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A COMPUTING METHOD OF THE POSITIONAL ACCURACY FOR THE R-R-R TYPE SERIAL ROBOT

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Abstract: *The accuracy is one of the main characteristics of the robots working in industry and in other domains. The industrial robots resolution, accuracy and repetability are influenced by some factors e. g. the actuators, type of command, sensors, the elements dimensions and masses, speed, the weight of manipulated objects. Using the mathematical procedures for solving nonlinear simultaneous equations and the inverse problem in the errors theory is presented one method that allow to compute the maximal values of the robot generalized coordinates deviation thus the operational coordinates values to be situated between the imposed limits. In the presented numerical example the method is applied for the study of possible attainted accuracy of the R-R-R type serial robot, often used in industrial applications.*

Key words: *the inverse problem in the error theory, simultaneous nonlinear equations, Newton-Raphson method, accuracy of serial robots, C programming*

1. INTRODUCTION

The robots are used in many working places, in manufacturing, for pieces displacements between two positions, in automobile production lines, especially to perform welding, in electronic industry, in medical fields and also at home, for cleaning, cutting grass, a. o.

The accuracy and repetability describe the ability of the robot to follow a imposed trajectory with little or no variance.

In [6] the authors define two kind of accuracy: absolute accuracy (the robot accuracy in returning to previously held position during the repetitive motion) and relative accuracy (robot's positioning accuracy for any point in the work place relative to a known coordinate frame).

Upon the past decades great strides have been made in the study and manufacturing of robots with high accuracy of movements.

The robots characteristics as resolution, accuracy and repetability are subject of many scientific papers [1], [2], [3], [4], [7], [8], [14], [28] solving different problems, regarding the influence of some variation of elements length, relative orientation of joints, friction, temperature, loading, manufacturing tolerances, actuator characteristics, a. o.

In figure 1 is shown the difference between accuracy and repetability.

Very important aspects are linked with the type of robots: serial or parallel, the parallel robots being more accurate than serial robots if are considered only the influence of the initial positions errors. If we take in account all the factors that influence the accuracy the answer is unknown, depending on the specified problem [10], [13].

Studiing robots accuracy it is obvious that the displacement of heavy masses, with different speeds, the elasticity of elements [11], [12], the backslash in joints, the joint clearances, the influence of inertia forces, including Coriolis inertia forces have to be considered [16], [22], [27].

Other aspects that have to be studied are the type of actuators, the sensors used to receive the information and of cause the command of robot [15].

The robot end-point have to cover the imposed trajectory observing the imposed accuracy. To obtain such displacement of end-effector it is necessary that each robot coordinate (determining the relative element positions) with known theoretically computed values has effective values contained in specified limits.

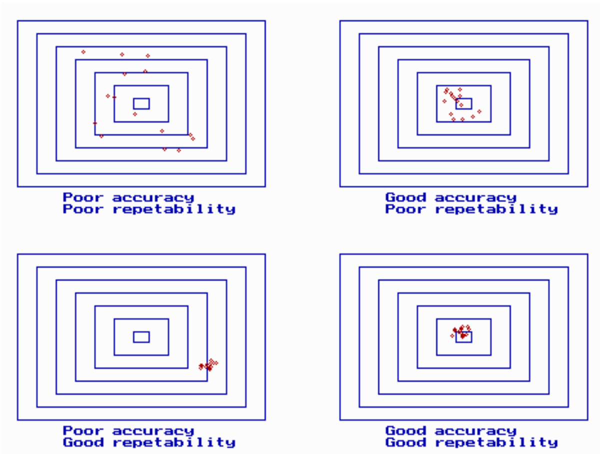

Fig. 1. Accuracy and repetability

This research uses the theory based on the inverse problem of error theory and the Newton-Raphson's method to solve simultaneous algebraic equations.

A serial R-R-R type robot will be considered, the end-effector movements being on parallel directions with the axes of fixed Cartesian frame.

