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THE GEOMETRIC MODELLING OF THE SERIAL MODULAR ROBOT TRTTRR1

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Abstract: *The paper present the direct geometrical model for the robot TRTTRR1. For establishing the geometrical model equations is necessary to determine the rotation matrix. Then is necessary to determine the independent parameters of the orientation, the vectors of position and, finally, following the results obtained result the direct geometrical model equations.* ***Key words:*** *geometric model, kinematic structure, modular robot, rotation matrix, independent parameters, column vector, characteristic point.*

1. INTRODUCTION

In this paper the authors present the direct geometric modeling of the TRTTRR1 robot using the rotation matrix presented in [1] and [2].

In the figure 1 is shown the mechanical structure of a serial robot with (n) degrees of freedom, having an open kinematic chain. Robot's mechanical structure is made of n+1 rigid elements linked together by (n) kinematic coupling of rotation (R) or translation (T). In the origin of each item k (k=1÷n) is attached a mobile reference system (T_k) and at the base of the robot is inserted the fixed reference system (T_0) in the point O_0.

The direct geometric modelling (DGM), according to [2] implies that the mechanical structure of the robot is in a known configuration, represented by the vector $\overline{q}$ of generalized coordinates.

The position of the reference system (T_n) which is jointly with the gripper of the robot, in relation with the fixed reference system (T_0), it can be determined as:

- determining the position of the origin O_n of the system (T_n) by the vector $\overline{p}_n = [p]^0$;
- using the rotation matrix, the orientation of each axis of the system (T_n) in relation with the system (T_0) it's determined.

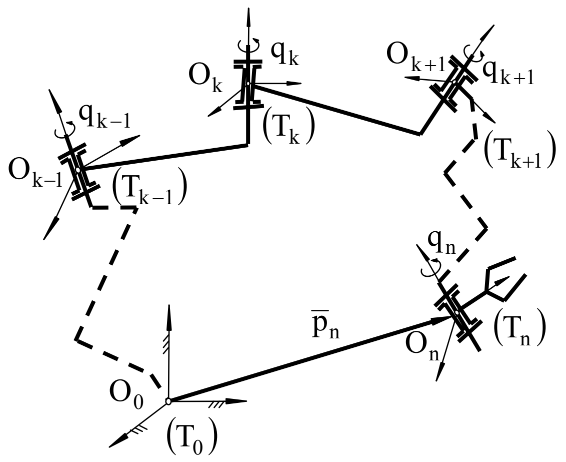

Fig. 1. The kinematic structure of a robot with (n) degrees of freedom

Using the successive iterations the problem of direct geometric modelling it can be solved. Thus considering the sequence of elements (q-1, k), k=1÷n, from the kinematic structure of the robot and corresponding to this sequence, are considered known the following:

$[R]_k^{k-1}$ - is the rotation matrix expressing the orientation axes of the system (T_k) to the system (T_{k-1}); it can be expressed mathematically as:

$$[R]_k^{k-1} = \left[\overline{x}_k^{k-1}\ \overline{y}_k^{k-1}\ \overline{z}_k^{k-1}\right] = \begin{cases} R(\{\overline{x}\ \overline{y}\ \overline{z}\}, q_k), \textit{if rotation torque} \\ J_3, \textit{if translation torque} \end{cases} \quad (1)$$

$\overline{r}_k^{k-1}$ - is the column position vector of the origin O_k of the system (T_k) in relation with the origin O_{k-1} of the system (T_{k-1}), which can be written as:

$$\overline{r}_k^{k-1} = \overline{p}_{k,k-1}^{k-1} = \begin{cases} \left[x_k^{k-1}\ y_k^{k-1}\ z_k^{k-1}\right], if\ rotation\ torque \\ r_k^{k-1}(q_k), if\ translation\ torque \end{cases} \quad (2)$$

