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THE DYNAMIC STUDY OF THE MOVING SYSTEM ACTUATED BY A FORCE

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Abstract: *The paper shows the numerical simulation of a mechanical system composed by a trolley, moving along a rigid bar, actuated by a force, which carries a load. The numerical results were obtained with the help of C program, using the Runge-Kutta numerical method for solving differential equations, for different parameter values.*
Key words: *moving system, trolley, load, non-linear differential equations, Runge-Kutta method.*

1. INTRODUCTION

The mechanical system composed by a trolley, moving along a rigid bar, actuated by a force, which carries a load, represents the one of the most used machinery in different areas of industry, such as: constructions, transports or in the manufacture or assembly of heavy components.

The runway design features of these mechanical systems depend on the area of use, such as: the weight and the type of load, location and geometric characteristics, as well as the environmental conditions.

The purpose of these systems is to place an object in one location, without having collisions with other objects and to fix it in the right place with the highest precision. Due the load, oscillations occur during relocation and this is a problem that should always be avoided.

The loads displacements on the cranes were very much studied in mechanical engineering and not only [1], [3]. The dynamical study of these mechanical systems is presented in many papers [11], [5], [6], some authors treats various particular cases [3], [4].

In this paper, was considered a mechanical system, shown in figure 1, the trolley of mass M, actuated by force F, carries a mobile load of mass m.

The equations of motion of the assembly were established using the Lagrange's method [7], [9], [10] and the numerical simulation was performed with the C program [8].

2. THEORETICAL BACKGROUND

The studied mechanical system (figure 1) is composed of a trolley of mass M, moving on the rigid bar, and the mobile load, suspended of the trolley, of mass m. This mobile load linked with mass M, is fixed to the trolley by a string considered inextensible of length ℓ.

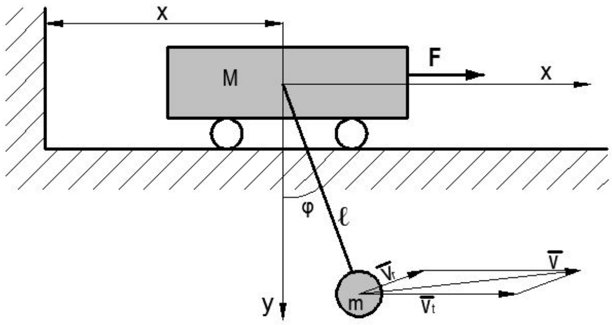

Fig. 1 The trolley with mobile load

The angle φ represents the deviation of the load to equilibrium positions, and x determines the horizontally trolley position.

The movement of the entire system is performed using force F, acting on the mass M, being constant or variable,

$$F(t)=\begin{cases}\frac{F_0}{t_0}t, & t<t_0\\ F_0, & t\ge t_0\end{cases}$$

In this case, the mechanical system has two degrees of freedom. There are two generalized coordinates, x and φ, that determine the position of system.

In case of x = x(t), the trolley speed is known and the system don't depend on this coordinate. The mechanical system has a single degree of freedom and exist a single generalized coordinate, the angle φ.

Further is studying the case in which exists a force F acting on the mass M.

The differential equations of mechanical system will be established using the method of Lagrange [7], [9], [10]. The function of Lagrange [2] has the following form:

$$L=E_c-E_p \qquad (1)$$

where E_c represents the total kinetic energy of the masses M and m, and E_p the potential energy.

The total kinetic energy can be written in the following form:

$$E_c=E_{cM}+E_{cm}=\frac{M\dot{x}^2}{2}+\frac{mv^2}{2} \qquad (2)$$

where v represents the absolute speed of the mass m, and has the form:

$$v=\sqrt{\dot{x}^2+\ell^2\dot{\varphi}^2+2\ell\dot{x}\dot{\varphi}\cos\varphi} \qquad (3)$$

the relation resulting from the composition of transport and the relative speeds.

