_1^2 - sq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_4^2 - 2sq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_1\dot{q}_4 - sq_4sq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5x}} - \\
& - 2sq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_1\dot{q}_4 - sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 - sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_4^2 - sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_1^2 - \\
& - sq_4cq_5M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 + 2sq_4cq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5z}} - 2sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_1\dot{q}_4 + cq_4M_5\cdot{}^5r_{C_{5x}}cq_5\ddot{q}_1 + \\
& + 2cq_4M_5\cdot{}^5r_{C_{5z}}\dot{q}_5cq_5\cdot\dot{q}_4 + 2cq_4M_5\cdot{}^5r_{C_{5z}}\dot{q}_5cq_5\cdot\dot{q}_1 - 2cq_4M_5\cdot{}^5r_{C_{5x}}\dot{q}_5sq_5\cdot\dot{q}_4 - \\
& - 2cq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_1\dot{q}_4 - 2cq_4M_5\cdot{}^5r_{C_{5x}}\dot{q}_5sq_5\cdot\dot{q}_1 - cq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_4^2 - cq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_1^2 - \\
& - cq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_4^2 + cq_4M_4\cdot{}^4r_{C_{4x}}\ddot{q}_1 + cq_4M_4\cdot{}^4r_{C_{4x}}\ddot{q}_4 - cq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_1^2 - sq_4M_4\cdot{}^4r_{C_{4y}}\ddot{q}_1 - \\
& - sq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_1^2 - sq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_4^2 + M_3\ddot{q}_1l_2 + M_2\dot{q}_1^2\cdot{}^2r_{C_{2x}} - M_2\dot{q}_1^2\cdot{}^2r_{C_{2y}} - M_5\dot{q}_1^2\cdot l_3 - \\
& - M_4\dot{q}_1^2\cdot l_4 + M_4\ddot{q}_1l_2 - M_5\dot{q}_1^2\cdot l_4 + M_5\ddot{q}_1l_2 - M_5\dot{q}_1^2\cdot q_3 + M_5\ddot{q}_3 - M_4\dot{q}_1^2\cdot l_3 - M_4\dot{q}_1^2\cdot q_3 \\
{}^2f_z = & M_5g - sq_5\cdot{}^6f_x + M_4\ddot{q}_2 + cq_5\cdot{}^6f_z + M_4g + sq_5M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 - sq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5z}} - \\
& - cq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5x}} - cq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 + M_5\ddot{q}_2 + M_3g + M_3\ddot{q}_2 + M_2g + M_2\ddot{q}_2
\end{aligned} \tag{39}$$

$$\begin{aligned}
{}^2f_z = & M_5g - sq_5\cdot{}^6f_x + M_4\ddot{q}_2 + cq_5\cdot{}^6f_z + M_4g + sq_5M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 - sq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5z}} - \\
& - cq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5x}} - cq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 + M_5\ddot{q}_2 + M_3g + M_3\ddot{q}_2 + M_2g + M_2\ddot{q}_2
\end{aligned} \tag{40}$$

The components of ${}^1\bar{f}_1$ are eventually represented by:

$$\begin{aligned}
{}^1f_x = & -M_2\dot{q}_1^2\cdot{}^2r_{C_{2x}} - M_2\ddot{q}_1\cdot{}^2r_{C_{2y}} - M_5\ddot{q}_1l_3 - M_4\ddot{q}_1q_3 - M_5\dot{q}_1^2\cdot l_2 - M_5\ddot{q}_1l_4 - 2M_5\dot{q}_3\dot{q}_1 - \\
& - 2M_4\dot{q}_3\dot{q}_1 - M_1\ddot{q}_1\cdot{}^1r_{C_{1y}} - M_1\dot{q}_1^2\cdot{}^1r_{C_{1x}} - cq_4sq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5x}} - cq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 - \\
& - cq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_4^2 - cq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_1^2 - cq_4cq_5M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 + cq_4cq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5z}} - \\
& - 2cq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_1\dot{q}_4 - 2cq_4M_5cq_5\cdot{}^5r_{C_{5z}}\dot{q}_1\dot{q}_4 - cq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_1^2 - \\
& - cq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_4^2 - 2cq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_1\dot{q}_4 + 2sq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_1\dot{q}_4 - 2sq_4M_5\cdot{}^5r_{C_{5z}}\dot{q}_5cq_5\dot{q}_1 - \\
& - sq_4M_5\cdot{}^5r_{C_{5z}}sq_5\ddot{q}_4 - sq_4M_5\cdot{}^5r_{C_{5z}}sq_5\ddot{q}_1 - sq_4M_5\cdot{}^5r_{C_{5x}}cq_5\ddot{q}_4 - sq_4M_5\cdot{}^5r_{C_{5x}}cq_5\ddot{q}_1 + \\
& + 2sq_4M_5\cdot{}^5r_{C_{5x}}\dot{q}_5sq_5\dot{q}_1 - 2sq_4M_5\cdot{}^5r_{C_{5z}}\dot{q}_5cq_5\dot{q}_4 + 2sq_4M_5\cdot{}^5r_{C_{5x}}\dot{q}_5sq_5\dot{q}_4 + \\
& + 2sq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_1\dot{q}_4 - sq_4\cdot{}^6f_y - M_3\ddot{q}_1q_3 - M_3\ddot{q}_1l_3 - M_3\dot{q}_1^2\cdot l_2 - 2M_3\dot{q}_3\dot{q}_1 - M_3\ddot{q}_1\cdot{}^3r_{C_{3y}} - \\
& - M_3\dot{q}_1^2\cdot{}^3r_{C_{3x}} + cq_4sq_5\cdot{}^6f_z + cq_4cq_5\cdot{}^6f_x - cq_4M_5\cdot{}^5r_{C_{5y}}\ddot{q}_4 - cq_4M_5\cdot{}^5r_{C_{5y}}\ddot{q}_1 - \\
& - cq_4M_4\cdot{}^4r_{C_{4y}}\ddot{q}_1 - cq_4M_4\cdot{}^4r_{C_{4y}}\ddot{q}_4 - cq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_1^2 - cq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_4^2 + sq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_1^2 + \\
& + sq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_4^2 - sq_4M_4\cdot{}^4r_{C_{4x}}\ddot{q}_1 - sq_4M_4\cdot{}^4r_{C_{4x}}\ddot{q}_4 + sq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_1^2 + sq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_4^2 - \\
& - M_4\ddot{q}_1l_3 - M_4\dot{q}_1^2\cdot l_2 - M_4\ddot{q}_1l_4 - M_5\ddot{q}_1q_3
\end{aligned} \tag{41}$$

