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THE DYNAMIC MODEL OF RTTRR SERIAL ROBOT – THE DETERMINATION OF THE GENERALIZED DRIVING FORCES

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Abstract: *The paper presents the final part of the determination of the dynamic model of RTTRR serial robot, consisting in the differential dynamic equations of the generalized driving forces, which take into account all the components of the mass distribution parameters and of the useful payload. The modeling process is accomplished using Newton-Euler's formulation, implemented into the generalized modeling application Robot_Symbolic [1] and the obtained equations are useful in the simulation of the dynamic behavior of the analyzed robot.*
Key words: *dynamic model, serial robot, symbolic computation, generalized driving forces.*

1. INTRODUCTION

It is not an easy task to determine the equations of the dynamic model of a robot with five or six degrees of freedom, especially when there are many rotational joint in the robot's structure. The aim of the dynamic model is to determine the generalized driving forces, taking into account the dynamic requirements of the task, expressed by the useful payload (forces and moments) and the mass distribution parameters, such as: masses, mass centers and inertial tensors. The dynamic model is therefore general and it pays attention to the causes that produce motion and also to the causes that restrict motion.

2. PRELIMINARY DATA

The generalized driving forces of the RTTRR serial robot, whose structure is described in [2], are determined by Newton-Euler's method [3], implemented into the module *Robot_Dynamics* [4] from the generalized modeling application *Robot_ Symbolic* [1]. The previously analyzed robots – TRR [5, 6, 7], TRT2 [8] and other robots with three degrees of freedom [9] RTTR [10], TRTR [11], VIPAS2 [12, 13], TRTRR [14], TRTTR1 [15], TRTTRR1 [16] – were modeled with some simplifying hypotheses, such as that a mass with a constant value was manipulated in the robot task, with no contact with the environment, or that the joint axes were the principal axes of inertia, leading to cancelling the centrifugal terms from the inertial tensors.

Unless the above mentioned robots, in the case of RTTRR robot, the mass center position vectors of links i with respect to the $\{i\}$ frame origin, as well as the generalized inertia tensors, corresponding to each link i, are expressed, each of them, generally, with none of the elements of the corresponding vectors or matrices to be considered as zero.

The useful payload is also defined by the general expressions of the force-moment vectors.

Using this input data, the external forces and their moments were determined, passing the mechanical structure by outwards iterations and the connection forces and their moments were calculated, by iterations inwards the robot's mechanical structure [17].

3. THE GENERALIZED DRIVING FORCES OF RTTRR SERIAL ROBOT

Knowing the connection forces and their moments, the generalized driving forces are determined as:

$$
\begin{aligned}
Q_m^1 = {} & {}^1J_z^*\ddot{q}_1 + {}^5J_x^*\ddot{q}_1 - {}^5J_x^*\ddot{q}_1 c^2 q_5 + 2l_4 M_5 \dot{q}_3\dot{q}_1 + {}^1r_{C_{1x}}^2 M_1\ddot{q}_1 + {}^1r_{C_{1y}}^2 M_1\ddot{q}_1 + {}^4J_z^*\ddot{q}_1 + {}^4J_z^*\ddot{q}_4 + cq_5\cdot{}^6n_z - \\
& - 2\cdot{}^5r_{C_{5z}} M_5\cdot{}^5r_{C_{5x}}\dot{q}_5\dot{q}_4 + {}^2J_z^*\ddot{q}_1 - sq_5 l_6\cdot{}^6f_z + sq_5\cdot{}^5r_{C_{5y}} M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 + sq_5\cdot{}^5r_{C_{5z}} M_5 cq_4\ddot{q}_3 + \\
& + 4\cdot{}^5r_{C_{5x}} M_5\cdot{}^5r_{C_{5z}}\dot{q}_5 c^2 q_5\dot{q}_1 + 2sq_5 M_5\cdot{}^5r_{C_{5z}}^2\dot{q}_5 cq_5\dot{q}_4 + 2sq_5\cdot{}^5r_{C_{5z}} M_5 cq_4\ddot{q}_1 l_2 + 2sq_5\cdot{}^5r_{C_{5z}} M_5 sq_4\ddot{q}_1 l_4 + \\
& + 2sq_5{}^5J_x^*\dot{q}_5 cq_5\dot{q}_1 + M_4\cdot{}^4r_{C_{4y}}^2\ddot{q}_1 + M_4\cdot{}^4r_{C_{4y}}^2\ddot{q}_4 + M_4\cdot{}^4r_{C_{4x}}^2\ddot{q}_1 + M_4\cdot{}^4r_{C_{4x}}^2\ddot{q}_4 + 2\cdot{}^4r_{C_{4x}} M_4 sq_4\ddot{q}_1 l_3 + \\
& + 2\cdot{}^4r_{C_{4x}} M_4 sq_4\dot{q}_3\dot{q}_1 + 2\cdot{}^4r_{C_{4x}} M_4 sq_4\ddot{q}_1 l_4 + 2\cdot{}^4r_{C_{4x}} M_4 cq_4\ddot{q}_1 l_2 + 2\cdot{}^4r_{C_{4y}} M_4 cq_4\ddot{q}_1 q_3 + \\
& + 2\cdot{}^4r_{C_{4y}} M_4 cq_4\ddot{q}_1 l_3 + 2\cdot{}^4r_{C_{4y}} M_4 cq_4\dot{q}_3\dot{q}_1 + 2\cdot{}^4r_{C_{4y}} M_4 cq_4\ddot{q}_1 l_4 - 2\cdot{}^4r_{C_{4y}} M_4 sq_4\ddot{q}_1 l_2 + \\
& + 2cq_5\cdot{}^5r_{C_{5x}} M_5 sq_4\ddot{q}_1 l_3 + 2\cdot{}^4r_{C_{4x}} M_4 sq_4\ddot{q}_1 q_3 - 2cq_5 M_5\cdot{}^5r_{C_{5x}}^2\dot{q}_5 sq_5\dot{q}_1 - 2cq_5 M_5\cdot{}^5r_{C_{5x}}^2\dot{q}_5 sq_5\dot{q}_4 + \\
& + 2cq_5\cdot{}^5r_{C_{5x}} M_5 sq_4\dot{q}_3\dot{q}_1 + 2cq_5\cdot{}^5r_{C_{5x}} M_5 cq_4\ddot{q}_1 l_2 + 2cq_5\cdot{}^5r_{C_{5x}} M_5 sq_4\ddot{q}_1 l_4 + 2cq_5\cdot{}^5r_{C_{5x}} M_5 sq_4\ddot{q}_1 q_3 + \\
& .+ 4\cdot{}^5r_{C_{5x}} M_5\cdot{}^5r_{C_{5z}}\dot{q}_5 c^2 q_5\dot{q}_4 + sq_5{}^5J_{xy}^*\ddot{q}_5 + {}^5J_z^* c^2 q_5\cdot\ddot{q}_4 - cq_5 l_6\cdot{}^6f_x + cq_5\dot{q}_5^2\cdot{}^5J_{xy}^* - \\
& - cq_5\cdot{}^5J_{zy}^*\ddot{q}_5 + {}^5J_z^* c^2 q_5\ddot{q}_1 + sq_5\cdot{}^5J_{xz}^* cq_5\ddot{q}_1 + 2\cdot{}^5J_{zx}^*\dot{q}_5 c^2 q_5\dot{q}_4 + 2c^2 q_5\dot{q}_5\cdot{}^5J_{xz}^*\dot{q}_1 + \\
& + 2\cdot{}^5J_{zx}^*\dot{q}_5 c^2 q_5\dot{q}_1 + 2c^2 q_5\dot{q}_5\cdot{}^5J_{xz}^*\dot{q}_4 - {}^4r_{C_{4y}} M_4 sq_4\ddot{q}_3 + {}^4r_{C_{4x}} M_4 cq_4\ddot{q}_3 - 2sq_5 cq_5\dot{q}_5\cdot{}^5J_z^*\dot{q}_1 - \\
& - 2sq_5 cq_5\dot{q}_5\cdot{}^5J_z^*\dot{q}_4 - sq_5\cdot{}^6n_x - cq_5\cdot{}^5r_{C_{5y}} M_5\ddot{q}_5\cdot{}^5r_{C_{5z}} + cq_5\cdot{}^5r_{C_{5y}} M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 + \\
& + cq_5\cdot{}^5r_{C_{5x}} M_5 cq_4\ddot{q}_3 + 2sq_5\cdot{}^5r_{C_{5z}} M_5 sq_4\ddot{q}_1 l_3 + 2sq_5\cdot{}^5r_{C_{5z}} M_5 sq_4\ddot{q}_1 q_3 + 2sq_5\cdot{}^5J_x^*\dot{q}_5 cq_5\dot{q}_4 + \\
& + sq_5\cdot{}^5r_{C_{5y}} M_5\ddot{q}_5\cdot{}^5r_{C_{5x}} + cq_5\cdot{}^5J_{zx}^* sq_5\ddot{q}_4 + 2sq_5\cdot{}^5r_{C_{5z}} M_5 sq_4\dot{q}_3\dot{q}_1 + 2sq_5\cdot{}^5r_{C_{5z}} M_5\cdot{}^5r_{C_{5x}} cq_5\ddot{q}_1 + \\
& + 2sq_5\cdot{}^5r_{C_{5z}} M_5\cdot{}^5r_{C_{5x}} cq_5\ddot{q}_4 + 2sq_5 M_5\cdot{}^5r_{C_{5x}}^2\dot{q}_5 cq_5\dot{q}_1 + sq_5\cdot{}^5J_{xz}^* cq_5\ddot{q}_4 + sq_5\dot{q}_5^2\cdot{}^5J_{zy}^* + \\
& + cq_5\cdot{}^5J_{zx}^* sq_5\ddot{q}_1 + M_5\cdot{}^5r_{C_{5x}}^2 c^2 q_5\ddot{q}_4 + M_5\cdot{}^5r_{C_{5x}}^2 c^2 q_5\ddot{q}_1 + {}^3J_z^*\ddot{q}_1 - {}^5r_{C_{5y}} M_5 sq_4\ddot{q}_3 - \\
& - 2\cdot{}^5r_{C_{5y}} M_5 sq_4\ddot{q}_1 l_2 + {}^5J_x^*\ddot{q}_4 - {}^5J_x^*\ddot{q}_4 c^2 q_5 - 2\cdot{}^5r_{C_{5z}} M_5\cdot{}^5r_{C_{5x}}\dot{q}_5\dot{q}_1 + 2\cdot{}^5r_{C_{5y}} M_5 cq_4\ddot{q}_1 l_3 - \\
