TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 59, Issue I, March, 2016

THE DYNAMIC STUDY AND MODELING OF IMPACT DAMPER

Iuliana Fabiola MOHOLEA, Ioan RADU, Nicolae URSU-FISCHER

Abstract: *The paper presents numerical simulation and graphical analysis of a mechanical vibro-impact system actuated by a harmonic force. The analytical study shows chaotic behavior of the mechanical system with two degrees of freedom and also the influence of initial conditions. Numerical results obtained using Runge-Kutta numerical method for solving differential equations were programmed in C, for different values of mechanical system parameters and initial conditions.*
Key words: *vibro-impact systems, nonlinear differential equations, Runge-Kutta method, chaos.*

1. INTRODUCTION

The impact damper systems are often utilized in many applications in engineering, such as soil compaction machines, drills and ball grinding equipment etc. It is important to study these mechanical systems and especially their dynamic behavior, because these movements are very important in the design and optimization of this vibrating machines and devices.

In this paper, we consider the mechanical system shown in figure 1, the mass m_1 being driven by a harmonic force. The mass m_2 is placed inside the mass m_1 and moves without friction and with limited amplitudes.

The movements of mechanical system with two degree of freedom are modeled with simultaneous differential equations of second order, written according the Newton's law. In the first part of this paper are presented a theoretical analysis of vibro-impact system, the determination the equations of motion and the numerical method used to solve the system of differential equations. The second part contains numerical results and graphs obtained considering different parameters of the mechanical system.

The chaotic motion of mechanical vibro-impact systems with one, two or more degrees of freedom, has been studied in many papers [3], [4], [7], and also in different chapters of books dealing with the mechanical vibrations [2], [5], [10], [11].

The bifurcation diagrams, the masses movements of dynamic systems with single or double impacts are presented in some papers belonging to Peterka et al as [1], [7], [8].

The damping characteristics of a vibro-impact system were studied in [17] also the phenomenon of the cutting tools durability. The masses weight ratio [4], [15], the bifurcation diagrams [6] and the coefficient of restitution can be used to perform optimal design of these systems.

2. THEORETICAL BACKGROUND

The studied mechanical system (Figure 1) is composed of a main system and an auxiliary system, the mass m_1 of the main system being actuated by a harmonic force. Elements of mass m_2 of auxiliary system have to be determined as that the vibration amplitude of the main system be minimal.

The mass m_1 is moved out of equilibrium position due to the harmonic force $F_0\cos\omega t$ and influenced by the spring elasticity coefficient k_1 and by the damper characteristic c_1.

The second mass moves in the longitudinal cavity of mass m_1 and it has the possibility to collide with the walls of the main mass. The two masses are connected by an elastic spring k_2 and a damper c_2

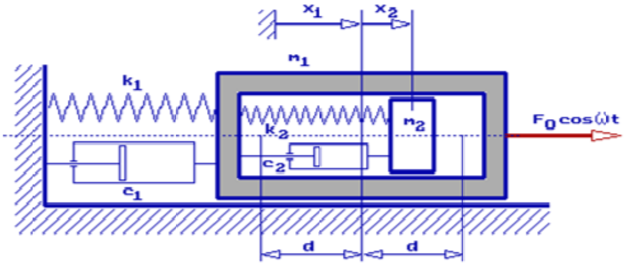

Fig. 1 The vibro-impact system

For writing the equations of motion on consider the following:

- the mechanical system has two degrees of freedom
- the mass m_2 is perfectly rigid
- damping phenomenon of the system is modeled as an elastic collision
- friction between the main mass and impact mass is negligible.

The linear and homogeneous equation of motion of main mass, in absence of the secondary mass, based on the second law of dynamics, can be writing as:

$$m\ddot{x}+c\dot{x}+kx=F_0\cos\omega t \tag{1}$$

The two differential equations corresponding to the two masses, written in matrix form are:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix}+\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix}+ +\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}=\begin{bmatrix} F_0\cos\omega t \\ 0 \end{bmatrix} \tag{2}$$

The absolute displacements of the two masses is denoted by z_1 and z_2 and considering the following form, $z_1=x_1$, $z_2=x_1+x_2$, therefore the equations of motions (2) become:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_1+\ddot{x}_2 \end{bmatrix}+\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\cdot \cdot\begin{bmatrix} \dot{x}_1 \\ \dot{x}_1+\dot{x}_2 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_1+x_2 \end{bmatrix}= =\begin{bmatrix} F_0\cos\omega t \\ 0 \end{bmatrix} \tag{3}$$