$\overline{p}^k$ - is the position of a point P in relation with the system (T_k), which can be written as:

$$\overline{p}^k = [p]^k = \left[p_x^k\ p_y^k\ p_z^k\right]^T \quad (3)$$

Next, using the matrix equation below, we calculate the position of the ponit P relative to the reference system (T_{k-1}):

$$\overline{p}^{k-1} = [R]_k^{k-1} \cdot \overline{p}^k = \left[p_x^{k-1}\ p_y^{k-1}\ p_z^{k-1}\right]^T. \quad (4)$$

The relation (4) is an iterative relation to obtain the following expressions, if the index k is from 1÷n:

$$\begin{bmatrix} \overline{p}^0 = \overline{p} \\ \overline{p}^1 \\ \cdots \\ \overline{p}^{k-1} \\ \cdots \\ p^{n-1} \end{bmatrix} = \begin{bmatrix} [R]_0^1 \cdot \overline{p}^1 \\ [R]_2^1 \cdot \overline{p}^2 \\ \cdots \\ [R]_k^{k-1} \cdot \overline{p}^k \\ \cdots \\ [R]_n^{n-1} \cdot \overline{p}^n \end{bmatrix} = \begin{bmatrix} \left[p_x\ p_y\ p_z\right]^T \\ [R]_2^1 \cdot \left[p_x^2\ p_y^2\ p_z^2\right]^T \\ \cdots\cdots \\ [R]_k^{k-1} \cdot \left[p_x^k\ p_y^k\ p_z^k\right]^T \\ \cdots\cdots \\ [R]_n^{n-1} \cdot \left[p_x^n\ p_y^n\ p_z^n\right]^T \end{bmatrix}. \quad (5)$$

For each k=1÷(n-1) and considering that $[\overline{p}]^k = [R]_{k+1}^k \cdot [\overline{p}]^{k+1}$, then the first relation from (5) becomes:

$$\overline{p}^0 = [R]_1^0 \cdot [R]_2^1 \ldots [R]_k^{k-1} \ldots [R]_n^{n-1} \cdot [p]^n \quad (6)$$

and this is the transformation matrix equation of the column vector $\overline{p}^n$, assumed known, the vector column $\overline{p}^0$. The equation above is equivalent to:

$$\overline{p}^0 = \overline{p} = [R]_n^0 \cdot \overline{p}^n \quad (7)$$

or

$$\left[p_x\ p_y\ p_z\right]^T = \left[\overline{x}_n\ \overline{y}_n\ \overline{z}_n\right] \cdot \left[p_x^n\ p_y^n\ p_z^n\right]^T.$$

To determine de rotation matrix, from the relations (6)-(7) it can be obtained a matrix relation that has the following form:

$$[R]_0^n = \prod_{k=1}^{n} [R]_k^{k-1} = \prod_{k=1}^{n} R(\{x\ y\ z\}, q_k) \quad (8)$$

or

$$\left[\overline{x}_n\ \overline{y}_n\ \overline{z}_n\right] = \prod_{k=1}^{n} \left[\overline{x}_k^{k-1}\ \overline{y}_k^{k-1}\ \overline{z}_k^{k-1}\right] = \begin{bmatrix} \alpha_{nx} & \alpha_{ny} & \alpha_{nz} \\ \beta_{nx} & \beta_{ny} & \beta_{nz} \\ \gamma_{nx} & \gamma_{ny} & \gamma_{nz} \end{bmatrix}. \quad (9)$$

The next step is to determine the column vector $\overline{r}_k^{k-1} = \overline{p}_{k,k-1}^{k-1}$ towards the fixed system (T_0), with the relation:

$$\overline{p}_{k,k-1} = [R]_{k-1}^0 \cdot \overline{r}_k^{k-1} = \begin{cases} \overline{r}_1^0, for\ k = 1 \\ \left\{ \prod_{j=1}^{k-1} [R]_j^{j-1} \right\} \cdot \overline{r}_k^{k-1}, for\ k = 2 \div (n+1). \end{cases} \quad (10)$$

