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ASPECTS REGARDING THE NUMERICAL MODELING OF THE TRAFFIC ACCIDENTS WITH MUTUAL VISIBILITY BETWEEN VEHICLES

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Abstract: *This paper evaluates the kinematic measures of a car accident with mutual visibility of the vehicles from the physical-mathematical perspective taking into account the parameters resulting from the primary research of the scene of the accident. The reconstruction of such accidents shall determine: the speeds of the vehicles before and after the collision based on the weight of the vehicles, based on the angle between streets (the angle between the longitudinal symmetry axes of the vehicles); the analysis of possibilities of avoiding the accident.*

The numerical modeling developed can be used to solve a large number of side-impact vehicle-vehicle accidents in order to establish the dynamics of their occurrence. The algorithm allows exchanging the entry data and getting results with graphical interpretation for various situations of side-impact between vehicles which facilitates the estimation and comparison of the various conditions taken into account.

The use of computerized analysis with its advantages (reduction of calculation time, simulation of various accident situations, etc.) becomes a useful and necessary tool for the technical experts and engineers who carry on their activity in the field of reconstruction of car accidents and development of car security systems.

Key words*: vehicle, passenger car, car accidents, mutual visibility, numerical modeling*

1. INTRODUCTION

The road transport plays a decisive role in our daily activity. It fulfills an important role both socially and economically as it ensures the circulation of people and goods within all socio-economic activities. The autovehicle plays an important part in the progress of our society but besides bringing along facilities and satisfactions, it also represents one of the artificial dangers that threatens the main causing human life losses and large material damages. The increasing number of vehicles led to an increasing number of car accidents and thus more and more victims.

Car accidents are global issues that affect all the fields of the society. They have an enormous impact on the health, economy and social life of the people involved, their families, the community and the country they belong to [11].

Delimitation of side collisions from head-on and rear-end collisions is made according to the direction of trajectory of the participants and the position of the point of impact (Fig. 1) [2, 6, 10, 11]. Thus, side collisions which usually take place in crowded intersections are those where the trajectory of the participants is following the hourly direction [6, 10, 11]:

- 1h, 5h, 7h or 11h, and the vehicle is hit on the side (dotted line – Fig. 1.a);
- 2h, 3h, 4h, 8h, 9h or 10h, and the vehicle is hit somewhere on the side (dotted line – Fig. 1.b).

2. THE NUMERICAL EVALUATION METHOD

In order to the determine the post-collision speeds when there is an oblique collision between two motor vehicles weighing m_1 and m_2

in a roundabout intersection (Fig. 2) where the angle of the two streets (the angle between the longitudinal symmetry axes of the passenger cars) is α_1, and the angle of the common normal of collision of the vehicles (n – the direction on which the percussions P act) with the axis of symmetry of the horizontal street is α_2, the initial speeds v_1 and v_2 of the vehicles are broken down following the direction of the common normal (coefficient n) and a perpendicular direction on it (coefficient t) (Fig. 3), resulting [10]:

$$\begin{cases} v_{1n} = v_1 \cdot \cos(\alpha_1 - \alpha_2) \\ v_{1t} = v_1 \cdot \sin(\alpha_1 - \alpha_2) \end{cases}, \quad (1)$$

$$\begin{cases} v_{2n} = v_2 \cdot \cos\alpha_2 \\ v_{2t} = v_2 \cdot \sin\alpha_2 \end{cases}. \quad (2)$$

Given the fact that there is no active force on the tangential direction, only the normal components of speeds will change after the collision (v'_{1n} and v'_{2n}), and the components (v'_{1t} and v'_{2t}) keep the same value they had before the impact ($v'_{1t} = v_{1t}$, $v'_{2t} = v_{2t}$) [10].

The components (v'_{1n} and v'_{2n}) of speeds (see Fig. 3) are determined according to one of the head-on collision cases (B, C, D – Table 1) according to the direction of the common normal of collision.

During a car accident there are three stages [1-12] (see Table 1): *pre-collision* – stage a; *collision* – stage b; *post-collision* – stage c.