Two problems will be solved. The first consists in computing of the generalized coordinates (the relative elements positions, joint coordinates), corresponding to successive end-effector's positions [9], [17], [18], [20], [25]. The second is the following: for each end-effector position that is affected with known absolute errors has to find the possible maximal errors of generalized coordinates.

2. MATHEMATICAL BACKGROUND

1. The solving of the algebraic nonlinear system of equations $F(X)=0$ with known initial approximate solution $\mathbf{X}^{(0)}$ are achived with the Newton-Raphson numerical method [5], [19], [23]. After each step on obtain a more accurate solution, $X^{(k+1)}=X^{(k)}+\Delta^{(k)}$, the vector $\mathbf{\Delta}^{(k)}$ containing the solution corrections. The new obtained solution $\mathbf{X}^{(k+1)}$ have to satisfy the equations $F\left(X^{(k+1)}\right)=F\left(X^{(k)}+\Delta^{(k)}\right)=0$. After expanding the functions using the Taylor series and considering only first two terms will result a linear system of equations

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}^{(k)} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \cdots \\ \delta_n \end{bmatrix}^{(k)} = - \begin{bmatrix} f_1 \\ f_2 \\ \cdots \\ f_n \end{bmatrix}^{(k)}$$

or $$J\left(X^{(k)}\right)\Delta^{(k)}=-F\left(X^{(k)}\right) \qquad (1)$$

where J ($X^{(k)}$) is the Jacoby's matrix. The values of the partial derivatives contained in this matrix may be computed using a numerical formula for functions derivatives [5], [24],

$$\frac{\partial f(x_1,x_2,..,x_j,..,x_n)}{\partial x_j} \approx$$

$$\approx \frac{1}{12h}[f(x_1,x_2,..,x_j-2h,..,x_n)- \\ -8f(x_1,x_2,..,x_j-h,..,x_n)+ \\ +8f(x_1,x_2,..,x_j+h,..,x_n)- \\ -f(x_1,x_2,..,x_j+2h,..,x_n)] \qquad (2)$$

If the following condition is accomplished $\left|J(X^{(k)})\right| \neq 0$, the column vector $\mathbf{\Delta}^{(k)}$ will result after the simultaneous equations (1) solving, using one of known exact method (the method of partial pivoting), the new solution being:

$$X^{(k+1)}=X^{(k)}+\Delta^{(k)} \qquad (3)$$

Using repeatedly this procedure will result the successive solutions, more and more accurate, $X^{(0)}$, $X^{(1)}$, $X^{(2)}$, ... , $X^{(k)}$, $X^{(k+1)}$, ... , $X^{(number\_of\_iterations)}$, the solutions' succession being convergent if some conditions are accomplished [5].

2. The second mathematical problems that have to be considered is the following: how to approximate compute the variation of some function when its arguments are modified (the direct theory of errors problem) and the inverse problem: which are the allowed modification of arguments values thus the function values have maximal imposed variations (the inverse problem of theory of errors).

A function with **n** variables is considered $v=f(u_1,u_2,\dots,u_n)$, its argument having values affected by errors, the absolute value of errors being known,

$$|\Delta_k|=|U_k-u_k|,\quad k=\overline{1,n} \tag{4}$$

where with $\mathbf{U_i}$ were denoted the exact values and with $\mathbf{u_i}$ the approximate values.