After some calculations, the equation (3) can be written as:

$$\begin{bmatrix} m_1 & 0 \\ m_2 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} c_1 & -c_2 \\ 0 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}+ +\begin{bmatrix} k_1 & -k_2 \\ 0 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} F_0\cos\omega t \\ 0 \end{bmatrix} \tag{4}$$

The matrix differential equation (4) can be written in analytical form:

$$\begin{cases} m_1\ddot{x}_1+c_1\dot{x}_1-c_2\dot{x}_2+k_1x_1-k_2x_2=F_0\cos\omega t \\ m_2\ddot{x}_1+m_2\ddot{x}_2+c_2\dot{x}_2+k_2x_2=0 \end{cases} \tag{5}$$

The inverse of inertial matrix is:

$$\begin{bmatrix} m_1 & 0 \\ m_2 & m_2 \end{bmatrix}^{-1}=\begin{bmatrix} \frac{1}{m_1} & 0 \\ -\frac{1}{m_1} & \frac{1}{m_2} \end{bmatrix} \tag{6}$$

Multiplying the matrix equation (4) with the previously computed inverse matrix on obtains:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}=\begin{bmatrix} \frac{1}{m_1} & 0 \\ -\frac{1}{m_1} & \frac{1}{m_2} \end{bmatrix}\cdot\left(-\begin{bmatrix} c_1 & -c_2 \\ 0 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}- -\begin{bmatrix} k_1 & -k_2 \\ 0 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} F_0\cos\omega t \\ 0 \end{bmatrix}\right) \tag{7}$$

After performing the multiplication of matrix inside of parentheses, we obtain:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}=\begin{bmatrix} \frac{1}{m_1} & 0 \\ -\frac{1}{m_1} & \frac{1}{m_2} \end{bmatrix}\cdot\left(-\begin{bmatrix} c_1\dot{x}_1-c_2\dot{x}_2 \\ c_2\dot{x}_2 \end{bmatrix}- -\begin{bmatrix} k_1x_1-k_2x_2 \\ k_2x_2 \end{bmatrix}+\begin{bmatrix} F_0\cos\omega t \\ 0 \end{bmatrix}\right) \tag{8}$$

After the successive calculations, results:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{m_1} & 0 \\ -\dfrac{1}{m_1} & \dfrac{1}{m_2} \end{bmatrix} \cdot \begin{bmatrix} -c_1 \dot{x}_1 + c_2 \dot{x}_2 - k_1 x_1 + k_2 x_2 + F_0 \cos\omega t \\ -c_2 \dot{x}_2 - k_2 x_2 \end{bmatrix} \tag{9}$$

The column matrix of the two masses acceleration is as follows:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-c_1 \dot{x}_1 + c_2 \dot{x}_2 - k_1 x_1 + k_2 x_2 + F_0 \cos\omega t}{m_1} \\ \dfrac{c_1 \dot{x}_1 - c_2 \dot{x}_2 + k_1 x_1 - k_2 x_2 - F_0 \cos\omega t}{m_1} - \dfrac{c_2 \dot{x}_2 + k_2 x_2}{m_2} \end{bmatrix} \tag{10}$$

By using the notations: $y_1 = x_1$, $y_2 = x_2$, $y_3 = \dot{x}_1$, $y_4 = \dot{x}_2$, the system of two differential equations of second order (10) may be written as a system of four first order differential equations:

$$\begin{cases} \dot{y}_1 = y_3 \\ \dot{y}_2 = y_4 \\ \dot{y}_3 = \dfrac{-c_1 y_3 + c_2 y_4 - k_1 y_1 + k_2 y_2 + F_0 \cos\omega t}{m_1} \\ \dot{y}_4 = \dfrac{c_1 y_3 - c_2 y_4 + k_1 y_1 - k_2 y_2 - F_0 \cos\omega t}{m_1} - \dfrac{c_2 y_4 + k_2 y_2}{m_2} \end{cases} \tag{11}$$

By replacing the above notations with the absolute displacements z_1 and z_2, one can write:

$$\begin{aligned} z_1 &= y_1, \quad z_2 = y_1 + y_2 \\ \dot{z}_1 &= \dot{y}_1, \quad \dot{z}_2 = \dot{y}_1 + \dot{y}_2 \end{aligned} \tag{12}$$

