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GEOMETRIC MODELING OF TRTRR SERIAL MODULAR ROBOT

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Abstract: *There is a modular serial robot noted TRTRR with two translation modules and three rotational modules, and that is inside the manufacturing cell. In this paper is presented the geometric modeling using matrix study. For the five modules are given step by step the position matrix and the final result if they work as an assembly.*
Keywords: *geometric modeling, TRTTR robot, serial modular robot.*

1. MECHANICAL STRUCTURE DESCRIPTION OF THE SERIAL MODULAR ROBOT

The robot kinematics diagram type industrial TRTRR, is shown in figure 1. This is made up of the following: The vertical translation module 1 (MTV), module 2 of robot arm rotation (MRB), the module 3 for translation in the horizontal direction (MT), module 4 for the horizontal rotation (MR), the guidance module 5 (MO) of the prehensiune tool noted 6.

In figure 1 are highlighted following notations: l_i – the constructive parameters of the robot, $i=1\div7$; q_k – generalized coordinates, $k=1\div5$.

2. GEOMETRIC MODEL DIRECTLY FROM THE TRTRR SERIAL MODULAR ROBOT ARRAY USING THE METHOD OF ROTATION

In the center of gravity of each module (i), $i=1\div6$, is attached Cartesian reference system $O_ix_iy_iz_i$. Mechanical structure of the robot is not in a known configuration, in accordance with [Isp04] and [Det07], represented by the vector column of the coordinates generalized, writing with matrix as follows:

$$[\overline{q}] = [q_i ; k = 1 \div 5]^T. \tag{1}$$

The rotation matrix $[R]_6^0$ is defined by

$$[R]_6^0 = [\overline{x}_6 \; \overline{y}_6 \; \overline{z}_6] = \begin{bmatrix} \alpha_{6x} & \alpha_{6y} & \alpha_{6z} \\ \beta_{6x} & \beta_{6y} & \beta_{6z} \\ \gamma_{6x} & \gamma_{6y} & \gamma_{6z} \end{bmatrix} \tag{2}$$

is the next step in determining the geometric model directly.

The point of the prehensiune in relation to fixed reference system associated at the robot base $A_0x_0y_0z_0$, is determined in accordance with [Kha02], using the following relationship of recurrence:

$$[\overline{p}] = [P]^0 = [\overline{p}]_5 + [\overline{p}]_{6,5} \tag{3}$$

Matrixes that expressed the orientation of the $O_ix_iy_iz_i$ system in relation to the $O_{i-1}x_{i-1}y_{i-1}z_{i-1}$ system have the following form:

$$[R]_1^0 = [I_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad [R]_2^1 = [\overline{z}_2 ; q_2] = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \tag{4}$$