& - \dot{q}_5\cdot{}^5J_{zx}^*\dot{q}_1 - \dot{q}_5\cdot{}^5J_{zx}^*\dot{q}_4 - {}^5J_{xz}^*\dot{q}_5\dot{q}_4 - {}^5J_{xz}^*\dot{q}_5\dot{q}_1 + M_5\cdot{}^5r_{C_{5y}}^2\ddot{q}_4 + M_5\cdot{}^5r_{C_{5y}}^2\ddot{q}_1 + \\
& + 2\cdot{}^5r_{C_{5y}} M_5 cq_4\ddot{q}_1 q_3 + M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_1 - M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_1 c^2 q_5 + M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_4 - M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_4 c^2 q_5 + \\
& + 2\cdot{}^5r_{C_{5y}} M_5 cq_4\dot{q}_3\dot{q}_1 + 2\cdot{}^5r_{C_{5y}} M_5 cq_4\ddot{q}_1 l_4 + {}^2r_{C_{2y}}^2 M_2\ddot{q}_1 + {}^2r_{C_{2x}}^2 M_2\ddot{q}_1 + M_4\ddot{q}_1 q_3^2 + M_5\ddot{q}_1 q_3^2 + \\
& + M_3\ddot{q}_1 q_3^2 + M_4\ddot{q}_1 l_3^2 + M_3\ddot{q}_1 l_3^2 + M_5\ddot{q}_1 l_3^2 + M_3\ddot{q}_1 l_2^2 + M_5\ddot{q}_1 l_2^2 + M_4\ddot{q}_1 l_2^2 + l_2 M_3\ddot{q}_3 + l_2 M_5\ddot{q}_3 + \\
& + l_2 M_4\ddot{q}_3 + q_3 sq_4\cdot{}^6f_y + l_3 sq_4\cdot{}^6f_y + l_2 cq_4\cdot{}^6f_y + 2l_3 sq_4 M_5\cdot{}^5r_{C_{5z}}\dot{q}_5 cq_5\dot{q}_4 + \\
& + 2l_3 sq_4 M_5\cdot{}^5r_{C_{5z}}\dot{q}_5 cq_5\dot{q}_1 - 2l_3 sq_4 M_5\cdot{}^5r_{C_{5x}}\dot{q}_5 sq_5\dot{q}_4 - 2l_3 sq_4 M_5\cdot{}^5r_{C_{5x}}\dot{q}_5 sq_5\dot{q}_1 - \\
& - 2l_3 sq_4 M_4\cdot{}^4r_{C_{4y}}\dot{q}_1\dot{q}_4 + l_3 cq_4 sq_5 M_5\cdot{}^5r_{C_{5z}}\dot{q}_4^2 + l_3 cq_4 cq_5 M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2 + 2l_3 cq_4 M_4{}^4r_{C_{4x}}\dot{q}_1\dot{q}_4 + \\
& + 2l_3 cq_4 M_5 cq_5\cdot{}^5r_{C_{5x}}\dot{q}_1\dot{q}_4 + l_3 sq_4 M_4{}^4r_{C_{4x}}\ddot{q}_4 + l_3 cq_4 M_5\cdot{}^5r_{C_{5y}}\ddot{q}_4 + l_3 cq_4 M_5\cdot{}^4r_{C_{4y}}\ddot{q}_4 + \\
& + l_3 cq_4 M_4\cdot{}^4r_{C_{4x}}\dot{q}_4^2 - l_3 sq_4 M_5\cdot{}^5r_{C_{5x}}\dot{q}_4^2 - l_3 sq_4 M_4\cdot{}^4r_{C4y}\dot{q}_4^2 - l_2 sq_4 sq_5 M_5\cdot{}^5r_{C_{5z}}\dot{q}_5^2 - \\
& - 2l_2 cq_4 M_5\cdot{}^5r_{C_{5y}}\dot{q}_1\dot{q}_4 + 2l_3 cq_4 sq_5 M_5\cdot{}^5r_{C_{5z}}\dot{q}_1\dot{q}_4 + 2l_2 cq_4 M_5\cdot{}^5r_{C_{5z}}\dot{q}_5 cq_5\dot{q}_4 + \\
& + 2l_2 cq_4 M_5\cdot{}^5r_{C_{5z}}\dot{q}_5 cq_5\dot{q}_1 - 2l_2 cq_4 M_5\cdot{}^5r_{C5x}\dot{q}_5 sq_5\dot{q}_1 - 2l_2 sq_4 M_5 cq_5\cdot{}^5r_{C5x}\dot{q}_1\dot{q}_4 - \\
& - l_2 sq_4 M_5 cq_5\cdot{}^5r_{C5x}\dot{q}_4^2 - 2l_2 sq_4 sq_5 M_5\cdot{}^5r_{Cz}\dot{q}_1\dot{q}_4 - l_2 sq_4 sq_5 M_5\cdot{}^5r_{C5z}\dot{q}_4^2 - \\
& - l_2 sq_4 cq_5 M_5\cdot{}^5r_{C5x}\dot{q}_5^2 - 2l_2 sq_4 M_4\cdot{}^4r_{C4x}\dot{q}_1\dot{q}_4 - l_2 sq_4 sq_5 M_5\ddot{q}_5\cdot{}^5r_{C5x} - \\
& - 2l_2 cq_4 M_5\cdot{}^5r_{C5x}\dot{q}_5 sq_5\dot{q}_4 - 2l_2 cq_4 M_4\cdot{}^4r_{C4y}\dot{q}_1\dot{q}_4 - l_2 cq_4 M_5\cdot{}^5r_{C5y}\dot{q}_4^2 - l_2 sq_4 M_5\cdot{}^5r_{C5y}\ddot{q}_4 - \\
& - l_2 sq_4 M_4\cdot{}^4r_{C4y}\dot{q}_4^2 - l_2 sq_4 M_4\cdot{}^4r_{C4x}\dot{q}_4^2 - l_2 cq_4 M_4\cdot{}^4r_{C4y}\dot{q}_4^2 + l_2 cq_4 M_5\cdot{}^5r_{C5z} sq_5\ddot{q}_4 + \\
& + l_2 cq_4 M_5\cdot{}^5r_{C5x} cq_5\ddot{q}_4 + l_2 sq_4 cq_4 M_5\ddot{q}_5\cdot{}^5r_{C5z} + l_2 cq_4 M_4\cdot{}^4r_{C4x}\ddot{q}_4 + 2l_3 sq_4 M_5\cdot{}^5r_{C5y}\dot{q}_1\dot{q}_4 + \\
& + l_2 sq_4 sq_5\cdot{}^6f_z - q_3 cq_4 sq_5\cdot{}^6f_z - q_3 cq_4 cq_5\cdot{}^6f_x - l_3 cq_4 sq_5\cdot{}^6f_z - l_3 cq_4 cq_5\cdot{}^6f_x + \\
& + q_3 cq_4 M_5 cq_5\cdot{}^5r_{C5x}\dot{q}_4^2 - q_3 cq_4 cq_5 M_5\ddot{q}_5\cdot{}^5r_{C5z} + q_3 cq_4 sq_5 M_5\ddot{q}_5\cdot{}^5r_{C5x} + \\