Considering from the Taylor series

$$f(u_1+\Delta_1,u_2+\Delta_2,\dots,u_n+\Delta_n)=f(u_1,u_2,\dots,u_n)+$$
$$+\sum_{i=1}^{n}\frac{\partial f}{\partial u_i}\Delta_i+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 f}{\partial u_i\,\partial u_j}\Delta_i\,\Delta_j+\dots$$

only first two terms and considering the absolute values, on obtains

$$|f(u_1+\Delta_1,u_2+\Delta_2,..,u_n+\Delta_n)-f(u_1,u_2,..,u_n)|=$$
$$=\left|\sum_{i=1}^{n}\frac{\partial f}{\partial u_i}\Delta_i\right|\le\sum_{i=1}^{n}\left|\frac{\partial f}{\partial u_i}\right||\Delta_i|$$

The function absolute error may be written as follows:

$$|\Delta_v|\le\sum_{i=1}^{n}\left|\frac{\partial f}{\partial u_i}\right||\Delta_i| \tag{5}$$

From (5) formula may be established also the relative error formula,

$$\delta_v=\frac{|\Delta_v|}{|v|}\le\frac{1}{|v|}\sum_{i=1}^{n}\left|\frac{\partial f}{\partial u_1}\right||\Delta_i|=\sum_{i=1}^{n}\left|\frac{\frac{\partial f}{\partial u_i}}{v}\right||\Delta_i|\le$$
$$\le\sum_{i=1}^{n}\left|\frac{\partial}{\partial u_i}\ln f(u_1,u_2,\dots,u_n)\right||\Delta_i| \tag{6}$$

Starting again from relation (5) the inverse problem of the error theory may be solved: if the maximal absolute errors of function are known, we have to compute the maximal allowed errors of function arguments.

It exists many possible methods to solve this problem [5], [23]. In our studied case the most adequate method is to consider all the function's absolute errors of arguments being equal (approximate equal) $|\Delta_1|\approx|\Delta_2|\approx\dots\approx|\Delta_n|$, resulting the formula

$$|\Delta_i|\approx\frac{|\Delta_v|}{\sum_{i=1}^{n}\left|\frac{\partial f}{\partial u_i}\right|},\quad i=\overline{1,n} \tag{7}$$

3. DIRECT AND INVERSE GEOMETRIC PROBLEM SOLVING FOR THE R-R-R TYPE SERIAL ROBOT

The study of accuracy was performed for the serial robot of type R-R-R, because such robots are wide spread in industry and other domains, including PUMA robots (figure 2) having first three joint of type R (rotational).

Fig. 2. The PUMA robot (Unimate)

It is obvious that the study about the positional accuracy have to start with the solving of the direct and inverse geometric problem.

The structural scheme of the R-R-R type robot is presented in figure 3.

The position of the manipulated body, linked with the last robot's element, with respect of fixed system of coordinates, is determined with six parameters (in general case), the three coordinates of mass center, x_C, y_C , z_C , and the three Euler's angles ψ,φ,θ. These parameters are the operational coordinates and the parameters that define the relative positions of robot elements are the generalized coordinates.

In our case, the vector of operational coordinates will contain only three elements, the coordinates of point P with respect of the fixed system $O_0x_0y_0z_0$ $[\,x_{P0}\,,\ y_{P0}\,,\ z_{P0}\,]^t$ and the vector of generalized coordinates will contain the three angles of rotation $\varphi_1,\varphi_2,\varphi_3$, that determine the relative positions of the robot elements.