Noting with $\dot{z}_1^{(init)}$ and $\dot{z}_2^{(init)}$ the absolute speeds of the m_1 and m_2 masses before the impact and with $\dot{z}_1^{(final)}$, $\dot{z}_2^{(final)}$ the speeds after impact, the following notations may be written [16]:

$$\begin{aligned} \dot{z}_1^{(final)} &= \dot{z}_1^{(init)} - \frac{(1+r)(\dot{z}_1^{(init)} - \dot{z}_2^{(init)})}{1 + \dfrac{m_1}{m_2}}, \\ \dot{z}_2^{(final)} &= \dot{z}_2^{(init)} + \frac{(1+r)(\dot{z}_1^{(init)} - \dot{z}_2^{(init)})}{1 + \dfrac{m_2}{m_1}} \end{aligned} \tag{13}$$

where r is the coefficient of restitution, which describes the semi elastic impact of the two masses, having values between 0 and 1. If the impact is elastic the value of coefficient of restitution will be considered 1.

During vibration, when the impact mass m_2 collides with the main mass m_1, occur impulsive forces acting on the two masses. Because the mechanical system executes nonlinear or linear motions, were considered two cases:

1. Case I: $-d < y_2^{(k+1)} < d$, the mass m_2 is free to move, without collisions. In this case the mechanical system executes combinations of linear or harmonic motions.

Using the fourth order numerical method of Runge–Kutta and solving the differential equations of system, it is obtained

$$\begin{bmatrix} y_1^{(k)} \\ y_2^{(k)} \\ y_3^{(k)} \\ y_4^{(k)} \end{bmatrix}_{(t)} \xrightarrow{(\text{Runge-Kutta})} \begin{bmatrix} y_1^{(k+1)} \\ y_2^{(k+1)} \\ y_3^{(k+1)} \\ y_4^{(k+1)} \end{bmatrix}_{(t+\Delta t)} \tag{14}$$

$$\text{if } -d < y_2^{(k+1)} < d \quad \xrightarrow{} \quad (\text{next step})$$

2. Case II: $y_2^{(k+1)} < -d$ or $y_2^{(k+1)} > d$, the mass m_2 collides with the mass m_1, in this case the mechanical system executes nonlinear chaotic movements.

In this case during the considered iteration will exist the following relations between initial and final speeds:

$$\begin{bmatrix} y_1^{(k)} \\ y_2^{(k)} \\ y_3^{(k)} \\ y_4^{(k)} \end{bmatrix}_{(t)} \xrightarrow{\ } (\text{Runge}-\text{Kutta})$$

$$\begin{bmatrix} y_1^{(k+1)} \\ y_2^{(k+1)} = -d \ \text{ or } \ y_2^{(k+1)} = d \\ y_3^{(k+1)} = z_1^{(\text{final})} \\ y_4^{(\text{final})} = z_2^{(\text{final})} - z_1^{(\text{final})} \end{bmatrix}_{(t+\Delta t)} = \begin{bmatrix} y_1^{(k+1)} \\ y_2^{(k+1)} \\ y_3^{(k+1)} \\ y_4^{(k+1)} \end{bmatrix}_{(t+\Delta t)} \tag{15}$$

3. NUMERICAL RESULTS

It was considered the following numerical values for the studied mechanical system:

Table 1 – The parameters used for the study of vibro-impact system movement

Main system	Auxiliary system
$F_0 = 120$ [N]	$m_2 = 5$ [kg]
$\omega = 12$ [rad/s]	$c_2 = 5$ [Ns/m]
$m_1 = 15$ [kg]	$k_2 = 200$ [N/m]
$c_1 = 10$ [Ns/m]	$d = 0.050$ [m]
$k_1 = 5000$ [N/m]	$r = 0.90$

It was required that the impact mass m_2 can execute oscillations inside the main mass m_1 with the maximum amplitude d. During the time t = 2 [s], the figure 2 shows graphs of movement of absolute mass m_1 and the relative displacement of the mass m_2. In this case the relative movement is not limited.