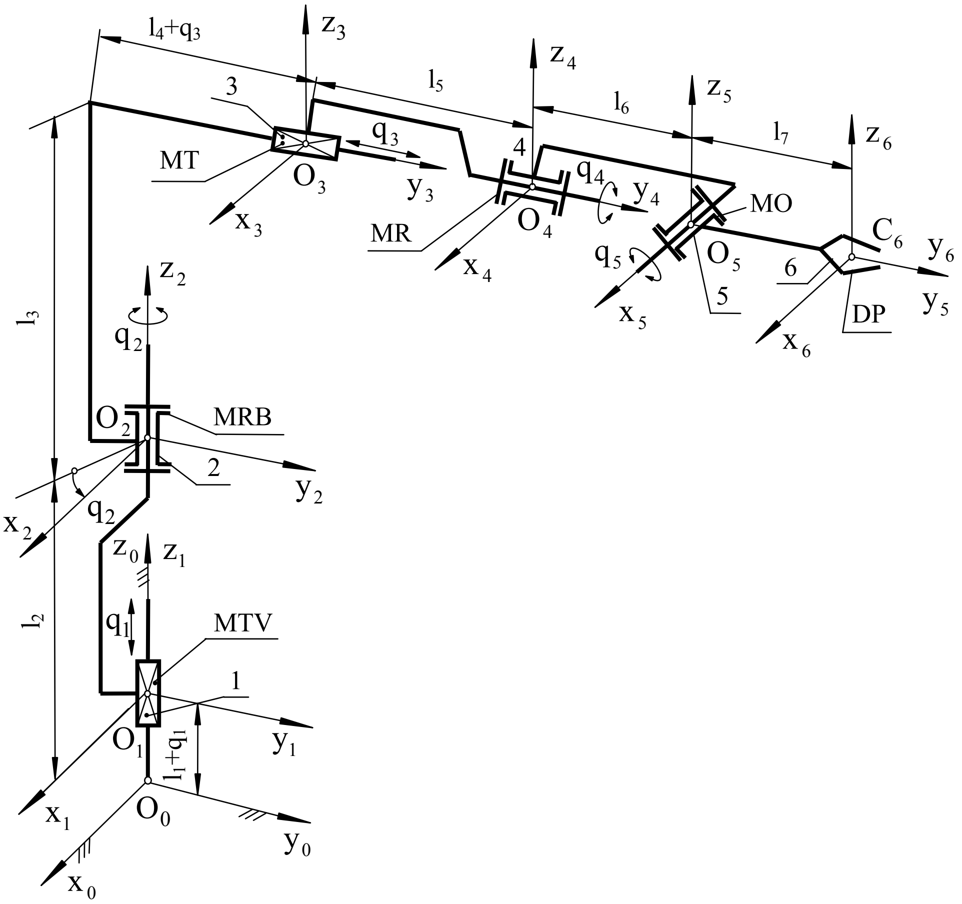

Fig.1. Structural Scheme of Kinematics for the Industrial Modular Serial Robot TRTRR

$$[R]_3^2=[I_3]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\ [R]_4^3=[\bar{y}_4;q_4]=\begin{bmatrix}cq_4&0&sq_4\\0&1&0\\-sq_4&0&cq_4\end{bmatrix};\qquad(5)$$

$$[R]_5^4=[\bar{x}_5;q_5]=\begin{bmatrix}1&0&0\\0&cq_5&-sq_5\\0&sq_5&cq_5\end{bmatrix};\ [R]_6^5=[I_6]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.\qquad(6)$$

Having regard to the written above can be determined following relations, which expresses each axis orientation $O_ix_iy_iz_i$ system in relation to the system $O_0x_0y_0z_0$:

$$[R]_2^0=[R]_1^0\cdot[R]_2^1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}=$$

$$=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix};\qquad(7)$$

$$[R]_3^0=[R]_2^0\cdot[R]_3^2=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=$$

$$=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix};\qquad(8)$$

$$[R]_4^0=[R]_3^0\cdot[R]_4^3=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}\cdot$$

$$\begin{bmatrix}cq_4&0&sq_4\\0&1&0\\-sq_4&0&cq_4\end{bmatrix}=\begin{bmatrix}cq_2cq_4&-sq_2&cq_2sq_4\\sq_2cq_4&cq_2&sq_2sq_4\\-sq_4&0&cq_4\end{bmatrix}\qquad(9)$$

$$[R]_5^0=[R]_4^0\cdot[R]_5^4=\begin{bmatrix}cq_2cq_4&-sq_2&cq_2sq_4\\sq_2cq_4&cq_2&sq_2sq_4\\-sq_4&0&cq_4\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&cq_5&-sq_5\\0&sq_5&cq_5\end{bmatrix}=$$

$$=\begin{bmatrix}cq_2cq_4&-sq_2cq_5+cq_2sq_4sq_5&sq_2sq_5+cq_2sq_4cq_5\\sq_2cq_4&cq_2cq_5+sq_2sq_4sq_5&-cq_2sq_5+sq_2sq_4cq_5\\-sq_4&cq_4sq_5&cq_4cq_5\end{bmatrix};\qquad(10)$$

$$[R]_6^0 = [R]_5^0 \cdot [R]_6^5 = \begin{bmatrix} cq_2cq_4 & -sq_2cq_5 + cq_2sq_4sq_5 & sq_2sq_5 + cq_2sq_4cq_5 \\ sq_2cq_4 & cq_2cq_5 + sq_2sq_4sq_5 & -cq_2sq_5 + sq_2sq_4cq_5 \\ -sq_4 & cq_4sq_5 & cq_4cq_5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} cq_2cq_4 & -sq_2cq_5 + cq_2sq_4sq_5 & sq_2sq_5 + cq_2sq_4cq_5 \\ sq_2cq_4 & cq_2cq_5 + sq_2sq_4sq_5 & -cq_2sq_5 + sq_2sq_4cq_5 \\ -sq_4 & cq_4sq_5 & cq_4cq_5 \end{bmatrix}. \quad (11)$$