& + 2q_3 cq_4 sq_5 M_5\cdot{}^5r_{C5z}\dot{q}_1\dot{q}_4 + l_2 sq_4 cq_5\cdot{}^6f_x + q_3 cq_4 sq_5 M_5\cdot{}^5r_{C5z}\dot{q}_5^2 + q_3 sq_4 M_5\cdot{}^5r_{C5z} sq_5\ddot{q}_4 + ...
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&\ldots + q_3 sq_4 M_5 \cdot^5 r_{C5x} cq_5 \ddot{q}_4 - 2q_3 sq_4 M_5 \cdot^5 r_{C5y} \dot{q}_1 \dot{q}_4 + 2q_3 sq_4 M_5 \cdot^5 r_{C5z} \dot{q}_5 cq_5 \dot{q}_4 + \\
&+ 2q_3 sq_4 M_5 \cdot^5 r_{C5z} \dot{q}_5 cq_5 \dot{q}_1 - 2q_3 sq_4 M_5 \cdot^5 r_{C5x} \dot{q}_5 sq_5 \dot{q}_4 - 2q_3 sq_4 M_5 \cdot^5 r_{C5x} \dot{q}_5 sq_5 \dot{q}_1 - \\
&- 2q_3 sq_4 M_4 \cdot^4 r_{C4y} \dot{q}_1 \dot{q}_4 + q_3 cq_4 sq_5 M_5 \cdot^5 r_{C5z} \dot{q}_4^2 + q_3 cq_4 cq_5 M_5 \cdot^5 r_{C5x} \dot{q}_5^2 + \\
&+ 2q_3 cq_4 M_4 \cdot^4 r_{C4x} \dot{q}_1 \dot{q}_4 + 2q_3 cq_4 M_5 cq_5 \cdot^5 r_{C5x} \dot{q}_1 \dot{q}_4 + q_3 sq_4 M_4 \cdot^4 r_{C4x} \ddot{q}_4 + q_3 cq_4 M_5 \cdot^5 r_{C_{5y}} \ddot{q}_4 + \\
&+ q_3 cq_4 M_4 \cdot^4 r_{C_{4y}} \ddot{q}_4 + q_3 cq_4 M_4 \cdot^4 r_{C_{4x}} \dot{q}_4^2 - q_3 sq_4 M_5 \cdot^5 r_{C_{5y}} \dot{q}_4^2 - q_3 sq_4 M_4 \cdot^4 r_{C_{4y}} \dot{q}_4^2 - \\
&- l_3 cq_4 cq_5 M_5 \ddot{q}_5 \cdot^5 r_{C_{5z}} + l_3 cq_4 sq_5 M_5 \ddot{q}_5 \cdot^5 r_{C5x} + l_3 cq_4 M_5 cq_5 \cdot^5 r_{C_{5x}} \dot{q}_4^2 + l_3 cq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5^2 + \ldots \\
&+ l_3 sq_4 M_5 \cdot^5 r_{C_{5z}} sq_5 \ddot{q}_4 + l_3 sq_4 M_5 \cdot^5 r_{C_{5x}} cq_5 \ddot{q}_4 + 2q_3 M_5 \dot{q}_3 \dot{q}_1 + 2q_3 M_4 \dot{q}_3 \dot{q}_1 + 2q_3 M_4 \ddot{q}_1 l_3 + \\
&+ 2q_3 M_3 \ddot{q}_1 l_3 + 2q_3 M_3 \dot{q}_3 \dot{q}_1 + 2q_3 M_5 \ddot{q}_1 l_3 + 2l_3 M_5 \dot{q}_3 \dot{q}_1 + 2l_3 M_4 \dot{q}_3 \dot{q}_1 + 2l_3 M_3 \dot{q}_3 \dot{q}_1 + \\
&+ 2 \cdot^3 r_{C_{3x}} M_3 \ddot{q}_1 l_2 + 2 \cdot^3 r_{C_{3y}} M_3 \ddot{q}_1 q_3 + 2 \cdot^3 r_{C_{3y}} M_3 \ddot{q}_1 l_3 + 2 \cdot^3 r_{C_{3y}} M_3 \dot{q}_3 \dot{q}_1 - l_4 cq_4 cq_5 \cdot^6 f_x - \\
&- l_4 cq_4 sq_5 \cdot^6 f_z + {}^3 r_{C_{3x}}^2 M_3 \ddot{q}_1 + {}^3 r_{C_{3x}} M_3 \ddot{q}_3 + {}^3 r_{C_{3y}}^2 M_3 \ddot{q}_1 + l_4 sq_4 \cdot^6 f_y - l_4 sq_4 M_5 \cdot^5 r_{C_{5y}} \dot{q}_4^2 - \\
&- 2l_4 sq_4 M_5 \cdot^5 r_{C_{5x}} \dot{q}_5 sq_5 \dot{q}_1 - 2l_4 sq_4 M_5 \cdot^5 r_{C_{5x}} \dot{q}_5 sq_5 \dot{q}_4 - 2l_4 sq_4 M_5 \cdot^5 r_{C_{5y}} \dot{q}_1 \dot{q}_4 + \\
&+ 2l_4 sq_4 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5 cq_5 \dot{q}_4 + 2l_4 sq_4 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5 cq_5 \dot{q}_1 - 2l_4 sq_4 M_4 \cdot^4 r_{C_{4y}} \dot{q}_1 \dot{q}_4 + l_4 sq_4 M_4 \cdot^4 r_{C_{4x}} \ddot{q}_4 - \\
&- l_4 sq_4 M_4 \cdot^4 r_{C_{4y}} \dot{q}_4^2 + l_4 sq_4 M_5 \cdot^5 r_{C_{5z}} sq_5 \ddot{q}_4 + l_4 sq_4 M_5 \cdot^5 r_{Cx} cq_5 \ddot{q}_4 + 2l_4 cq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_1 \dot{q}_4 + \\
&+ l_4 cq_4 M_5 \cdot^5 r_{C_{5y}} \ddot{q}_4 + 2l_4 cq_4 M_4 \cdot^4 r_{C_{4x}} \dot{q}_1 \dot{q}_4 + l_4 cq_4 M_4 \cdot^4 r_{C_{4y}} \ddot{q}_4 + l_4 cq_4 M_4 \cdot^4 r_{Cx} \dot{q}_4^2 - \\
&- l_4 cq_4 cq_5 M_5 \ddot{q}_5 \cdot^5 r_{C_{5z}} + l_4 cq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5^2 + l_4 cq_4 sq_5 M_5 \ddot{q}_5 \cdot^5 r_{C_{5x}} + l_4 cq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_4^2 + \\
&+ l_4 cq_4 cq_5 M_5 \cdot^5 r_{C_{5x}} \dot{q}_5^2 + l_4 cq_4 M_5 cq_5 {}^5 r_{C_{5x}} \dot{q}_4^2 + 2 l_4 cq_4 M_5 cq_5 \cdot^5 r_{C_{5x}} \dot{q}_1 \dot{q}_4 + l_4^2 M_5 \ddot{q}_1 + l_4^2 M_4 \ddot{q}_1 + \\
&+ 2l_4 M_5 \ddot{q}_1 q_3 + 2l_4 M_5 \ddot{q}_1 l_3 + 2l_4 M_4 \ddot{q}_1 q_3 + 2l_4 M_4 \ddot{q}_1 l_3 + 2l_4 M_5 \dot{q}_3 \dot{q}_1
\end{aligned}
$$