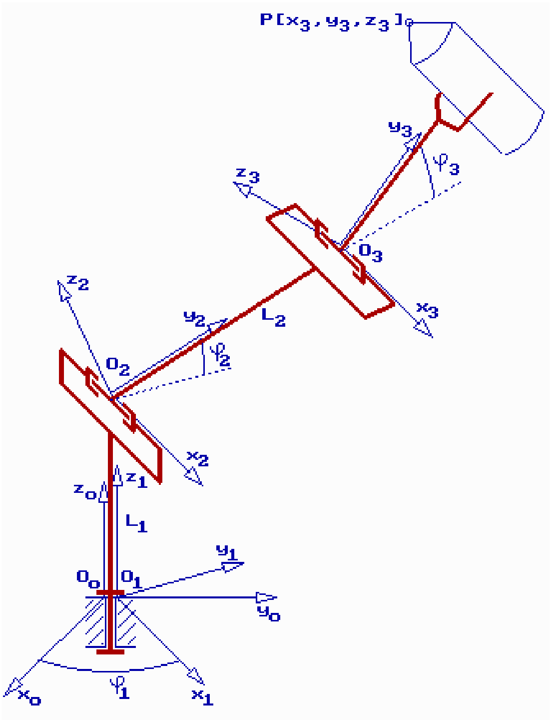

Fig. 2 The R-R-R robot structural scheme

If the generalized coordinates $\varphi_1, \varphi_2, \varphi_3$ have known values (the relative positions of robot elements are known) we can obtain the values of operational vector of coordinates after some matrix multiplications. Performing these operations we can solve the direct geometric problem.

The object of the inverse geometric problem consists in finding the generalized coordinate values thus the manipulated object reach the desired position (the values of the operational coordinates will be equal with the imposed values).

The following homogeneous 4x4 matrices are used to express the relations between coordinates of the same point with respect to two successive system of coordinates:

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

$$^{(0)}V_P = {}^{(0)}_{(1)}T \; ^{(1)}V_P \qquad (10)$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_2 & -\sin\varphi_2 & L_1 \\ 0 & \sin\varphi_2 & \cos\varphi_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}$$

$$^{(1)}V_P = {}^{(2)}_{(1)}T \; ^{(2)}V_P \qquad (11)$$

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_3 & -\sin\varphi_3 & L_2 \\ 0 & \sin\varphi_3 & \cos\varphi_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix}$$

$$^{(2)}V_P = {}^{(2)}_{(3)}T \; ^{(3)}V_P \qquad (12)$$

The matrix relation between vectors $^{(0)}V_P$ and $^{(3)}V_P$ can be write as follows

$$^{(0)}V_P = {}^{(0)}_{(1)}T \; {}^{(1)}_{(2)}T \; {}^{(2)}_{(3)}T \; ^{(3)}V_P \qquad (13)$$

or

$$^{(0)}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_P = \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_2 & -\sin\varphi_2 & 0 \\ 0 & \sin\varphi_2 & \cos\varphi_2 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi_3 & -\sin\varphi_3 & L_2 \\ 0 & \sin\varphi_3 & \cos\varphi_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^{(3)}\begin{bmatrix} x_P \\ y_P \\ z_P \\ 1 \end{bmatrix}$$

As a result of matrix multiplications one can obtain the expressions of coordinates x_{P0}, y_{P0} şi z_{P0}, therefore the relations of direct geometric problem:

$$x_{P0} = x_{P3}\cos\varphi_1 - [(y_{P3}\cos\varphi_3 - z_{P3}\sin\varphi_3 + L_2)\cos\varphi_2 - \\ -(y_{P3}\sin\varphi_3 + z_{P3}\cos\varphi_3)\sin\varphi_2]\sin\varphi_1$$

$$y_{P0} = x_{P3}\sin\varphi_1 + [(y_{P3}\cos\varphi_3 - z_{P3}\sin\varphi_3 + L_2)\cos\varphi_2 - \\ -(y_{P3}\sin\varphi_3 + z_{P3}\cos\varphi_3)\sin\varphi_2]\cos\varphi_1$$

$$z_{P0} = (y_{P3}\cos\varphi_3 - z_{P3}\sin\varphi_3 + L_2)\sin\varphi_2 + \\ +(y_{P3}\sin\varphi_3 + z_{P3}\cos\varphi_3)\cos\varphi_2 + L_1 \qquad (14)$$

Considering different values for the robot coordinates $\varphi_1, \varphi_2, \varphi_3$, using the relations (14), will result the values of operational coordinates x_{P0}, y_{P0} and z_{P0} .