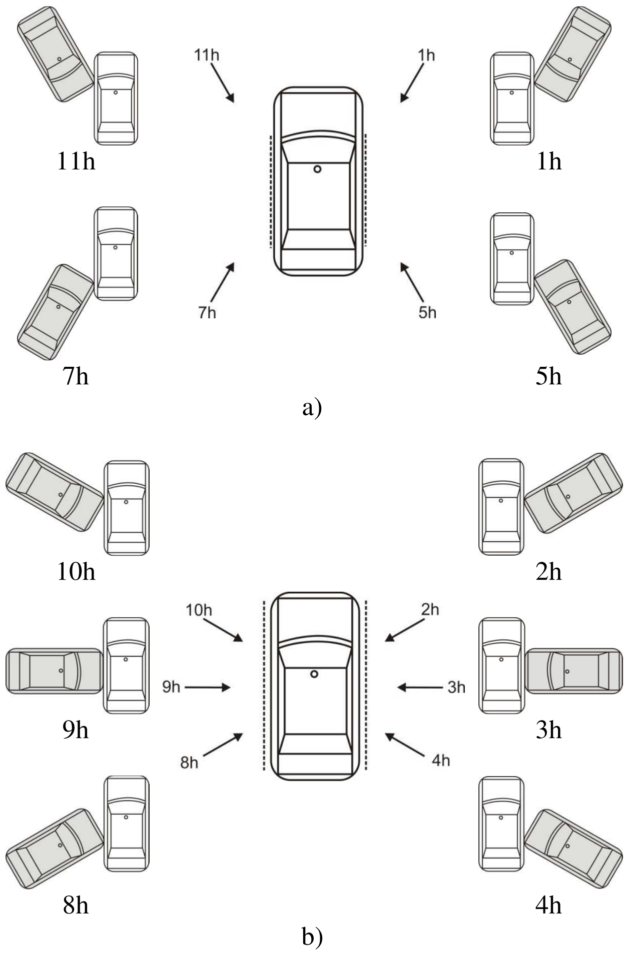

Fig. 1. The trajectory of the participants in side collisions.

Table 1

Collision cases and their stages

Stages / Cases	a		b	c	
	v_1	v_2	$v_{c,d}$	v'_1	v'_2
A	→	→	→	→	→
B	→	←	In a head-on collision between two vehicles running on different directions, the direction of the common speed "v" is established by the direction of the vehicle for which the product "$m_i \cdot v_i$" is bigger (the coefficient "i" refers to the motor vehicle "i").	→	→
C	→	←		←	←
D	→	←		←	→

The positive direction considered for all the collision cases (A, B, C, D) is from left to right: ⊕⟶