Also, it's calculated the position of each origins O_k of the system (T_k) in relation to (T_0), with the relations:

$$\overline{p}_k = \overline{p}_{k-1} + \overline{p}_{k,k-1} = \sum_{j=1}^{k} \overline{p}_{j,j-1}, for\ k = 1 \div (n+1), \quad (11)$$

with which the column vectors are obtained:

$$\overline{p}_n = \overline{p}_{n-1} + \overline{p}_{n,n-1} = \sum_{k=1}^{n} \overline{p}_{k,k-1} = \left[p_{xn}\ p_{yn}\ p_{zn}\right]^T \quad (12)$$

$$\overline{p} = \overline{p}_{n+1} = \overline{p}_n + \overline{p}_{n+1,n} = \sum_{k=1}^{n+1} \overline{p}_{k,k-1} = \left[p_x\ p_y\ p_z\right]^T. \quad (13)$$

The points O_n and P belong to the mobile system (T_n), thus implicitly to the gripper, so that the relations (12)-(13) determines the coordinates in relation to the fixed system (T_0).

In view of (8), (12) and (13), can write the equations:

$$\left.\begin{matrix} \left[\overline{p}_n^T\ \overline{x}_n^T\ \overline{y}_n^T\ \overline{z}_n^T\right]^T \\ \left[\overline{p}^T\ \overline{x}_n^T\ \overline{y}_n^T\ \overline{z}_n^T\right]^T \end{matrix}\right\} = \left[f_j(q_k, k = 1 \div n), j = 1 \div 12\right]^T. \quad (14)$$

The equations above are the direct geometric model equations (DGM). From these equations, only six are independent because three parameters are needed for guidance, so that we can write the identity matrix:

$$[R]_n^0(q_k, k = 1 \div n) = R(\alpha, \beta, \gamma), \quad (15)$$

where R (α, β, γ) is the orientation matrix corresponding to a set of Euler angles. For example the matrix $R(\alpha_z - \beta_x - \gamma_y)$, and from the identity matrix (15) we obtain the independent parameters of the orientation and for $\gamma_{nz} \neq \pm 1$,

$$\begin{aligned} \alpha_z &= A\tan 2(\alpha_{nz}, -\beta_{nz}) \\ \beta_x &= A\tan 2(\alpha_{nz} s\alpha_z - \beta_{nz} c\alpha_z, \gamma_{nz}) \\ \gamma_z &= A\tan 2(-\alpha_{ny} c\alpha_z - \beta_{ny} s\alpha_z, \alpha_{nx} c\alpha_z + \beta_{nx} s\alpha_z). \end{aligned} \quad (16)$$

According to the equations (14), we can write the column vector of the operational coordinates:

$$\overline{X}^0 = \left[p_x\ p_y\ p_z\ \alpha_z\ \beta_y\ \gamma_z\right]^T = \\ = \left[f_j\left(q_k, k = 1 \div n\right), j = 1 \div 6\right]^T. \qquad (17)$$

The relation (17) defines the position of the robot's gripper with the fixed system (T_0) by: coordinates p_x, p_y, p_z of a point and the α_z, β_x, γ_z elements of the rotation matrix R, defining its orientation.

2. THE GEOMETRIC MODELLING OF THE SERIAL MODULAR ROBOT TRTTRR1

2.1. The description of the mechanical structure of the TRTTRR1 robot

The robot with six degrees of freedom shown in figure 2.5, has a kinematic structure consists of: module 1 is the horizontal translation module of the whole robot; the module 2 is the rotation module of the robot's arm; the module 3 is the vertical translation module; the module 4 is the horizontal translation module of the robot's arm; the 5 and 6 module, together form the orientation module of the gripper 7.

In the figure 2.5 are defined as follows:
l_i – the constructive parameters of the robot, $i=1\div7$ and q_k – the generalized coordinates, $k=1\div6$.