In accordance with [Neg08], may be determined the orientation independent parameters, based on this equation matrix:

$$[R]_6^0 = R(\alpha_z - \beta_y - \gamma_x), \quad (12)$$

in which the matrix $R(\alpha\ \beta\ \gamma)$ is:

$$R(\alpha_z - \beta_y - \gamma_x) = \begin{bmatrix} c\alpha_z c\beta_y & c\alpha_z s\beta_y s\gamma_x - s\alpha_z c\gamma_x & c\alpha_z s\beta_y c\gamma_x + s\alpha_z s\gamma_x \\ s\alpha_z c\beta_y & s\alpha_z s\beta_y s\gamma_x + c\alpha_z c\gamma_z & s\alpha_z s\beta_y c\gamma_x - c\alpha_z s\gamma_x \\ -s\beta_y & c\beta_y s\gamma_x & c\beta_y c\gamma_x \end{bmatrix} \quad (13)$$

By identifying matrix elements (11) with matrix elements (13), the consequence of independent parameters results the orientation vector, in accordance with [Mar93] and [Ghi04]:

$$[\alpha_z \quad \beta_y \quad \gamma_x]^T = [q_2 \quad q_4 \quad q_5]^T \quad (14)$$

Position vectors of each front couplers of the system above, are shown with matrix as follows:

$$[\bar{r}]_1^0 = \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix}; \ [\bar{r}]_2^1 = \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix}; \ [\bar{r}]_3^2 = \begin{bmatrix} 0 \\ l_4 + q_3 \\ l_3 \end{bmatrix};$$

$$[\bar{r}]_4^3 = \begin{bmatrix} 0 \\ l_5 \\ 0 \end{bmatrix}; \ [\bar{r}]_5^4 = \begin{bmatrix} 0 \\ l_6 \\ 0 \end{bmatrix}; \ [\bar{r}]_6^5 = \begin{bmatrix} 0 \\ l_7 \\ 0 \end{bmatrix}; \quad (15)$$

By determining Cartesian components of the translation vectors relative position can be calculated $O_ix_iy_iz_i$ system in relation to the system $O_0x_0y_0z_0$. So:

$$[\bar{p}]_{10} = [\bar{r}]_1^0 = \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix}; [\bar{p}]_{21} = [R]_1^0 \cdot [\bar{r}]_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} \quad (16)$$

$$[\bar{p}]_{32} = [R]_2^0 \cdot [\bar{r}]_3^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_4 + q_3 \\ l_3 \end{bmatrix} = \begin{bmatrix} -(l_4 + q_3)sq_2 \\ (l_4 + q_3)cq_2 \\ l_3 \end{bmatrix} \quad (17)$$

$$[\bar{p}]_{43} = [R]_3^0 \cdot [\bar{r}]_4^3 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_5 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_5sq_2 \\ l_5cq_2 \\ 0 \end{bmatrix}; \quad (18)$$

$$[\bar{p}]_{54} = [R]_4^0 \cdot [\bar{r}]_5^4 = \begin{bmatrix} cq_2cq_4 & -sq_2 & cq_2sq_4 \\ sq_2cq_4 & cq_2 & sq_2sq_4 \\ -sq_4 & 0 & cq_4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_6 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_6sq_2 \\ l_6cq_2 \\ 0 \end{bmatrix}. \quad (19)$$

$$[\bar{p}]_{65} = [R]_5^0 \cdot [\bar{r}]_6^5 =$$

$$\begin{bmatrix} cq_2cq_4 & -sq_2cq_5 + cq_2sq_4sq_5 & sq_2sq_5 + cq_2sq_4cq_5 \\ sq_2cq_4 & cq_2cq_5 + sq_2sq_4sq_5 & -cq_2sq_5 + sq_2sq_4cq_5 \\ -sq_4 & cq_4sq_5 & cq_4cq_5 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_7 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} l_7(-sq_2cq_5 + cq_2sq_4sq_5) \\ l_7(cq_2cq_5 + sq_2sq_4sq_5) \\ l_7cq_4sq_5 \end{bmatrix}. \quad (20)$$