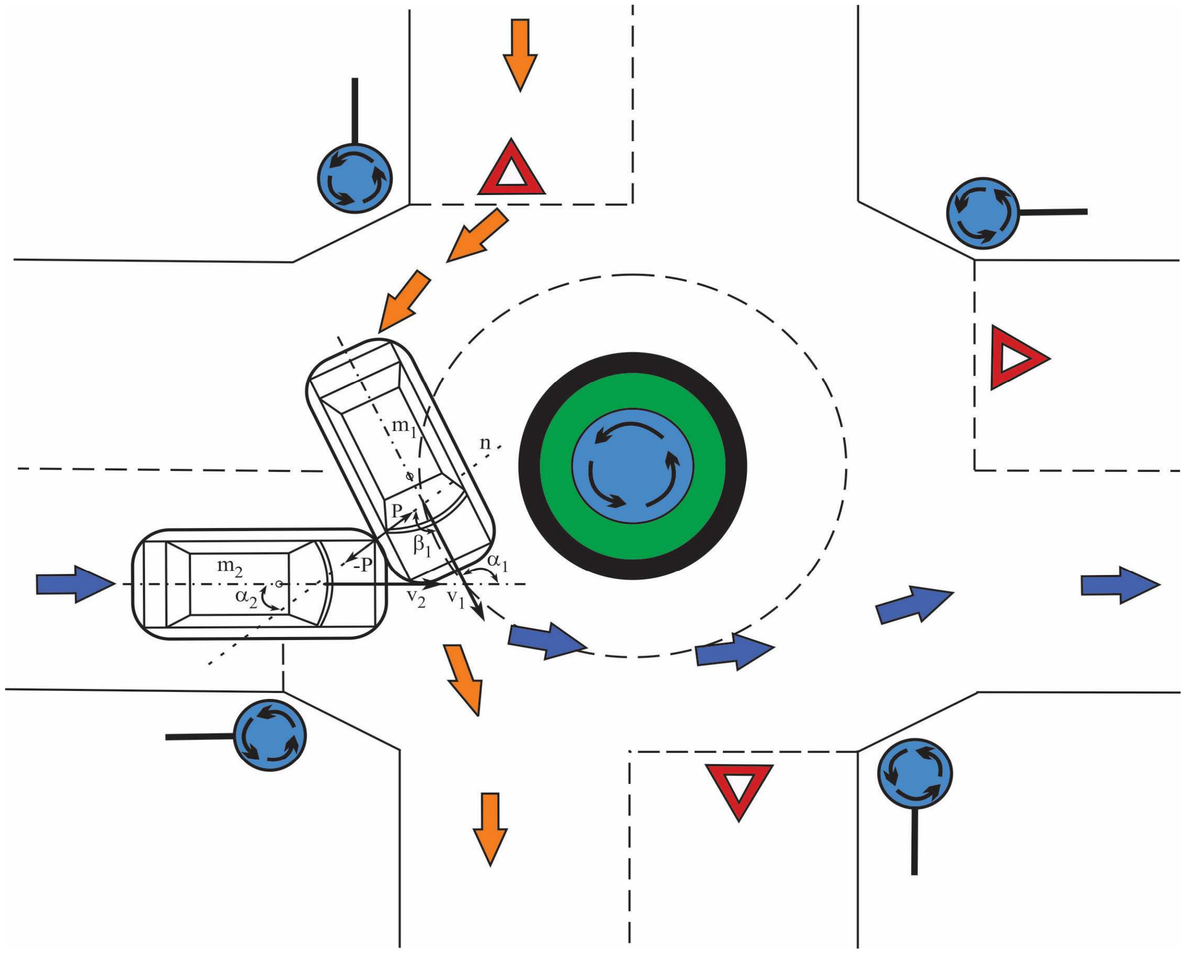

Fig. 2. The figure of side collision between two vehicles in a roundabout.

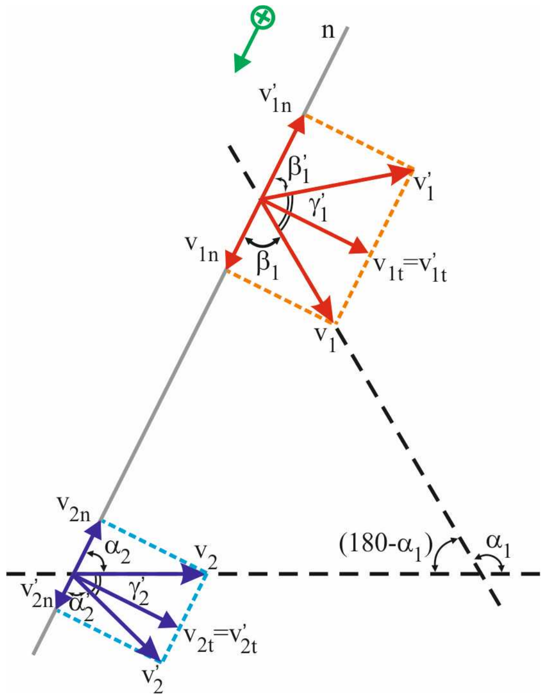

Fig. 3. The speeds of the vehicles when there is an oblique collision.