The same relations (14), considered as a system of nonlinear equations with unknowns $\varphi_1, \varphi_2, \varphi_3$

$$\begin{cases} f_1(\varphi_1, \varphi_2, \varphi_3) = 0 \\ f_2(\varphi_1, \varphi_2, \varphi_3) = 0 \\ f_3(\varphi_1, \varphi_2, \varphi_3) = 0 \end{cases} \qquad (15)$$

may be considered as the mathematic model of the inverse geometric problem. The approximate numerical solving of the system (15) will be performed using the Newton-Raphson method.

As part of the inverse geometric problem we consider that the point P, belonging to the manipulated object, covers some trajectories parallel with the system of coordinate frames (figure 4), along the segments AB (parallel to Ox), CD (parallel to Oy) and DE (parallel to Oz)

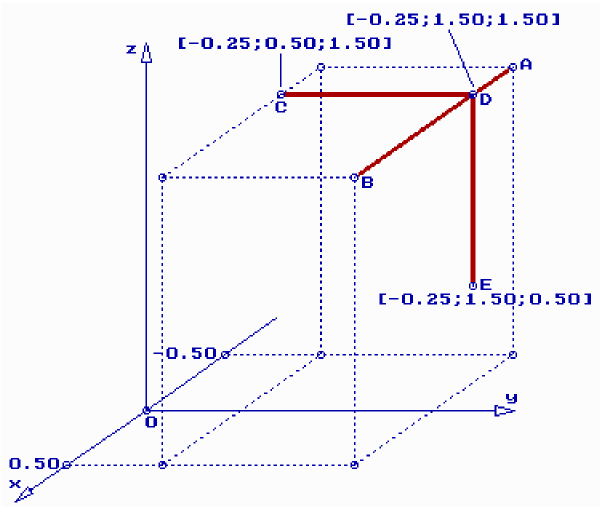

Fig. 4. The considered trajectories of end-effector, AB, CD and DE

The obtained numerical results (the variation of the angles φ_1, φ_2, φ_3) are presented in figures 5, 6 and 7, each of them corresponding to the displacements lengthways of the specified segments.

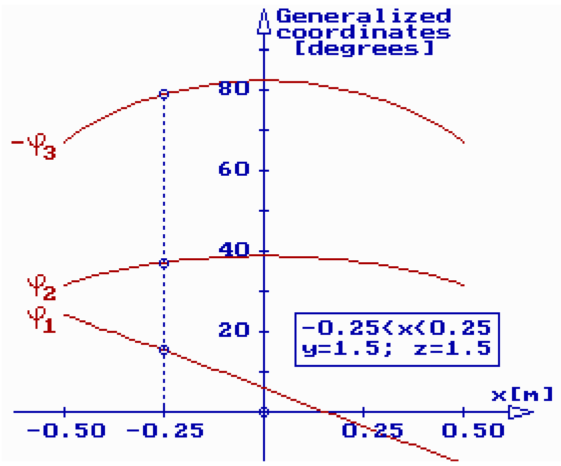

Fig. 5 The variation of generalized robot coordinates, the displacement of end-effector is along of AB direction

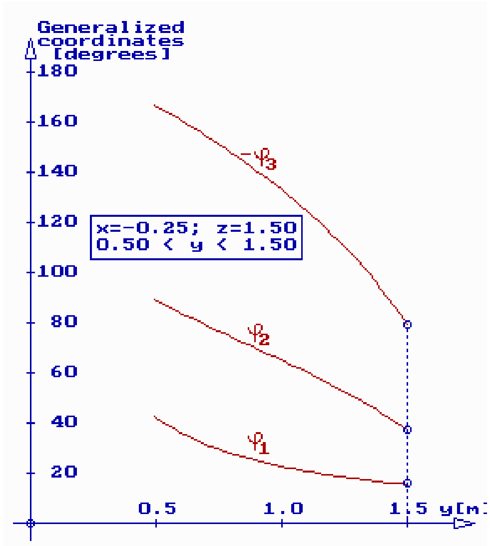

Fig. 6 The variation of generalized robot coordinates, the displacement of end-effector is along of CD direction