2.2. The direct geometric modelling of the robot TRTTRR1 using the rotation matrices method

The orientation matrices which espress the relative orientation of each system compared with the previuos system, are:

$$[R]_1^0 = [I_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; [R]_2^1 = [\overline{z}_2; q_2] = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \qquad (18)$$

$$[R]_3^2 = [I_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; [R]_4^3 = [I_4] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \qquad (19)$$

$$[R]_5^4 = R[\overline{x}_5;\ q_5] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & -sq_5 \\ 0 & sq_5 & cq_5 \end{bmatrix}; \qquad (20)$$

$$[R]_6^5 = [\overline{y}_6; q_6] = \begin{bmatrix} cq_6 & 0 & sq_6 \\ 0 & 1 & 0 \\ -sq_6 & 0 & cq_6 \end{bmatrix}; \qquad (21)$$

$$[R]_7^6 = I_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (22)$$

The relative position vectors of the origins O_i of the reference systems $O_ix_iy_iz_i$, $i=0\div7$, compared with the previous system, have the following matrix expression:

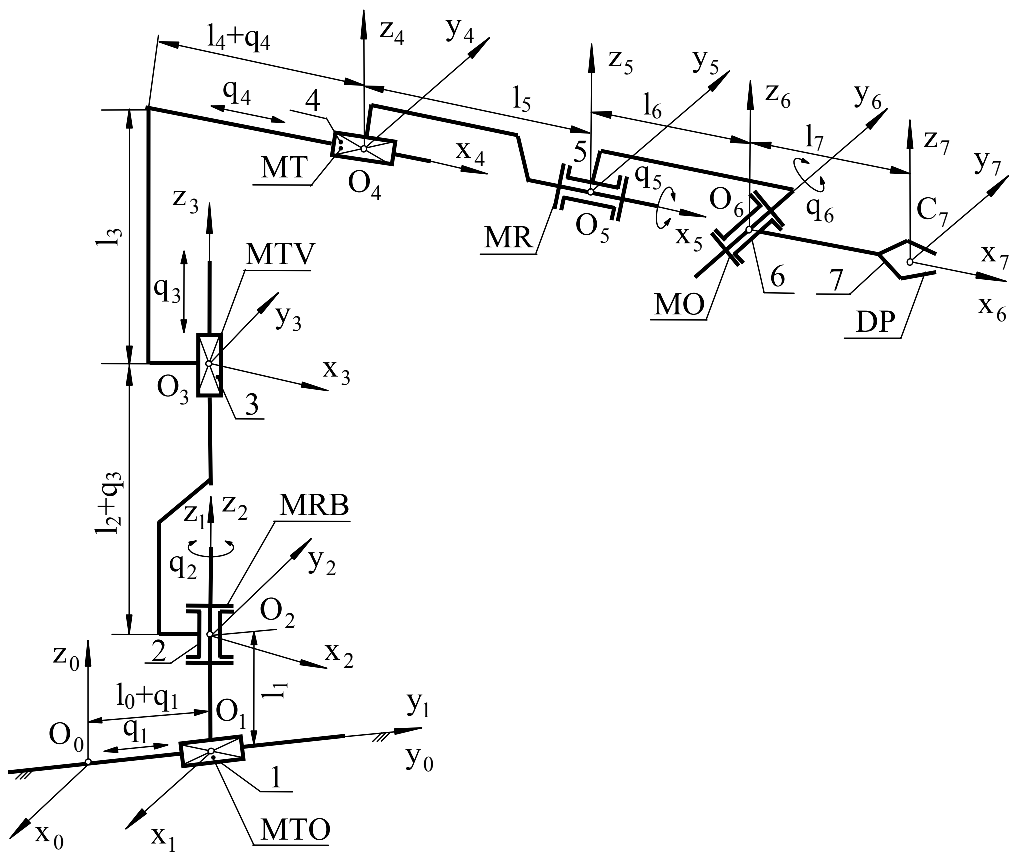

Fig. 2. The kinematic scheme of the serial modular TRTTRR1 robot

$$\bar{r}_1^0=\begin{bmatrix}0\\ l_0+q_1\\ 0\end{bmatrix};\ \bar{r}_2^1=\begin{bmatrix}0\\ 0\\ l_1\end{bmatrix};\ \bar{r}_3^2=\begin{bmatrix}0\\ 0\\ l_2+q_3\end{bmatrix}; \quad (23)$$