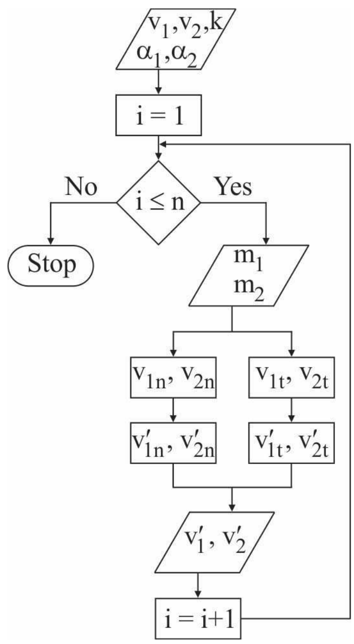

Fig. 4. The sketch for the numerical modeling of the vehicle-vehicle accident. “i” takes into account the vehicle “i” and “n” refers to the number of vehicles involved in the accident.

It is considered that the collision takes place by sequence of two stages: the *compression* stage and the *expansion* stage (or *relaxation* stage). The compression stage takes place from the beginning of the collision to the equaling of the speeds of the two vehicles (Table 1 – stages a-b). During this period some of the kinetic energy of the vehicles becomes deformation energy. During the collision between two vehicles, the compression and expansion stage is very short (Table 1 – stages b-c), and the compression speed "v_c" of the vehicles is considered approximately equal to the expansion speed "v_d". The common speed during the maximum compression (Table 1 – stage b) is marked with "v". During the expansion stage some of the deformation energy which was gathered by the deformation of the two vehicles is restored.

Thus, the speeds of the two vehicles will change reaching the final values v_1', and v_2' [1, 3, 4, 6, 8, 9, 10, 11].

Taking into account that in the case of a head-on back-up collision (case A – Table 1) the post-collision speeds are given by the following relations [1, 3, 4, 10]:

$$\begin{cases} v_{1A}' = v_{1A} - \dfrac{m_2}{m_1 + m_2} \cdot (v_{1A} - v_{2A}) \cdot (1+k) \\ v_{2A}' = v_{2A} + \dfrac{m_1}{m_1 + m_2} \cdot (v_{1A} - v_{2A}) \cdot (1+k) \end{cases}, \quad (3A)$$

the calculation relations of the post-collision speeds for the head-on collision cases B, C, D (see Table 1) are similar to the relations (3A), but each direction of speed is taken into account like this:

$$\begin{cases} v_{1B}' = v_{1B} - \dfrac{m_2}{m_1 + m_2} \cdot (v_{1B} + v_{2B}) \cdot (1+k) \\ v_{2B}' = -v_{2B} + \dfrac{m_1}{m_1 + m_2} \cdot (v_{1B} + v_{2B}) \cdot (1+k) \end{cases}; \quad (3B)$$

$$\begin{cases} -v_{1C}' = v_{1C} - \dfrac{m_2}{m_1 + m_2} \cdot (v_{1C} + v_{2C}) \cdot (1+k) \\ -v_{2C}' = -v_{2C} + \dfrac{m_1}{m_1 + m_2} \cdot (v_{1C} + v_{2C}) \cdot (1+k) \end{cases}; \quad (3C)$$

$$\begin{cases} -v_{1D}' = v_{1D} - \dfrac{m_2}{m_1 + m_2} \cdot (v_{1D} + v_{2D}) \cdot (1+k) \\ v_{2D}' = -v_{2D} + \dfrac{m_1}{m_1 + m_2} \cdot (v_{1D} + v_{2D}) \cdot (1+k) \end{cases}, \quad (3D)$$

where k is the *restoring coefficient* given by the report of the values of percussions for the two collision stages (extension and compression) [1, 2, 3, 4, 5, 6, 9, 10]:

$$k = \frac{P_d}{P_c} = \frac{v_2' - v_1'}{v_1 - v_2}. \quad (4)$$

The limit values of the restoring coefficient are usually 0 (without restitution of the initial form – *perfectly inelastic* collision) and 1 (with complete restitution of the initial form – *perfectly elastic* collision) [2, 3, 10, 11] but in the case of collisions between vehicles $0 < k \leq 0.3$ [10, 11].