$$
\begin{aligned}
Q_m^2 = {} & cq_5 \cdot^6 f_z - sq_5 \cdot^6 f_x + sq_5 M_5 \cdot^5 r_{C_{5x}} \dot{q}_5^2 - sq_5 M_5 \ddot{q}_5 \cdot^5 r_{C_{5z}} - cq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5^2 - cq_5 M_5 \ddot{q}_5 \cdot^5 r_{C_{5x}} + \\
& + M_5 g + M_5 \ddot{q}_2 + M_4 \ddot{q}_2 + M_4 g + M_3 g + M_3 \ddot{q}_2 + M_2 g + M_2 \ddot{q}_2
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
Q_m^3 = {} & M_3 \ddot{q}_3 - 2cq_4 M_5 \cdot^5 r_{C_{5y}} \dot{q}_1 \dot{q}_4 + 2cq_4 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5 cq_5 \dot{q}_4 + 2cq_4 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5 cq_5 \dot{q}_1 + \\
& + cq_4 M_5 \cdot^5 r_{C_{5x}} cq_5 \ddot{q}_4 + cq_4 M_5 \cdot^5 r_{C_{5x}} cq_5 \ddot{q}_1 + cq_4 \cdot^6 f_y - M_3 \dot{q}_1^2 q_3 - M_3 \dot{q}_1^2 l_3 + M_3 \ddot{q}_1 {}^3 r_{C_{3x}} - \\
& - M_3 \dot{q}_1^2 \cdot^3 r_{C_{3y}} + sq_4 cq_5 \cdot^6 f_x + sq_4 sq_5 \cdot^6 f_z + cq_4 M_5 \cdot^5 r_{C_{5z}} sq_5 \ddot{q}_1 - cq_4 M_5 \cdot^5 r_{C_{5y}} \dot{q}_4^2 - \\
& - cq_4 M_5 \cdot^5 r_{C_{5y}} \dot{q}_1^2 + M_3 \ddot{q}_1 l_2 - 2cq_4 M_5 \cdot^5 r_{C_{5x}} \dot{q}_5 sq_5 \dot{q}_1 - sq_4 M_5 \cdot^5 r_{C_{5y}} \ddot{q}_4 - sq_4 M_4 \cdot^4 r_{C_{4y}} \ddot{q}_1 - \\
& - sq_4 M_4 \cdot^4 r_{C4y} \ddot{q}_4 - sq_4 M_4 \cdot^4 r_{C_{4x}} \dot{q}_1^2 - sq_4 M_4 \cdot^4 r_{C_{4x}} \dot{q}_4^2 - sq_4 M_5 \cdot^5 r_{C_{5y}} \ddot{q}_1 - \\
& - sq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_5^2 - 2sq_4 M_5 cq_5 \cdot^5 r_{C_{5x}} \dot{q}_1 \dot{q}_4 + sq_4 cq_5 M_5 \ddot{q}_5 \cdot^5 r_{C_{5z}} - sq_4 M_5 cq_5 \cdot^5 r_{C_{5x}} \dot{q}_4^2 - \\
& - 2sq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_1 \dot{q}_4 - sq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_1^2 - sq_4 sq_5 M_5 \cdot^5 r_{C_{5z}} \dot{q}_4^2 - sq_4 cq_5 M_5 \cdot^5 r_{C_{5x}} \dot{q}_5^2 - \\
& - sq_4 M_5 cq_5 \cdot^5 r_{C_{5x}} \dot{q}_1^2 - 2sq_4 M_4 \cdot^4 r_{C_{4x}} \dot{q}_1 \dot{q}_4 - sq_4 sq_5 M_5 \ddot{q}_5 \cdot^5 r_{C_{5x}} + cq_4 M_5 \cdot^5 r_{C_{5z}} sq_5 \ddot{q}_4 - \\
& - 2cq_4 M_5 \cdot^5 r_{C_{5x}} \dot{q}_5 sq_5 \dot{q}_4 - 2cq_4 M_4 \cdot^4 r_{C_{4y}} \dot{q}_1 \dot{q}_4 + cq_4 M_4 \cdot^4 r_{C_{4x}} \ddot{q}_1 + cq_4 M_4 \cdot^4 r_{C_{4x}} \ddot{q}_4 - \\
& - cq_4 M_4 \cdot^4 r_{C_{4y}} \dot{q}_1^2 - cq_4 M_4 \cdot^4 r_{C_{4y}} \dot{q}_4^2 - M_5 \dot{q}_1^2 q_3 + M_5 \ddot{q}_3 + M_4 \ddot{q}_3 - M_4 \dot{q}_1^2 q_3 - M_4 \dot{q}_1^2 l_3 + \\
& + M_5 \ddot{q}_1 l_2 - M_4 \dot{q}_1^2 l_4 + M_4 \ddot{q}_1 l_2 - M_5 \dot{q}_1^2 l_3 - M_5 \dot{q}_1^2 l_4
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
Q_m^4 = {} & {}^5 J_x^* \ddot{q}_1 - {}^5 J_x^* \ddot{q}_1 c^2 q_5 + {}^5 r_{C_{5y}} M_5 sq_4 \dot{q}_1^2 \cdot l_3 + {}^4 J_z^* \ddot{q}_1 + {}^4 J_z^* \ddot{q}_4 + cq_5 \cdot^6 n_z - 2 \cdot^5 r_{C_{5z}} M_5 \cdot^5 r_{C_{5x}} \dot{q}_5 \dot{q}_4 - sq_5 l_6 \cdot^6 f_z + \\
& + sq_5 \cdot^5 r_{C_{5y}} M_5 \cdot^5 r_{C_{5z}} \dot{q}_5^2 + sq_5 \cdot^5 r_{C_z} M_5 cq_4 \ddot{q}_3 + 4 \cdot^5 r_{C_{5x}} M_5 \cdot^5 r_{C_{5z}} \dot{q}_5 c^2 q_5 \dot{q}_1 + 2sq_5 M_5 \cdot^5 r_{C_{5z}}^2 \dot{q}_5 cq_5 \dot{q}_4 + \\
& + sq_5 \cdot^5 r_{C_z} M_5 cq_4 \ddot{q}_1 l_2 + sq_5 \cdot^5 r_{C_{5z}} M_5 sq_4 \ddot{q}_1 l_4 + 2sq_5 \cdot^5 J_x^* \dot{q}_5 cq_5 \dot{q}_1 + M_4 \cdot^4 r_{C_{4y}}^2 \ddot{q}_1 + M_4 \cdot^4 r_{C_{4y}}^2 \ddot{q}_4 + \\
& + M_4 \cdot^4 r_{C_{4x}}^2 \ddot{q}_1 + M_4 \cdot^4 r_{C_{4x}}^2 \ddot{q}_4 + {}^4 r_{C_{4x}} M_4 sq_4 \ddot{q}_1 l_3 + {}^4 r_{C_{4x}} M_4 sq_4 \dot{q}_1^2 l_2 + 2 \cdot^4 r_{C_{4x}} M_4 sq_4 \dot{q}_3 \dot{q}_1 + {}^4 r_{C_{4x}} M_4 sq_4 \ddot{q}_1 l_4 + \\