$$\bar{r}_4^3=\begin{bmatrix}l_4+q_4\\ 0\\ l_3\end{bmatrix};\ \bar{r}_5^4=\begin{bmatrix}l_5\\ 0\\ 0\end{bmatrix};\ \bar{r}_6^5=\begin{bmatrix}l_6\\ 0\\ 0\end{bmatrix};\ \bar{r}_7^6=\begin{bmatrix}l_7\\ 0\\ 0\end{bmatrix}. \quad (24)$$

According to [3] and [4], the absolute rotation matrices which express the orientation of each mobile system in relation to the fixed system, are obtained with the following relations:

$$[R]_2^0=[R]_1^0\cdot[R]_2^1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}= \quad (25)$$
$$=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix};$$

$$[R]_3^0=[R]_2^0\cdot[R]_3^2=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}= \quad (26)$$
$$=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix};$$

$$[R]_4^0=[R]_3^0\cdot[R]_4^3=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}= \quad (27)$$
$$=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix};$$

$$[R]_5^0=[R]_4^0\cdot[R]_5^4=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&0&0\\0&cq_5&-sq_5\\0&sq_5&cq_5\end{bmatrix}= \quad (28)$$
$$=\begin{bmatrix}cq_2&-sq_2cq_5&sq_2sq_5\\sq_2&cq_2cq_5&-cq_2sq_5\\0&sq_5&cq_5\end{bmatrix};$$

$$[R]_6^0=[R]_5^0\cdot[R]_6^5=$$
$$=\begin{bmatrix}cq_2&-sq_2cq_5&sq_2sq_5\\sq_2&cq_2cq_5&-cq_2sq_5\\0&sq_5&cq_5\end{bmatrix}\cdot\begin{bmatrix}cq_6&0&sq_6\\0&1&0\\-sq_6&0&cq_6\end{bmatrix}=$$
$$=\begin{bmatrix}cq_2cq_6-sq_2sq_5sq_6&-sq_2cq_5&cq_2sq_6+sq_2sq_5cq_6\\sq_2cq_6+cq_2sq_5sq_6&cq_2cq_5&sq_2sq_6-cq_2sq_5cq_6\\-cq_5sq_6&sq_5&cq_5cq_6\end{bmatrix}; \quad (29)$$

$$[R]_7^0=[R]_6^0\cdot[R]_7^6=$$
$$=\begin{bmatrix}cq_2cq_6-sq_2sq_5sq_6&-sq_2cq_5&cq_2sq_6+sq_2sq_5cq_6\\sq_2cq_6+cq_2sq_5sq_6&cq_2cq_5&sq_2sq_6-cq_2sq_5cq_6\\-cq_5sq_6&sq_5&cq_5cq_6\end{bmatrix}\cdot$$
$$\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=$$
$$=\begin{bmatrix}cq_2cq_6-sq_2sq_5sq_6&-sq_2cq_5&cq_2sq_6+sq_2sq_5cq_6\\sq_2cq_6+cq_2sq_5sq_6&cq_2cq_5&sq_2sq_6-cq_2sq_5cq_6\\-cq_5sq_6&sq_5&cq_5cq_6\end{bmatrix}. \quad (30)$$

According to [5], the set of independent parameters of the orientation ($\alpha_z-\beta_x-\gamma_y$), is determined by identifying the elements 22, 32 and 33 from the matrix relation (30) with the same elements of the relation (31):

$$R(\alpha_z-\beta_x-\gamma_y)=$$
$$=\begin{bmatrix}-s\alpha_zs\beta_xs\gamma_y+c\alpha_zc\gamma_y&-s\alpha_zc\beta_x&s\alpha_zs\beta_xc\gamma_y+c\alpha_zs\gamma_y\\c\alpha_zs\beta_xs\gamma_y+s\alpha_zc\gamma_y&c\alpha_zc\beta_x&-c\alpha_zs\beta_xc\gamma_y+s\alpha_zs\gamma_y\\-c\beta_xs\gamma_y&s\beta_x&c\beta_xc\gamma_y\end{bmatrix} \quad (31)$$