If the post-collision speeds are known and the pre-collision speeds shall be determined taking into consideration the collision case A (see Table 1), these can be determined using the relations [10]:

$$\begin{cases} v_{1A} = \dfrac{m_2 \cdot (1+k)}{k \cdot (m_1 + m_2)} \cdot v_{2A}' - \dfrac{m_2 - k \cdot m_1}{k \cdot (m_1 + m_2)} \cdot v_{1A}' \\ v_{2A} = \dfrac{m_1 \cdot (1+k)}{k \cdot (m_1 + m_2)} \cdot v_{1A}' - \dfrac{m_1 - k \cdot m_2}{k \cdot (m_1 + m_2)} \cdot v_{2A}' \end{cases}, \quad (5)$$

they can be adapted to the collision cases B, C or D (see Table 1).

This paper aims to determine the post-collision speeds assuming that the pre-collision speeds are known. In order to do this for any of the cases the relations (3B, 3C, 3D) shall be taken into account according to the situation and if the initial calculation result in negative values of the post-collision speeds then some changes will be made on the initial sketch and the directions of the negative speeds will be reversed and their values in calculations will be considered the same, but positive. For example, if case B is taken into account but the initial calculations v'_{1B} show negative results, then the real case will be D and the speed considered will be v'_{1D} (see relation 3D), ($v'_{1D} = -v'_{1B}$).

In this case, taking into consideration the collision case D following the direction of the common normal, the normal components (v'_{1n} and v'_{2n}) of the speeds shall be given by the calculation relations:

$$\begin{cases} -v'_{1n} = v_{1n} - \dfrac{m_2}{m_1 + m_2} \cdot (v_{1n} + v_{2n}) \cdot (1 + k) \\ v'_{2n} = -v_{2n} + \dfrac{m_1}{m_1 + m_2} \cdot (v_{1n} + v_{2n}) \cdot (1 + k) \end{cases} . \quad (6)$$

The post-collision speeds (v'_1 and v'_2) of the two vehicles (see Fig. 3) can be calculated using the relations:

$$\begin{cases} v'_1 = \sqrt{(v'_{1n})^2 + (v'_{1t})^2} \\ v'_2 = \sqrt{(v'_{2n})^2 + (v'_{2t})^2} \end{cases} . \quad (7)$$

If the angles α_1 and α_2 are considered to be known (see Fig. 2 and Fig. 3) then the angle β_1 is the result of the difference ($\beta_1 = \alpha_1 - \alpha_2$) and the directions of the speed vectors after collision can be determined by the relations:

$$\begin{cases} \beta'_1 = \mathrm{arctg}\left(\dfrac{v'_{1t}}{v'_{1n}}\right) \\ \alpha'_2 = \mathrm{arctg}\left(\dfrac{v'_{2t}}{v'_{2n}}\right) \end{cases} . \quad (8)$$

3. OBTAINED RESULTS

As an example, in the numerical calculation model developed in MathCAD software the post-collision speeds shall be determined taking into account the following entry data: the speed of the first vehicle before the collision is 75 km/h and the speed of the second vehicle is 50 km/h; the restoring coefficient, k = 0.3; different values of the angles α_1 and α_2 (Table 2, Table 3); different values of the weight m_1 and m_2 (Table 2, Table 3), the variation being considered between its own weight and the maximum authorized weight (Table 3).

Table 2

The post-collision speeds of the vehicles and angles α_1 and α_2

No. crt.	v'_1, [km/h]	v'_2, [km/h]	α_1, [°]	α_2, [°]	m_1, [kg]	m_2, [kg]
I1	61.625	46.019	115	60	1090	1285
I2	68.821	41.611	120	55	1090	1285
I3	74.378	38.327	125	50	1090	1285
I4	78.154	36.842	130	45	1090	1285
I5	80.102	37.515	135	40	1090	1285

Table 3

The post-collision speeds of the vehicles and their weight

No. crt.	v'_1, [km/h]	v'_2, [km/h]	m_1, [kg]	m_2, [kg]	α_1, [°]	α_2, [°]
L1	61.625	41.019	1090	1285	115	60
L2	61.598	46.142	1220	1415	115	60
L3	61.587	46.477	1350	1500	115	60
L4	61.575	46.262	1480	1690	115	60
L5	61.551	46.394	1630	1830	115	60

Based on the entry data, considering the sketch from figure 4 and the numerical calculation model, the following results are obtained: the variation of the post-collision speeds of the two vehicles depending on the angle α_1 (Table 2, Fig. 5); the variation of the post-collision speeds of the two vehicles depending on their weight (Table 3, Fig. 6).