& + {}^4 r_{C_{4x}} M_4 cq_4 \ddot{q}_1 l_2 - {}^4 r_{C_{4x}} M_4 cq_4 \dot{q}_1^2 q_3 - {}^4 r_{C_{4x}} M_4 cq_4 \dot{q}_1^2 l_3 - {}^4 r_{C_{4x}} M_4 cq_4 \dot{q}_1^2 l_4 + {}^4 r_{C_{4y}} M_4 cq_4 \ddot{q}_1 q_3 + \ldots
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&...+{}^4r_{C_{4y}}M_4cq_4\ddot{q}_1l_3+{}^4r_{C_{4y}}M_4cq_4\dot{q}_1^2l_2+2\cdot{}^4r_{C_{4y}}M_4cq_4\dot{q}_3\dot{q}_1+{}^4r_{C_{4y}}M_4cq_4\ddot{q}_1l_4-{}^4r_{C_{4y}}M_4sq_4\ddot{q}_1l_2+\\
&+{}^4r_{C_{4y}}M_4sq_4\dot{q}_1^2\cdot q_3+{}^4r_{C_{4y}}M_4sq_4\dot{q}_1^2\cdot l_3+cq_5\cdot{}^5r_{C_{5x}}M_5sq_4\ddot{q}_1l_3-cq_5\cdot{}^5r_{C_{5x}}M_5cq_4\dot{q}_1^2\cdot l_4+\\
&+{}^4r_{C_{4x}}M_4sq_4\ddot{q}_1q_3-cq_5\cdot{}^5r_{C_{5x}}M_5cq_4\dot{q}_1^2\cdot l_3-cq_5\cdot{}^5r_{C_{5x}}M_5cq_4\dot{q}_1^2q_3-2cq_5M_5\cdot{}^5r_{C_{5x}}^2\dot{q}_5sq_5\dot{q}_1-\\
&-2cq_5M_5\cdot{}^5r_{C_{5x}}^2\dot{q}_5sq_5\dot{q}_4+2cq_5\cdot{}^5r_{C_{5x}}M_5sq_4\dot{q}_3\dot{q}_1+cq_5\cdot{}^5r_{C_{5x}}M_5cq_4\ddot{q}_1l_2+\\
&+cq_5\cdot{}^5r_{C_{5x}}M_5sq_4\ddot{q}_1l_4+cq_5\cdot{}^5r_{C_{5x}}M_5sq_4\ddot{q}_1q_3+cq_5\cdot{}^5r_{C_{5x}}M_5sq_4\dot{q}_1^2\cdot l_2+\\
&+4\cdot{}^5r_{C_{5x}}M_5\cdot{}^5r_{C_{5z}}\dot{q}_5c^2q_5\dot{q}_4+{}^4r_{C_{4y}}M_4sq_4\dot{q}_1^2\cdot l_4+sq_5\cdot{}^5J_{zy}^*\ddot{q}_5+{}^5J_z^*c^2q_5\ddot{q}_4-\\
&-cq_5l_6\cdot{}^6f_x+cq_5\dot{q}_5^2\cdot{}^5J_{xy}^*-cq_5\cdot{}^5J_{zy}^*\ddot{q}_5+{}^5J_z^*c^2q_5\ddot{q}_1+sq_5\cdot{}^5J_{xz}^*cq_5\ddot{q}_1+2\cdot{}^5J_{zx}^*\dot{q}_5c^2q_5\dot{q}_4+\\
&+2c^2q_5\dot{q}_5{}^5J_{xz}^*\dot{q}_1+2\cdot{}^5J_{zx}^*\dot{q}_5c^2q_5\dot{q}_1+2c^2q_5\dot{q}_5{}^5J_{xz}^*\dot{q}_4-{}^4r_{C_{4y}}M_4sq_4\ddot{q}_3+{}^4r_{C_{4x}}M_4cq_4\ddot{q}_3-\\
&-sq_5\cdot{}^5r_{C_{5z}}M_5cq_4\dot{q}_1^2\cdot l_3-sq_5\cdot{}^5r_{C_{5z}}M_5cq_4\dot{q}_1^2q_3-sq_5\cdot{}^5r_{C_{5z}}M_5cq_4\dot{q}_1^2l_4-\\
&-2sq_5cq_5\dot{q}_5\cdot{}^5J_z^*\dot{q}_1-2sq_5cq_5\dot{q}_5\cdot{}^5J_z^*\dot{q}_4-sq_5\cdot{}^6n_x-cq_5\cdot{}^5r_{C_{5y}}M_5\ddot{q}_5\cdot{}^5r_{C_{5z}}+\\
&+cq_5\cdot{}^5r_{C_{5y}}M_5\cdot{}^5r_{C_{5x}}\dot{q}_5^2+cq_5\cdot{}^5r_{C_{5x}}M_5\cdot cq_4\ddot{q}_3+sq_5\cdot{}^5r_{C_{5z}}M_5sq_4\dot{q}_1^2l_2+\\
&+sq_5\cdot{}^5r_{C_{5z}}M_5sq_4\ddot{q}_1l_3+sq_5\cdot{}^5r_{C_{5z}}M_5sq_4\ddot{q}_1q_3+2sq_5\cdot{}^5J_x^*\dot{q}_5cq_5\dot{q}_4+\\
&+sq_5\cdot{}^5r_{C_{5y}}M_5\ddot{q}_5\cdot{}^5r_{C_{5x}}+cq_5\cdot{}^5J_{zx}^*sq_5\ddot{q}_4+2sq_5\cdot{}^5r_{C_{5z}}M_5sq_4\dot{q}_3\dot{q}_1+\\
&+2sq_5\cdot{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5x}}cq_5\ddot{q}_1+2sq_5\cdot{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5x}}cq_5\ddot{q}_4+2sq_5M_5\cdot{}^5r_{C_{5z}}^2\dot{q}_5cq_5\dot{q}_1+\\
&+sq_5\cdot{}^5J_{xz}^*cq_5\ddot{q}_4+sq_5\dot{q}_5^2\cdot{}^5J_{zy}^*+cq_5\cdot{}^5J_{zx}^*sq_5\ddot{q}_1+M_5\cdot{}^5r_{C_{5x}}^2c^2q_5\ddot{q}_4+\\
&+M_5\cdot{}^5r_{C_{5x}}^2c^2q_5\ddot{q}_1-{}^5r_{C_{5y}}M_5sq_4\ddot{q}_3-{}^5r_{C_{5y}}M_5sq_4\ddot{q}_1l_2+{}^5J_x^*\ddot{q}_4-{}^5J_x^*\ddot{q}_4c^2q_5-\\
&-2\cdot{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5x}}\dot{q}_5\dot{q}_1+{}^5r_{C_{5y}}M_5cq_4\ddot{q}_1l_3+{}^5r_{C_{5y}}M_5cq_4\dot{q}_1^2\cdot l_2-\dot{q}_5\cdot{}^5J_{zx}^*\dot{q}_1-\dot{q}_5\cdot{}^5J_{zx}^*\dot{q}_4-\\
&-{}^5J_{xz}^*\dot{q}_5\dot{q}_4-{}^5J_{xz}^*\dot{q}_5\dot{q}_1+M_5\cdot{}^5r_{C_{5y}}^2\ddot{q}_4+M_5\cdot{}^5r_{C_{5y}}^2\ddot{q}_1+{}^5r_{C_{5y}}M_5cq_4\ddot{q}_1q_3+M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_1-\\
&-M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_1c^2q_5+M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_4-M_5\cdot{}^5r_{C_{5z}}^2\ddot{q}_4c^2q_5+2\cdot{}^5r_{C_{5y}}M_5cq_4\dot{q}_3\dot{q}_1+\\
&+{}^5r_{C_{5y}}M_5sq_4\dot{q}_1^2\cdot l_4+{}^5r_{C_{5y}}M_5sq_4\dot{q}_1^2q_3+{}^5r_{C_{5y}}M_5cq_4\ddot{q}_1l_4
\end{aligned}
$$