Summing the kinetic energy of the trolley of mass M and of the mobile load of mass m, we obtain the expression of the total kinetic energy:

$$E_c=\frac{1}{2}\left(M\dot{x}^2+m\dot{x}^2+m\ell^2\dot{\varphi}^2\right)+ \\ +m\ell\dot{x}\dot{\varphi}\cos\varphi \qquad (4)$$

The potential energy can be written as:

$$E_p=mg\ell(1-\cos\varphi) \qquad (5)$$

Subtracting relation (4) of (5) on obtains the Lagrange's functions, depending of the generalized coordinates x and φ:

$$L=\frac{1}{2}\left(M\dot{x}^2+m\dot{x}^2+m\ell^2\dot{\varphi}^2\right)+ \\ +m\ell\dot{x}\dot{\varphi}\cos\varphi-mg\ell(1-\cos\varphi) \qquad (6)$$

and the two Lagrange equations are:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)-\frac{\partial L}{\partial x}=Q_x\ , \\ \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right)-\frac{\partial L}{\partial\varphi}=Q_\varphi \qquad (7)$$

where generalized forces are Q_x = F and Q_φ = 0. The system of differential equations, after some calculi, can be written as:

$$\begin{cases}(M+m)\ddot{x}+m\ell\ddot{\varphi}\cos\varphi=F+m\ell\dot{\varphi}^2\sin\varphi\\ \ddot{x}\cos\varphi+\ell\ddot{\varphi}=-g\sin\varphi\end{cases} \qquad (8)$$

with initial conditions:

$t=0$, $x=x_0$, $\varphi=\varphi_0$ and $\dot{x}=\dot{x}_0$, $\dot{\varphi}=\dot{\varphi}_0$.

The system (8) can be written in the following form:

$$\begin{cases}a_{11}\ddot{x}+a_{12}\ddot{\varphi}=b_1\\ a_{21}\ddot{x}+a_{22}\ddot{\varphi}=b_2\end{cases}, \qquad (9)$$

where the terms $\ddot{x}$ and $\ddot{\varphi}$ are unknowns. The following notations were used:

$$a_{11}=M+m\ ,\quad a_{12}=m\ell\cos\varphi \qquad (10) \\ a_{21}=\cos\varphi\ ,\quad a_{22}=\ell$$

$$b_1=F+m\ell\dot{\varphi}^2\sin\varphi \qquad (11) \\ b_2=-g\sin\varphi$$

From linear algebraic system of equations (9), can be determined the unknowns (the second order derivatives), using Cramer's rule:

$$\ddot{x} = \frac{b_1 a_{22} - b_2 a_{12}}{\Delta}, \quad \ddot{\varphi} = \frac{b_2 a_{11} - b_1 a_{21}}{\Delta} \quad (12)$$

where $\Delta = a_{11}\, a_{22} - a_{12}\, a_{21}$.

Using the following notations in the linear system (9), $y_1 = x$, $y_2 = \varphi$, $y_3 = \dot{x}$, $y_4 = \dot{\varphi}$, is obtained the system composed by four first order differential equations:

$$\begin{cases} \dot{y}_1 = y_3 \\ \dot{y}_2 = y_4 \\ \dot{y}_3 = \dfrac{b_1\, a_{22} - b_2\, a_{12}}{\Delta} \\ \dot{y}_4 = \dfrac{b_2\, a_{11} - b_1\, a_{21}}{\Delta} \end{cases} \quad (13)$$

with following initial conditions: $y_1 = x_0$, $y_2 = \varphi_0$ and $y_3 = \dot{x}_0$, $y_4 = \dot{\varphi}_0$.

3. NUMERICAL RESULTS

The numerical solving of the system of differential equations (13), was performed with a C program [8]. In the program were considered the following numerical values: M + m = 500 [kg], ℓ = 1.5 [m], F = 100 [N], t = 5 [s] and g = 9.81 [m/s^2].

Considering a mass M + m actuated by a force F_0 on obtains the following formula for the performed displacement, depending on time:

$$x_{(M+m)} = \frac{F_0\, t^2}{2(M+m)}$$

After the numerical solving of the system (8) of differential equations, we obtain the values of displacement $x_{(M)}$. In the figure 2 are shown the diagrams of differences $x_{(M)} - x_{(M+m)}$ corresponding to four considered cases, noticing the influence of mass m values on the shape of the diagrams.

In figures 3, 4 and 5 are presented the speeds of mass M in three variants of considered laws of variation of force F.

We notice that if the value of t_0 grows up the difference between the diagrams are diminished.