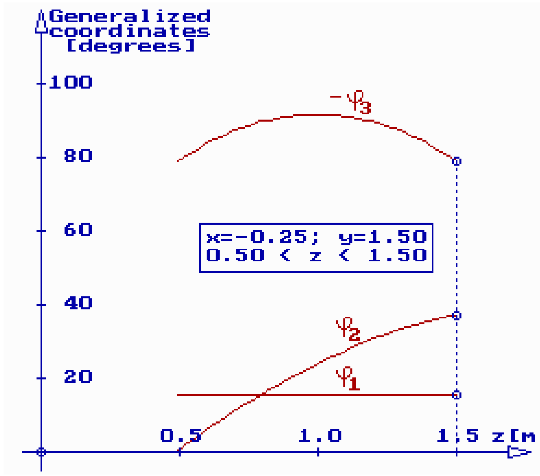

Fig. 7 The variation of generalized robot coordinates, the displacement of end-effector is along of DE direction

4. THE DETERMINATION OF THE MAXIMAL ABSOLUTE ERRORS OF THE GENERALIZED COORDINATES φ_1 , φ_2 AND φ_3 THUS THE OBTAINED ABSOLUTE ERRORS OF THE OPERATIONAL COORDINATES x_{P0} , y_{P0} AND z_{P0} WILL BE IN IMPOSED LIMITS

The possible maximal deviations of the three operational coordinates x_{P0} , y_{P0} and z_{P0} with respect of the exact values are noted with $|\Delta_{xP0}|, |\Delta_{yP0}|, |\Delta_{zP0}|$. The maximal absolute allowed errors of the generalized coordinates $\varphi_1, \varphi_2, \varphi_3$ have to be computed thus the operational coordinate values to be situated between the desired limits.

There has been established the following: considering a function $y=f(x_1, x_2, \ldots, x_n)$ that depend on **n** variables, if the absolute error of function is $|\Delta f|$ results that the maximal permissible errors of variables are computed with formula

$$|\Delta x_i| \approx \frac{|\Delta f|}{\sum_{i=1}^{n}\left|\frac{\partial f}{\partial x_i}\right|} \quad . \quad i=\overline{1,n} \qquad (16)$$

supposing that all errors of variables are equal.

In the frame of direct geometric problem the relations (14) were determined, establishing links between operational coordinates $\mathbf{x_{P0}}$, $\mathbf{y_{P0}}$ and $\mathbf{z_{P0}}$ and generalized coordinates $\varphi_1, \varphi_2, \varphi_3$.

Considering the following values of point P coordinates related to mobile system $\mathbf{O_3\ x_3\ y_3\ z_3}$ and the system $\mathbf{O\ x_0\ y_0\ z_0}$, linked to the basement,

$$\begin{bmatrix} x_{P0} \\ y_{P0} \\ z_{p0} \end{bmatrix} = \begin{bmatrix} -0.25 \\ 1.50 \\ 1.50 \end{bmatrix}, \quad \begin{bmatrix} x_{P3} \\ y_{P3} \\ z_{p3} \end{bmatrix} = \begin{bmatrix} 0.15 \\ 0.60 \\ 0.40 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix},$$

$$L_1=1.0; \; L_2=1.0$$

after solving the inverse geometric problem we will obtain two results, one of them being the following :

$$\begin{bmatrix} 15.123 \\ 36.940 \\ -78.679 \end{bmatrix}^{(degrees)} = \begin{bmatrix} 0.02674333 \\ 0.64472462 \\ -1.37320760 \end{bmatrix}^{(radians)}$$

The following values of operational coordinates maximal errors are considered:

$$|\Delta_{xP0}|=0.0075\ [m], \quad |\Delta_{yP0}|=0.0075\ [m],$$
$$|\Delta_{zP0}|=0.0125\ [m],$$

and the problem consist in determination of maximal errors of generalized coordinates $\boldsymbol{\varphi_1}$, $\boldsymbol{\varphi_2}$ and $\boldsymbol{\varphi_3}$ (all equals, $|\Delta_{\varphi 1}|=|\Delta_{\varphi 2}|=|\Delta_{\varphi 3}|$), assuming that the point **P** displacement follow some certain direction in space.