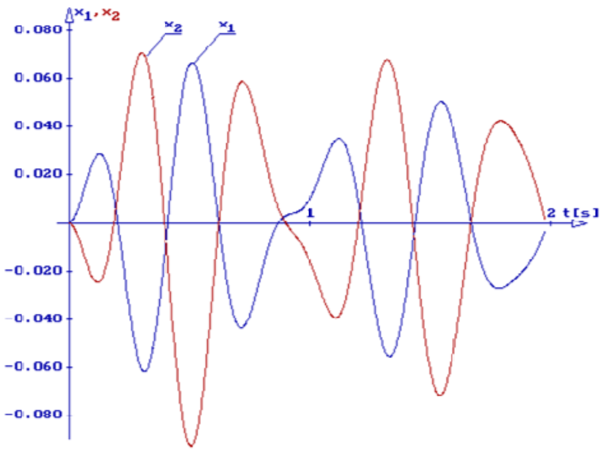

Fig. 2. The movement of the mechanical system with two degrees of freedom. The relative movement is not limited.

In the figure 3 is represented the graph of the absolute displacement of the main mass m_1 and the graph of the relative movement of the impact mass m_2. The relative movement is limited in the interval [-d; d].

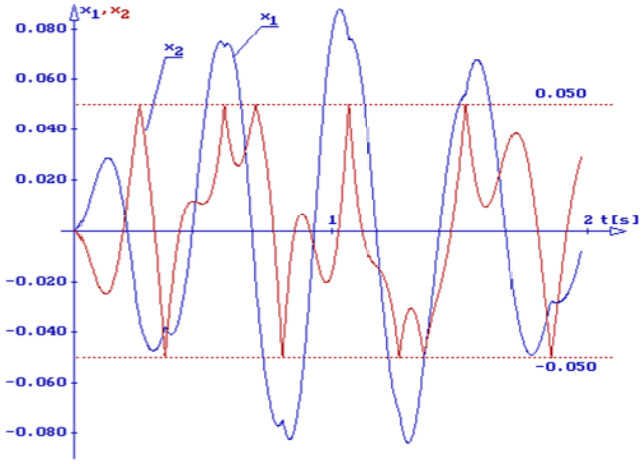

Fig. 3. The movement of the mechanical system with two degrees of freedom. The relative movement of mass m_2 is limited in interval [-0.050, 0.050].

In the figure 4 is represented the graph of displacement of the main mass m_1, in the absence of the auxiliary mass m_2, during time t = 4 [s].

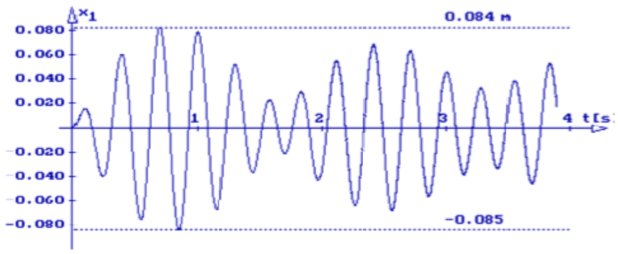

Fig. 4. The movement of the mechanical system with one degree of freedom. The numerical values of parameters are: F_0=110 [N], ω=22 [rad/s], m_1=15 [kg], c_1=10 [Ns/m], k_1=5000 [N/m]

In the following graphs shown in figures 5-10, are presented the movements of mass m_1 (absolute displacements) and the movements of mass m_2 (relative displacements) belonging to the two degrees of freedom mechanical system. Were considered the numerical values: F_0 = 110 [N] and ω = 22 [rad/s], the other values of parameters being unchanged (Table 1). The value of distance d is considered between 10 mm and 60 mm.

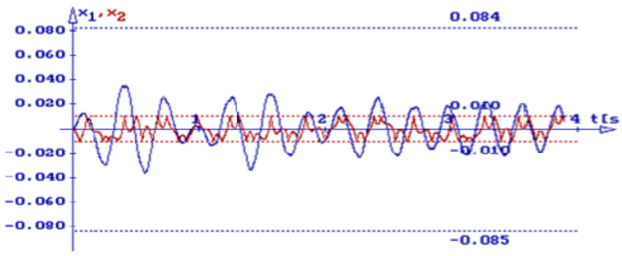

Fig. 5. The distance d = 0.010 [m]

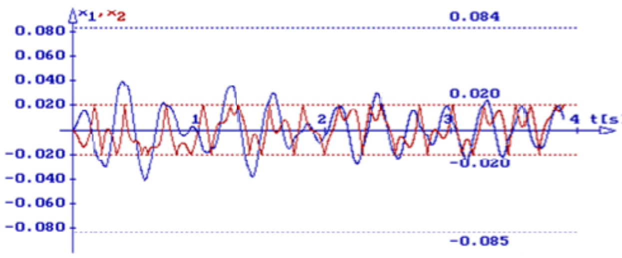

Fig. 6. The distance d = 0.020 [m]