The independent parameters of orientation ($\alpha_z-\beta_x-\gamma_y$) are described in the relation:

$$\begin{bmatrix}\alpha_z&\beta_x&\gamma_y\end{bmatrix}^T=\begin{bmatrix}q_2&q_5&q_6\end{bmatrix}^T. \quad (32)$$

The relations below, called the translational relative vectors, are used to express the posititon of the origin of each reference system, in relation to the previous system:

$$\bar{p}_{10}=[\bar{r}]_1^0=\begin{bmatrix}0\\ l_0+q_1\\ 0\end{bmatrix}; \quad (33)$$

$$\bar{p}_{21}=[R]_1^0\cdot[\bar{r}]_2^1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\l_1\end{bmatrix}=\begin{bmatrix}0\\0\\l_1\end{bmatrix}; \quad (34)$$

$$\bar{p}_{32}=[R]_2^0\cdot[\bar{r}]_3^2=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\l_2+q_3\end{bmatrix}=\begin{bmatrix}0\\0\\l_2+q_3\end{bmatrix}; \quad (35)$$

$$\bar{p}_{43}=[R]_3^0\cdot[\bar{r}]_4^3=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}l_4+q_4\\0\\l_3\end{bmatrix}= \quad (36)$$
$$=\begin{bmatrix}(l_4+q_4)\cdot cq_2\\(l_4+q_4)\cdot sq_2\\l_3\end{bmatrix};$$

$$\overline{p}_{54}=[R]_4^0\cdot[\overline{r}]_5^4=\begin{bmatrix}cq_2 & -sq_2 & 0\\ sq_2 & cq_2 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}l_5\\0\\0\end{bmatrix}= \tag{37}$$

$$=\begin{bmatrix}l_5cq_2\\ l_5sq_2\\ 0\end{bmatrix};$$

$$[\overline{p}]_{65}=[R]_5^0\cdot[\overline{r}]_6^5=$$

$$=\begin{bmatrix}cq_2 & -sq_2cq_5 & sq_2sq_5\\ sq_2 & cq_2cq_5 & -cq_2sq_5\\ 0 & sq_5 & cq_5\end{bmatrix}\cdot\begin{bmatrix}l_6\\0\\0\end{bmatrix}=\begin{bmatrix}l_6cq_2\\ l_6sq_2\\ 0\end{bmatrix}; \tag{38}$$

$$[\overline{p}]_{76}=[R]_6^0\cdot[\overline{r}]_7^6=$$

$$=\begin{bmatrix}cq_2cq_6-sq_2sq_5sq_6 & -sq_2cq_5 & cq_2sq_6+sq_2sq_5cq_6\\ sq_2cq_6+cq_2sq_5sq_6 & cq_2cq_5 & sq_2sq_6-cq_2sq_5cq_6\\ -cq_5sq_6 & sq_5 & cq_5cq_6\end{bmatrix}\cdot$$

$$\cdot\begin{bmatrix}l_7\\0\\0\end{bmatrix}=\begin{bmatrix}(cq_2cq_6-sq_2sq_5sq_6)l_7\\ (sq_2cq_6+cq_2sq_5sq_6)l_7\\ -cq_5sq_6l_7\end{bmatrix}. \tag{39}$$