The equations (14) may be written in the form:

$$x_{P0} = f_1(\varphi_1, \varphi_2, \varphi_3) = p_1 \cos\varphi_1 - $$
$$-[(p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\cos\varphi_2 - $$
$$-(p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\sin\varphi_2]\sin\varphi_1$$

$$y_{P0} = f_2(\varphi_1, \varphi_2, \varphi_3) = p_1 \sin\varphi_1 + $$
$$+[(p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\cos\varphi_2 - $$
$$-(p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\sin\varphi_2]\cos\varphi_1$$

$$z_{P0} = f_3(\varphi_1, \varphi_2, \varphi_3) = $$
$$=(p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\sin\varphi_2 + $$
$$+(p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\cos\varphi_2 + L_1$$

First, we consider that the point P displacement is take place on direction, parallel with $\mathbf{Ox_0}$ axe. The allowed maximal deviations are noted with $|\boldsymbol{\Delta_{xP0}}|$. According to formula (16) one can compute the equal possible errors of generalized coordinates thus the obtained error of the operational coordinate $\mathbf{x_{P0}}$ be smaller than the imposed value $|\boldsymbol{\Delta_{xP0}}|$.

The partial derivatives are computed,

$$\frac{\partial x_{P0}}{\partial \varphi_1} = -p_1 \sin\varphi_1 - $$
$$-[(p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\cos\varphi_2 - $$
$$-(p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\sin\varphi_2]\cos\varphi_1$$

$$\frac{\partial x_{P0}}{\partial \varphi_2} = [(p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\sin\varphi_2 + $$
$$+(p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\cos\varphi_2]\sin\varphi_1$$

$$\frac{\partial x_{P0}}{\partial \varphi_3} = [(p_2 \sin\varphi_3 + p_3 \cos\varphi_3 - L_2)\cos\varphi_2 + $$
$$+(p_2 \cos\varphi_3 - p_3 \sin\varphi_3)\sin\varphi_2]\sin\varphi_1$$

and the maximal error values of generalized coordinates result

$$|\Delta_{\varphi 1}|_{(x-\mathrm{var}iabl\,e)} \approx |\Delta_{\varphi 2}|_{(x-\mathrm{var}iable)}$$
$$\approx |\Delta_{\varphi 3}|_{(x-\mathrm{var}iable)} \approx$$
$$\approx \frac{|\Delta_{xP0}|}{\sum_{j=1}^{3}\left|\frac{\partial x_{P0}}{\partial \varphi_j}\right|}$$

In a similar way, we can use this procedure considering that the P point executes displacements on the directions of $\mathbf{Oy_0}$ and $\mathbf{Oz_0}$ axes, the maximal allowed errors being $|\boldsymbol{\Delta_{yP0}}|$ and $|\boldsymbol{\Delta_{zP0}}|$. According to the same formula (16) the equal error of generalized coordinates are computed, that yield errors of operational coordinates $\mathbf{y_{P0}}$ and $\mathbf{z_{P0}}$ in the desired limits.

Again, the partial derivatives are computed,

$$\frac{\partial y_{P0}}{\partial \varphi_1} = p_1 \cos\varphi_1 - [(p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\cos\varphi_2 - (p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\sin\varphi_2]\sin\varphi_1$$

$$\frac{\partial y_{P0}}{\partial \varphi_2} = [-(p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\sin\varphi_2 - (p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\cos\varphi_2]\cos\varphi_1$$

$$\frac{\partial y_{P0}}{\partial \varphi_3} = [(-p_2 \sin\varphi_3 - p_3 \cos\varphi_3)\cos\varphi_2 - (p_2 \cos\varphi_3 - p_3 \sin\varphi_3)\sin\varphi_2]\cos\varphi_1$$