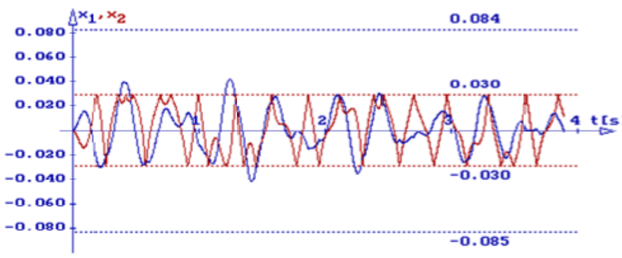

Fig. 7. The distance d = 0.030 [m]

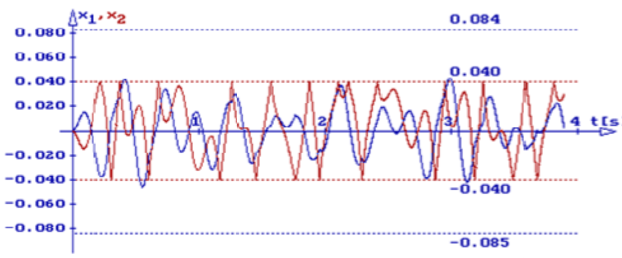

Fig. 8. The distance d = 0.040 [m]

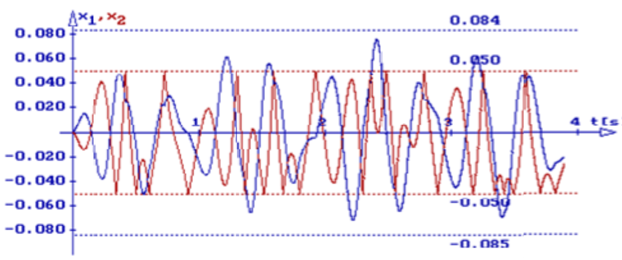

Fig. 9. The distance d = 0.050 [m]

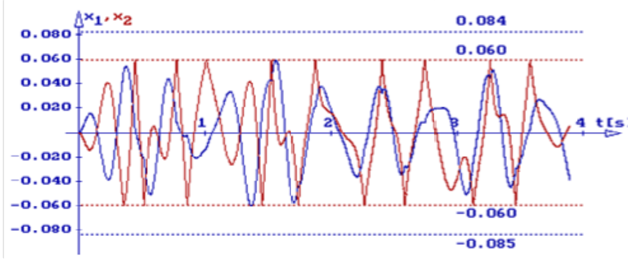

Fig. 10. The distance d = 0.060 [m]

4. CONCLUSION

The vibro-impact mechanical systems have been widely studied as examples of nonlinear systems. Such dynamic systems are of interest because a great number of behaviors mechanical systems are nonlinear or chaotic. With this mechanical systems can revealed apparition of bifurcations, instability and sensitivity to initial conditions.

Using the C program developed by authors [12], it was performed a mechanical vibro-impact displacement simulating for different values of mass m_2.

By consulting the obtained graphs we can notice that the amplitude of movement m_1 is small if the distance d is smaller.

Another important aspect of vibro-impact systems dynamics is the oscillation control. The random and unpredictable chaotic vibrations are generally regarded as being undesirable or even damaging phenomena when they occur. This chaos control in vibro-impact systems has practical applications, such as the impact dampers, the rotary hammers, the pneumatic hammers etc.

5. REFERENCES

[1] Blazejczyk-Okolewska, Barbara, Peterka, F., *An investigation of the dynamic system with impacts*, Chaos, Solitons & Fractals, 1998, Vol. 9, No. 8, pp. 1321-1338

[2] Brândeu, L., *Vibrații şi vibropercuții. Metode şi dezvoltări analitice*, Editura Politehnică, Timişoara, 2005, 161 pp., ISBN 973-625-221-3

[3] Egle, D. M., *An investigation of an impact vibration absorber*, Transactions of the

ASME, Journal of Engineering for Industry, 1967, pp. 653-661.