The results show that with the growth of the angle α_1 at the interval of 20º (the angle of the two streets), the post-collision speeds of the first vehicle (v_1') are in inverse ratio to the post-collision speeds of the second vehicle (v_2') (Tabelul 2, Fig. 5).

It is also noticed that the bigger the weight of the vehicles is, their post-collision speeds are lower, the post-collision speeds are in inverse ratio to the weight of the vehicles (Table 3, Fig.6).

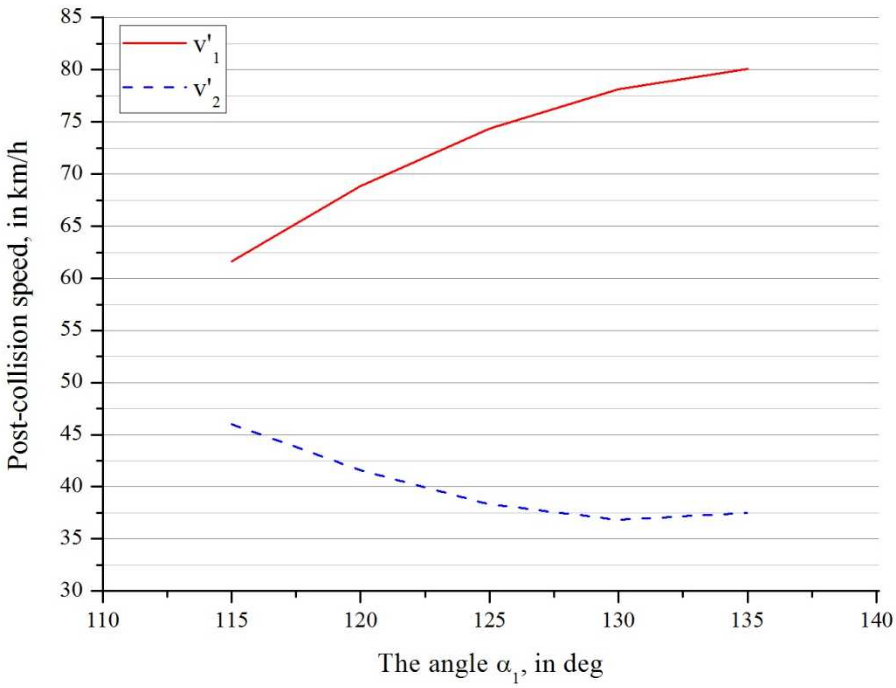

Fig. 5. The post-collision speeds of the two vehicles depending on the angle of the two streets of the intersection (the angle between the longitudinal symmetry axes of the passenger cars) α_1 (I1…I5 – Table 2).