$$
\begin{aligned}
Q_m^5=&-M_5cq_5sq_5\cdot{}^5r_{C_{5z}}^2\dot{q}_4^2-{}^5J_{yz}^*cq_5\ddot{q}_1-{}^5J_{yz}^*cq_5\ddot{q}_4-c^2q_5\cdot{}^5J_{xz}^*\dot{q}_1^2-c^2q_5\cdot{}^5J_{xz}^*\dot{q}_4^2+{}^5J_{yx}^*sq_5\ddot{q}_1+\\
&+{}^5J_{yx}^*sq_5\ddot{q}_4-{}^5J_{zx}^*\dot{q}_1^2c^2q_5-{}^5J_{zx}^*\dot{q}_4^2c^2q_5+{}^5J_{yz}^*\dot{q}_5sq_5\dot{q}_1-\dot{q}_5cq_5\cdot{}^5J_{xy}^*\dot{q}_4+{}^5J_{yz}^*\dot{q}_5cq_5\dot{q}_4+\\
&+2\cdot{}^5J_{zx}^*\dot{q}_1\dot{q}_4-2c^2q_5\cdot{}^5J_{xz}^*\dot{q}_1\dot{q}_4-2\cdot{}^5J_{zx}^*\dot{q}_1\dot{q}_4c^2q_5+2cq_5sq_5\cdot{}^5J_z^*\dot{q}_1\dot{q}_4-2sq_5cq_5\cdot{}^5J_x^*\dot{q}_1\dot{q}_4-\\
&-sq_5cq_5\cdot{}^5J_x^*\dot{q}_1^2-sq_5cq_5\cdot{}^5J_x^*\dot{q}_4^2-\dot{q}_5cq_5\cdot{}^5J_{xy}^*\dot{q}_1+{}^5J_{yx}^*\dot{q}_5cq_5\dot{q}_1+cq_5sq_5\cdot{}^5J_z^*\dot{q}_4^2+\\
&+{}^5J_{yz}^*\dot{q}_5sq_5\dot{q}_4+cq_5sq_5\cdot{}^5J_z^*\dot{q}_1^2-\dot{q}_5sq_5\cdot{}^5J_{zy}^*\dot{q}_1-\dot{q}_5sq_5\cdot{}^5J_{zy}^*\dot{q}_4+{}^5J_y^*\ddot{q}_5+{}^6n_y+M_5\ddot{q}_5\cdot{}^5r_{C_{5z}}^2+\\
&+M_5\ddot{q}_5\cdot{}^5r_{C_{5x}}^2+{}^5J_{zx}^*\dot{q}_1^2+{}^5J_{zx}^*\dot{q}_4^2-{}^5r_{C_{5z}}M_5cq_5cq_4\dot{q}_1^2\cdot l_2-2\cdot{}^5r_{C_{5z}}M_5cq_5cq_4\dot{q}_3\dot{q}_1-\\
&-{}^5r_{C_{5z}}M_5cq_5cq_4\ddot{q}_1l_4+{}^5r_{C_{5z}}M_5cq_5sq_4\ddot{q}_1l_2+2M_5cq_5sq_5\cdot{}^5r_{C_{5z}}^2\dot{q}_1\dot{q}_4-{}^5r_{C_{5z}}M_5cq_5sq_4\dot{q}_1^2\cdot l_3-\\
&-2\cdot{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5x}}c^2q_5\dot{q}_4^2-{}^5r_{C_{5z}}M_5cq_5cq_4\ddot{q}_1q_3-{}^5r_{C_{5z}}M_5cq_5sq_4\dot{q}_1^2\cdot l_4-{}^5r_{C_{5z}}M_5cq_5cq_4\ddot{q}_1l_3-\\
&-{}^5r_{C_{5z}}M_5cq_5sq_4\dot{q}_1^2q_3+{}^5r_{C_{5x}}M_5sq_5sq_4\dot{q}_1^2q_3+{}^5r_{C_{5z}}M_5sq_5cq_4\ddot{q}_1l_4+2\cdot{}^5r_{C_{5x}}M_5sq_5cq_5\dot{q}_3\dot{q}_1+\\
&+{}^5r_{C_{5x}}M_5\cdot{}^5r_{C_{5y}}sq_5\ddot{q}_4-{}^5r_{C_{5z}}M_5sq_5sq_4\ddot{q}_1l_2+{}^5r_{C_{5x}}M_5\cdot{}^5r_{C_{5y}}sq_5\ddot{q}_1-{}^5r_{C_{5z}}M_5sq_5sq_4\ddot{q}_3+\\
&+{}^5r_{C_{5x}}M_5sq_5sq_4\dot{q}_1^2\cdot l_4+{}^5r_{C_{5z}}M_5sq_5cq_4\ddot{q}_1q_3+{}^5r_{C_{5z}}M_5sq_5cq_4\ddot{q}_1l_3+{}^5r_{C_{5x}}M_5sq_5cq_4\dot{q}_1^2\cdot l_2+\\
&+{}^5r_{C_{5x}}M_5sq_5sq_4\dot{q}_1^2\cdot l_3+M_5cq_5sq_5\cdot{}^5r_{C_{5x}}^2\dot{q}_1^2+M_5cq_5sq_5\cdot{}^5r_{C_{5x}}^2\dot{q}_4^2-2\cdot{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5x}}c^2q_5\cdot\dot{q}_1^2-\\
&-{}^5r_{C_{5x}}M_5cq_5\cdot g-{}^5r_{C_{5x}}M_5cq_5\ddot{q}_2-{}^5r_{C_{5z}}M_5sq_5\cdot g-{}^5r_{C_{5z}}M_5sq_5\ddot{q}_2-2M_5cq_5sq_5\cdot{}^5r_{C_{5x}}^2\dot{q}_1\dot{q}_4-\\
&-{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5y}}cq_5\ddot{q}_1+{}^5r_{C_{5z}}M_5cq_5sq_5\ddot{q}_3-4\cdot{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5x}}c^2q_5\cdot\dot{q}_1\dot{q}_4-{}^5r_{C_{5z}}M_5\cdot{}^5r_{C_{5y}}cq_5\ddot{q}_4-\\
&-M_5cq_5sq_5\cdot{}^5r_{C_{5z}}^2\dot{q}_1^2+2\cdot{}^5r_{C_{5x}}M_5\cdot{}^5r_{C_{5z}}c^2q_5\cdot\dot{q}_1\dot{q}_4+{}^5r_{C_{5x}}M_5\cdot{}^5r_{C_{5z}}\dot{q}_4^2+{}^5r_{C_{5x}}M_5\cdot{}^5r_{C_{5z}}\dot{q}_1^2
\end{aligned}
\qquad . \qquad (5)
$$