To obtain the origin position of each system to the fixed system $O_0x_0y_0z_0$ from the base of the robot, are used the relations (33)-(39). Thus, we can write:

$$[\overline{p}]_1=[\overline{p}]_{10}=\begin{bmatrix}0\\ l_0+q_1\\ 0\end{bmatrix}; \tag{40}$$

$$[\overline{p}]_2=[\overline{p}]_1+[\overline{p}]_{21}=\begin{bmatrix}0\\ l_0+q_1\\ 0\end{bmatrix}+\begin{bmatrix}0\\0\\l_1\end{bmatrix}=\begin{bmatrix}0\\ l_0+q_1\\ l_1\end{bmatrix}; \tag{41}$$

$$[\overline{p}]_3=[\overline{p}]_2+[\overline{p}]_{32}=\begin{bmatrix}0\\ l_0+q_1\\ l_1+l_2+q_3\end{bmatrix}; \tag{42}$$

$$[\overline{p}]_4=[\overline{p}]_3+[\overline{p}]_{43}=\begin{bmatrix}(l_4+q_4)cq_2\\ l_0+q_1+(l_4+q_4)sq_2\\ l_1+l_2+l_3+q_3\end{bmatrix}; \tag{43}$$

$$[\overline{p}]_5=[\overline{p}]_4+[\overline{p}]_{54}=\begin{bmatrix}(l_4+l_5+q_4)cq_2\\ l_0+q_1+(l_4+l_5+q_4)sq_2\\ l_1+l_2+l_3+q_3\end{bmatrix}; \tag{44}$$

$$[\overline{p}]_6=[\overline{p}]_5+[\overline{p}]_{65}=$$

$$=\begin{bmatrix}(l_4+l_5+l_6+q_4)cq_2\\ l_0+q_1+(l_4+l_5+l_6+q_4)sq_2\\ l_1+l_2+l_3+q_3\end{bmatrix}; \tag{45}$$

$$[\overline{p}]_7=[\overline{p}]_6+[\overline{p}]_{76}=$$

$$=\begin{bmatrix}(l_4+l_5+l_6+q_4)cq_2+(cq_2cq_6-sq_2sq_5sq_6)l_7\\ l_0+q_1+(l_4+l_5+l_6+q_4)sq_2+(sq_2cq_6+cq_2sq_5sq_6)l_7\\ l_1+l_2+l_3+q_3-cq_5sq_6l_7\end{bmatrix}. \tag{46}$$

The column vector of the operational coordinates which defines the position of the robot's gripper relative to the fixed system, by the coordinates $p_{x_7}, p_{y_7}, p_{z_7}$ of a point and the elements orientation α_z, β_x, γ_y of the matrix R, is expressed by the relation:

$$[\overline{X}]^0=\begin{bmatrix}p_{x_7}\\ p_{y_7}\\ p_{z_7}\\ \dots\\ \alpha_z\\ \beta_x\\ \gamma_y\end{bmatrix}=\begin{bmatrix}(l_4+l_5+l_6+q_4)cq_2+(cq_2cq_6-sq_2sq_5sq_6)l_7\\ l_0+q_1+(l_4+l_5+l_6+q_4)sq_2+(sq_2cq_6+cq_2sq_5sq_6)l_7\\ l_1+l_2+l_3+q_3-cq_5sq_6l_7\\ \dots\dots\dots\dots\\ q_2\\ q_5\\ q_6\end{bmatrix}. \tag{47}$$

3. CONCLUSION

To effectuate the direct geometric modelling of the TRTTRR1 robot are required the constructive parameters and the generalized coordinates.

Having the rotation matrix that expressed the relative orientation of each system in relation to the previous system and relative position vectors, the position and orientation of the robot's gripper relative to the fixed system it can be determined after following some steps.

The direct geometrical modelling defines the position and orientation of the gripper, expressed by the coordinates of the characteristic point and relative to this point the orientation of the gripper.

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Modelarea geometrică a robotului serial modular TRTTRR1

Rezumat:

Lucrarea prezintă modelul geometric direct pentru robotul TRTTRR1.

Pentru stabilirea ecuațiilor modelului geometric este necesar determinarea matricei de rotație. Apoi este necesar determinarea parametrilor independenți ai orientării, vectorii de poziție şi, în final, urmărind rezultatele obținute rezultă ecuațiile modelului geometric direct.

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