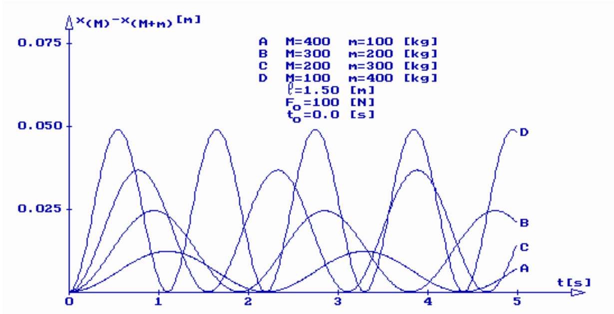

Fig. 2 The difference of displacements

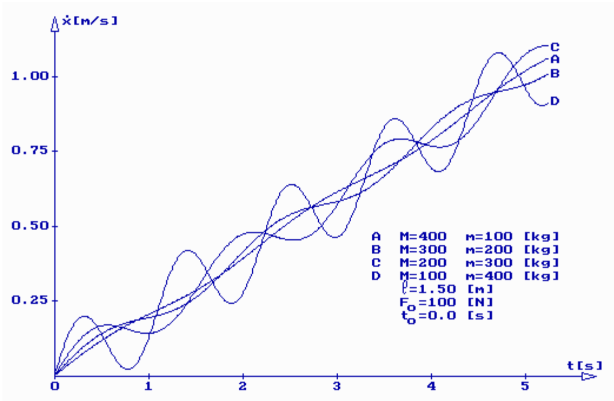

Fig. 3 The speeds of mass M. The force F is constant

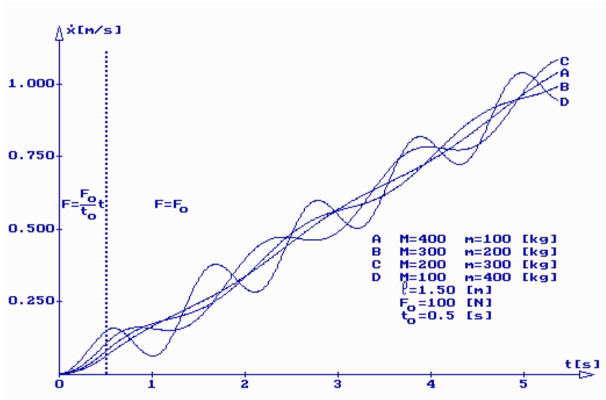

Fig. 4 The speeds of mass M. The force F is linear in time interval [0;0.5] [s] and constant if t > 0.5 [s]

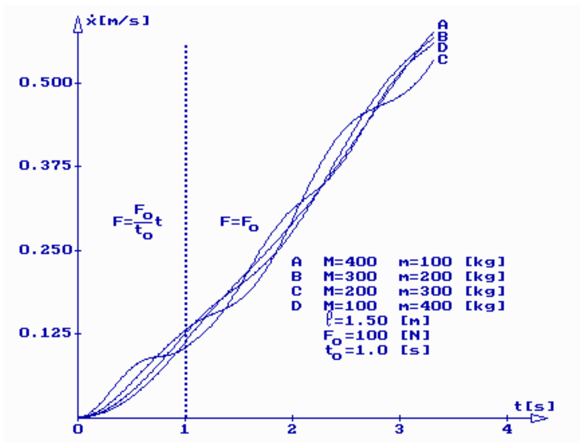

Fig. 4 The speeds of mass M. The force F is linear in time interval [0;1.0] [s] and constant if t > 1.0 [s]

4. CONCLUSION

The paper deals with dynamic behaviors of moving system actuated by a force, corresponding to the real case of trolleys movements on the cranes. Equations of motions for a given mechanical model are shown, and numerical results were obtained using the Runge-Kutta numerical method for solving differential equations.

Numerical calculus indicates that the movement of trolley of mass M depends on the weight of mass m (figure 2), for different variants of M + m (A, B, C and D cases).

Comparing the graphical representations in figures 3, 4 and 5, we can notice that influence of time t_0 on the speed trolley of mass M, for cases when force F is constant or linear in particular time intervals.

The aim of this works is to emphasize the moving mechanical system composed by a trolley, moving along a rigid bar, actuated by a force, which carries a load. The numerical results can be applied to model of crane bridge with various structural types.

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Studiul dinamic al unui sistem mobil acţionat de o forţă

Abstract: *Lucrarea prezintă simularea numerică a unui sistem mecanic complex, format dintr-un cărucior care transportă o sarcină mobilă, acţionat de o forţă, deplasându-se pe o bară rigidă. Rezultatele numerice s-au obţinut cu ajutorul programului C, folosindu-se metoda numerică de rezolvare a ecuaţiilor diferenţiale Runge-Kutta, pentru diferite valori ale parametrilor introduşi.*

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