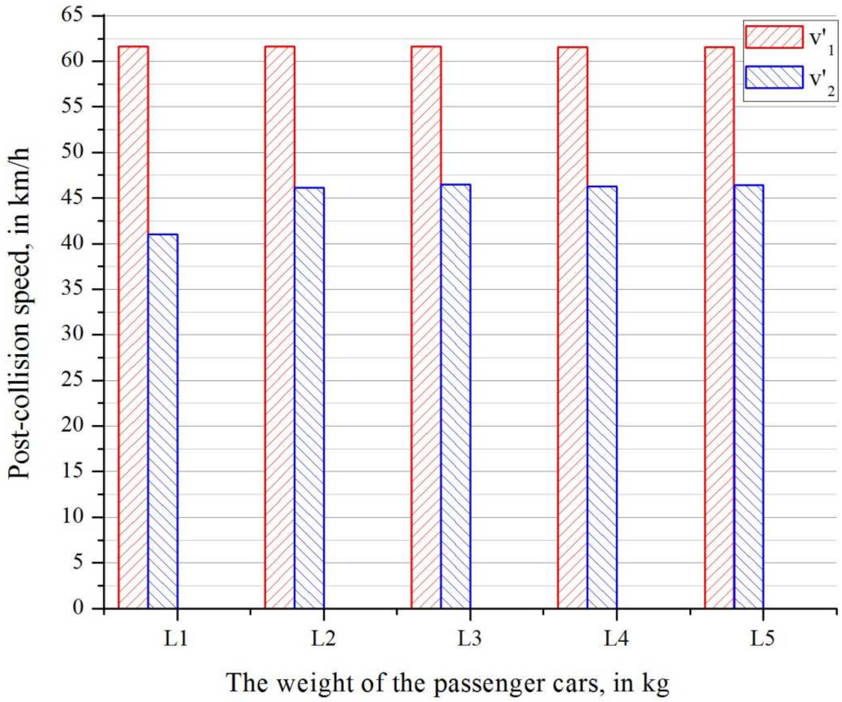

Fig. 6. The post-collision speeds of the two vehicles depending on their weight (L1…L5 – Table 3).

4. CONCLUSIONS

The development of such physical and numerical models in order to reconstruct vehicle-vehicle side collisions leads to the following conclusions:

- the analytical models allow the reconstruction of car accidents and the numerical and graphical simulation of these accidents;
- one of the advantages of the physical-mathematical modeling is that if the initial speed and position of the traffic participants is known, their speed and position after the collision, as well as their trajectory can be determined. The method can also be applied backwards, that is to say that knowing the post-collision speed and the final positions of the traffic participants allow us to determine the pre-collision speed and the initial positions;
- the algorithm developed allows changing the entry data and getting results with graphical interpretation for any other situation of impact of the vehicles, this facilitates the estimation and comparison of the various conditions taken into account;
- the method can also be adapted to the multiple collision situations, for example when three vehicles collide at the same time, or when after the collision between two vehicles, one of them collides with a third one, this is when the initial impact speed of the vehicle which collides with the third one is the post-collision speed resulted from the collision of the first two vehicles;
- the study can be extended to the reconstruction of car accidents when the vehicles undergo a translational motion or a rotational motion around the vertical axes which pass through the center of gravity by applying the conservation of the quantity of movement law;
- furthermore, the method developed can be extended to the situations where an energy balance between the pre-collision and post-collision stage during a car accident can be produced.

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ASPECTE CU PRIVIRE LA MODELAREA NUMERICĂ A ACCIDENTELOR RUTIERE CU VIZIBILITATE RECIPROCĂ ÎNTRE AUTOVEHICULE

Rezumat: În lucrare se evaluează, din punct de vedere fizico-matematic, mărimile cinematice ale unui accident rutier cu vizibilitate reciprocă între autovehicul-autovehicul, ținând seama de parametrii rezultați din cercetarea primară a locului faptei. Prin reconstituirea unor asemenea accidente rutiere se caută să se determine: vitezele antecoliziune/postcoliziune ale autovehiculelor, în funcție de masele acestora, în funcție de unghiul celor două străzi (unghiul dintre axele longitudinale de simetrie ale autovehiculelor); analiza posibilităţile de evitare a accidentului.

Modelul numeric dezvoltat poate fi aplicat la soluționarea unui număr mare de cazuri de accidente rutiere, de tip autovehicul-autovehicul în impact lateral, pentru a stabili dinamica producerii acestora. Algoritmul de lucru permite schimbarea datelor de intrare şi obținerea rezultatelor cu interpretare grafică pentru diferite situații de impact lateral între autovehicule, ceea ce facilitează aprecierea şi compararea diferitelor condiții luate în studiu.

Utilizarea analizei computerizate, prin avantajele pe care le oferă (reducerea timpilor de calcul, simularea diferitelor situaţii de accident etc.) devine un instrument util şi necesar experţilor tehnici şi inginerilor care îşi desfăşoară activitatea în cadrul reconstituirii accidentelor rutiere şi dezvoltării sistemelor de siguranță ale autovehiculelor.

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