In the end, determine the translation vectors absolute in relation to fixed system $O_0x_0y_0z_0$. Expression of their matrix is:

$$[\bar{p}]_1 = [\bar{p}]_{10} = \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix};$$

$$[\bar{p}]_2 = [\bar{p}]_1 + [\bar{p}]_{21} = \begin{bmatrix} 0 \\ 0 \\ l_1 + q_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 + l_2 + q_1 \end{bmatrix}; \quad (21)$$

$$[\bar{p}]_3 = [\bar{p}]_2 + [\bar{p}]_{32} = \begin{bmatrix} 0 \\ 0 \\ l_1 + l_2 + q_1 \end{bmatrix} +$$

$$+ \begin{bmatrix} -(l_4 + q_3)sq_2 \\ (l_4 + q_3)cq_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} -(l_4 + q_3)sq_2 \\ (l_4 + q_3)cq_2 \\ l_1 + l_2 + l_3 + q_1 \end{bmatrix} \quad (22)$$

$$[\bar{p}]_5 = [\bar{p}]_4 + [\bar{p}]_{54} = \begin{bmatrix} -(l_4 + l_5 + q_3)sq_2 \\ (l_4 + l_5 + q_3)cq_2 \\ l_1 + l_2 + l_3 + q_1 \end{bmatrix} +$$

$$+ \begin{bmatrix} -l_6sq_2 \\ l_6cq_2 \\ 0 \end{bmatrix} = \begin{bmatrix} -(l_4 + l_5 + l_6 + q_3)sq_2 \\ (l_4 + l_5 + l_6 + q_3)cq_2 \\ l_1 + l_2 + l_3 + q_1 \end{bmatrix}; \quad (23)$$

$$[\bar{p}]_5 = [\bar{p}]_4 + [\bar{p}]_{54} = \begin{bmatrix} -(l_4 + l_5 + q_3)sq_2 \\ (l_4 + l_5 + q_3)cq_2 \\ l_1 + l_2 + l_3 + q_1 \end{bmatrix} +$$

$$+ \begin{bmatrix} -l_6sq_2 \\ l_6cq_2 \\ 0 \end{bmatrix} = \begin{bmatrix} -(l_4 + l_5 + l_6 + q_3)sq_2 \\ (l_4 + l_5 + l_6 + q_3)cq_2 \\ l_1 + l_2 + l_3 + q_1 \end{bmatrix}; \quad (24)$$

$$[\bar{p}]_6 = [\bar{p}]_5 + [\bar{p}]_{65} = \begin{bmatrix} -(l_4+l_5+l_6+q_3)sq_2 \\ (l_4+l_5+l_6+q_3)cq_2 \\ l_1+l_2+l_3+q_1 \end{bmatrix} + \begin{bmatrix} l_7(-sq_2cq_5+cq_2sq_4sq_5) \\ l_7(cq_2cq_5+sq_2sq_4sq_5) \\ l_7cq_4sq_5 \end{bmatrix} =$$

$$= \begin{bmatrix} -(l_4+l_5+l_6+q_3)sq_2+l_7(-sq_2cq_5+cq_2sq_4sq_5) \\ (l_4+l_5+l_6+q_3)cq_2+l_7(cq_2cq_5+sq_2sq_4sq_5) \\ l_1+l_2+l_3+q_1+l_7cq_4sq_5 \end{bmatrix}. \tag{25}$$

As a result of results obtained with relations (14) and (25), obtain the vector column of the coordinates operational, whose expression matrix is as follows:

$$[\bar{X}]^0 = \begin{bmatrix} p_{x_6} \\ p_{y_6} \\ p_{z_6} \\ \cdots \\ \alpha_z \\ \beta_y \\ \gamma_x \end{bmatrix} = \begin{bmatrix} -(l_4+l_5+l_6+q_3)sq_2+l_7(-sq_2cq_5+cq_2sq_4sq_5) \\ (l_4+l_5+l_6+q_3)cq_2+l_7(cq_2cq_5+sq_2sq_4sq_5) \\ l_1+l_2+l_3+q_1+l_7cq_4sq_5 \\ \cdots\cdots\cdots\cdots \\ q_2 \\ q_4 \\ q_5 \end{bmatrix}. \tag{26}$$

3. CONCLUSIONS

In accordance with [Isp06], [Isp04], [Kha02] and [Det07], have been obtained equations geometric model direct relationship (26) for the robot TRTRR, which expresses the position of the tool prehensiune 6 through a point and orientation matrix elements by its guidance.

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MODELAREA GEOMETRICĂ A ROBOTULUI SERIAL MODULAR TRTRR

Rezumat: Lucrarea prezintă modelarea geometrică a unui robot serial modular cu cinci grade de mobilitate, care este compus din trei module de rotație şi două de translație. Robotul, astfel conceput, face parte dintr-o celulă de fabricație, pentru care structura este impusă de mişcările necesare din punct de vedere funcțional, pentru încadrarea în structura propusă.

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