$$\begin{aligned}
{}^1f_y = & M_4\ddot{q}_3 + sq_4sq_5\cdot{}^6f_z + sq_4cq_5\cdot{}^6f_x - M_3\dot{q}_1^2\cdot{}^3r_{C_{3y}} + M_3\ddot{q}_1\cdot{}^3r_{C_{3x}} - M_3\dot{q}_1^2\cdot l_3 - M_3\dot{q}_1^2\cdot q_3 + \\
& + M_3\ddot{q}_3 + cq_4M_5\cdot{}^5r_{C_{5z}}sq_5\ddot{q}_4 + cq_4M_5\cdot{}^5r_{C_{5z}}sq_5\ddot{q}_1 + cq_4M_5\cdot{}^5r_{C_{5x}}cq_5\ddot{q}_4 - \\
& - 2cq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_1\dot{q}_4 - sq_4M_4\cdot{}^4r_{C_{4y}}\ddot{q}_4 + cq_4\cdot{}^6f_y - sq_4M_5\cdot{}^5r_{C_{5y}}\ddot{q}_4 - sq_4M_5\cdot{}^5r_{C_{5y}}\ddot{q}_1 - \\
& - sq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_1^2 - sq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_4^2 - 2sq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_1\dot{q}_4 - sq_4sq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5x}} - \\
& - 2sq_4M_5cq_5\cdot{}^5r_{C_{5x}}\dot{q}_1\dot{q}_4 - sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 - sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_4^2 - sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_1^2 - \\
& - sq_4cq_5M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 + sq_4cq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5z}} - 2sq_4sq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_1\dot{q}_4 + cq_4M_5\cdot{}^5r_{C_{5x}}cq_5\cdot\ddot{q}_1 + \\
& + 2cq_4M_5\cdot{}^5r_{C_{5z}}\dot{q}_5cq_5\cdot\dot{q}_4 + 2cq_4M_5\cdot{}^5r_{C_{5z}}\dot{q}_5cq_5\cdot\dot{q}_1 - 2cq_4M_5\cdot{}^5r_{C_{5x}}\dot{q}_5sq_5\cdot\dot{q}_4 - \\
& - 2cq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_1\dot{q}_4 - 2cq_4M_5\cdot{}^5r_{C_{5x}}\dot{q}_5sq_5\dot{q}_1 - cq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_4^2 - cq_4M_4\cdot{}^4r_{C_{4y}}\dot{q}_1^2 - \\
& - cq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_4^2 + cq_4M_4\cdot{}^4r_{C_{4x}}\ddot{q}_1 + cq_4M_4\cdot{}^4r_{C_{4x}}\ddot{q}_4 - cq_4M_5\cdot{}^5r_{C_{5y}}\dot{q}_1^2 - sq_4M_4\cdot{}^4r_{C_{4y}}\ddot{q}_1 - \\
& - sq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_1^2 - sq_4M_4\cdot{}^4r_{C_{4x}}\dot{q}_4^2 + M_3\ddot{q}_1l_2 + M_2\dot{q}_1^2\cdot{}^2r_{C_{2x}} - M_2\dot{q}_1^2\cdot{}^2r_{C_{2y}} - M_5\dot{q}_1^2\cdot l_3 - \\
& - M_4\dot{q}_1^2\cdot l_4 + M_4\ddot{q}_1l_2 - M_5\dot{q}_1^2\cdot l_4 + M_1\ddot{q}_1\cdot{}^1r_{C_{1x}} - M_1\dot{q}_1^2\cdot{}^1r_{C_{1y}} + M_5\ddot{q}_1l_2 - M_5\dot{q}_1^2\cdot q_3 + \\
& + M_5\ddot{q}_3 - M_4\dot{q}_1^2\cdot l_3 - M_4\dot{q}_1^2\cdot q_3
\end{aligned} \tag{42}$$

$$\begin{aligned}
{}^1f_z = & M_5g - sq_5\cdot{}^6f_x + M_4\ddot{q}_2 + cq_5\cdot{}^6f_z + M_4g + sq_5M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 - sq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5z}} - \\
& - cq_5M_5\ddot{q}_5\cdot{}^5r_{C_{5x}} - cq_5M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 + M_5\ddot{q}_2 + M_3g + M_3\ddot{q}_2 + M_2g + M_2\ddot{q}_2 + M_1g
\end{aligned} \tag{43}$$

The connection forces and their moments are useful for determining the system of dynamic differential equations that characterizes the dynamic model of RTTRR serial robot.

5. CONCLUSIONS

The equation complexity of the connection forces and their moments is justified by the large number of degrees of freedom of the analyzed robot and is also due to the fact that, apart from the simplified models, where the centrifugal moments of inertia are usually taken as zero, they can be found here symbolically and they can be substituted by numerical values taken from the CAD model of the robot.

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Modelul dinamic al robotului serial de tip RTTRR – iterații spre exteriorul și spre interiorul structurii mecanice a robotului

Rezumat: Lucrarea prezintă primul pas în procesul de determinare a modelului dinamic al robotului serial RTTRR. După stabilirea datelor de intrare, sunt determinate forțele externe și momentele acestora, în urma parcurgerii structurii mecanice a robotului prin iterații spre exterior. Lucrarea se încheie cu iterațiile spre interiorul structurii mecanice a robotului, cu scopul de a determina forțele de legătură și momentele acestora.

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