The relations (1)-(5) represent the system of dynamic differential equations that features the dynamic model of the RTTRR serial modular robot.

3. CONCLUSIONS

The equations complexity of RTTRR dynamic model is justified by the fact that, apart from the simplified models, where the centrifugal moments of inertia are usually taken as zero, they can be found here symbolically and they can be substituted by numerical values taken from the CAD model of the robot. Additionally to the most frequent models from the literature, where the *z*-component of the manipulation force is the only non-zero component, corresponding to the weight of the manipulated body, this approach considers all the six components of the force-moment vectors $({}^6\bar{f}_6, {}^6\bar{n}_6)$ associated to the useful payload. Therefore, the determined dynamic model can be applied even in the operations requiring the contact of the end-effecter with the surrounding environment, such as insertion operations with friction on different directions, processing operations by cutting, painting, reconditioning, cleaning, and all these operations need specialized end-effecters or tools, different from a simple gripper.

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Modelarea dinamică a robotului serial de tip RTTRR – determinarea forţelor generalizate

Rezumat: Lucrarea prezintă ultima parte a metodei de determinare a modelului dinamic al robotului serial RTTRR, alcătuit din ecuaţiile diferenţiale dinamice ale forţelor generalizate motoare, care iau în considerare toate componentele parametrilor de distribuţie a maselor si ale vectorilor sarcină utilă. Procesul de modelare este îndeplinit folosind formalismul Newton-Euler, implementat în aplicaţia generalizată de modelare *Robot_Symbolic* [1], iar ecuaţiile obţinute sunt utile în simularea compotamentului dinamic al robotului analizat.

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