[4] Farshi, B., Assadi, A., *Development of a chaotic nonlinear tuned mass damper for optimal vibration response*, Commun. Nonlinear Sci Numer Simulat, 2011, pp. 4514 – 4523

[5] Korenev, B. G., Reznikov, L. M., *Dynamic Vibration Absorbers. Theory and Technical Applications*, New York, John Willey, 1993, 312 pp., ISBN 0 471 92850 X

[6] Luo, G.W., Lv, X.H., *Controlling bifurcation and chaos of a plastic impact oscillator*, Nonlinear Analysis: Real World Applications, 2009, pp. 2047 – 2061

[7] Peterka, F., *Dynamics of double impact oscillators*, FACTA UNIVERSITATIS, *Series*: Mechanics, Automatic Control and Robotics, 2000, Vol. 2, No. 10, pp. 1177 – 1190

[8] Peterka, F., *More detail view on the dynamics of the impact damper*, FACTA UNIVERSITATIS, Series: Mechanics, Automatic Control and Robotics, 2003, Vol. 3, No. 14, pp. 907 – 920

[9] Radu, I., *Dinamica*, Editura Mirton, Timişoara, 2001, 210 pp.

[10] Silas, G., Brândeu, L., *The vibro-impact systems (in Romanian), Bucureşti*, Editura Tehnică, 1986, 456 pp.

[11] Ursu-Fischer, N., *Vibrations of Mechanical Systems. Theory and Applications (in Romanian),* Casa Cărţii de Ştiinţă, Cluj-Napoca, 1998, 452 pp., ISBN 973-9404-05-7

[12] Ursu-Fischer, N., Ursu, M., *Programming with C in Engineering (in Romanian)*, Casa Cărţii de Ştiinţă, Cluj-Napoca, 2001, 405 pp., ISBN 973-686-227-5

[13] Ursu-Fischer, N., Ursu, M., *Complements of Mathematics with Engineering Applications (in Romanian)*, Casa Cărţii de Ştiinţă, Cluj-Napoca, 2010, 373 pp., ISBN 978-973-133-728-9

[14] Ursu-Fischer, N., *Numerical Methods in Engineering and Programs in C. Differential Equations and Systems with Initial and Boundary Conditions (in Romanian)*, Casa Cărţii de Ştiinţă, Cluj-Napoca, 2013, 454 pp., ISBN 978-606-17-0306-7

[15] Vinayaravi, R., Kumaresan, D., Jayaraj, K., Asraff, A.K., Muthukumar, R., *Experimental investigation and theoretical modeling of an impact damper,* Journal of Sound and Vibration, 2013, pp.1324–1334

[16] Vâlcovici, V., Bălan, Şt., Voinea, R., *Theoretical Mechanics* (in Romanian), ed. II, Editura Tehnică, Bucureşti, 1963, 1007 pp.

[17] Wiercigroch, M., *Chaotic vibration of a simple model of the machine tool-cutting process system,* Transaction of ASME, Journal of Vibration and Acoustics, 1997, Vol. 119, pp. 468-475

Studiul dinamic şi modelarea sistemelor vibropercutante

Abstract: *Lucrarea prezintă simularea numerică şi analiza grafică a unui sistem mecanic vibropercutant acţionat de o forţă perturbatoare armonică. Studiul analitic prezentat atestă comportamentul haotic al sistemului mecanic cu două grade de libertate, precum şi condiţiile iniţiale pentru care sistemul ar avea un comportament haotic. Rezultatele numerice sunt obţinute cu ajutorul unor programe C, folosindu-se metoda numerică Runge-Kutta de rezolvare a ecuaţiilor diferenţiale, pentru diferite valori ale parametrilor de calcul.*

Iuliana Fabiola MOHOLEA, PhD student, Technical University of Cluj-Napoca, Faculty of Machine Building, Department of Mechanical Systems Engineering, E-mail: iuliana.stef@mep.utcluj.ro, Office Phone 0264.401.781

Ioan RADU, Prof. Dr. Eng., "Aurel Vlaicu" University of Arad, Faculty of Engineering, Department of Automatics, Industrial Engineering, Textiles and Transport, E-mail: raduioanuav@gmail.com, Office Phone 0257.283.010.

Nicolae URSU-FISCHER, Prof. Dr. Eng. Math., Technical University of Cluj-Napoca, Faculty of Machine Building, Department of Mechanical Systems Engineering, Email: nic_ursu@yahoo.com, Office Phone 0264.401.659.