$$\frac{\partial z_{P0}}{\partial \varphi_1} = 0$$

$$\frac{\partial z_{P0}}{\partial \varphi_2} = (p_2 \cos\varphi_3 - p_3 \sin\varphi_3 + L_2)\cos\varphi_2 - (p_2 \sin\varphi_3 + p_3 \cos\varphi_3)\sin\varphi_2$$

$$\frac{\partial z_{P0}}{\partial \varphi_3} = (-p_2 \sin\varphi_3 - p_3 \cos\varphi_3)\sin\varphi_2 + (p_2 \cos\varphi_3 - p_3 \sin\varphi_3)\cos\varphi_2$$

then the maximal allowed errors of generalized coordinates,

$$|\Delta_{\varphi 1}|_{(y-\text{variabe})} \approx |\Delta_{\varphi 2}|_{(y-\text{variable})} \approx |\Delta_{\varphi 3}|_{(y-\text{variable})} \approx \frac{|\Delta_{yP0}|}{\sum_{j=1}^{3}\left|\frac{\partial y_{P0}}{\partial \varphi_j}\right|}$$

$$|\Delta_{\varphi 1}|_{(z-\text{variable})} \approx |\Delta_{\varphi 2}|_{(z-\text{variable})} \approx |\Delta_{\varphi 3}|_{(z-\text{variable})} \approx \frac{|\Delta_{zP0}|}{\sum_{j=1}^{3}\left|\frac{\partial z_{P0}}{\partial \varphi_j}\right|}$$

The following numerical results were obtained:

$$|\Delta_{\varphi 1}|_{(x-\text{variable})} \approx |\Delta_{\varphi 2}|_{(x-\text{variable})} \approx |\Delta_{\varphi 3}|_{(x-\text{variable})} \approx \approx 0.259 \text{ [degrees]}$$

$$|\Delta_{\varphi 1}|_{(y-\text{variable})} \approx |\Delta_{\varphi 2}|_{(y-\text{variable})} \approx |\Delta_{\varphi 3}|_{(y-\text{variable})} \approx \approx 0.518 \text{ [degrees]}$$

$$|\Delta_{\varphi 1}|_{(z-\text{variable})} \approx |\Delta_{\varphi 2}|_{(z-\text{variable})} \approx |\Delta_{\varphi 3}|_{(z-\text{variable})} \approx \approx 0.322 \text{ [degrees]}$$

The correctness of these results may be verified after computing the operational coordinate values considering different values of generalized coordinates, to exact values adding or subtracting the error values.

In Table no. 1 are presented the eight possible values of generalized coordinates values and the obtained values of the operational coordinate $\mathbf{x_{P0}}$.

As we notice the equal value of absolute errors of the three angles is 0.259 [degrees] , added or subtracted to exact values of generalized coordinates, and the obtained operational coordinate $\mathbf{x_{P0}}$ has values that satisfy the imposed condition.

In a similar way are obtained the numerical values presented in the Tables no. 2 and 3.

The considered operational coordinates are $\mathbf{y_{P0}}$ and $\mathbf{z_{P0}}$ (the maximal absolute errors are $|\Delta_{yP0}|$=0.0075 and $|\Delta_{zP0}|$=0.0125) and the maximal absolute errors of the two angles are 0.518 [degrees] and 0.322 [degrees].

Table no. 1

φ1	**φ2**	**φ3**	Min. $\mathbf{x_{P0}}$	$\mathbf{x_{P0}}$ (obtained)	Max. $\mathbf{x_{P0}}$
15.123-0.259	36.940-0.259	-78.679-0.259	**-0.2575**	**-0.243665**	**-0.2425**
15.123-0.259	36.940-0.259	-78.679+0.259		**-0.243907**	
15.123-0.259	36.940+0.259	-78.679-0.259		**-0.242513**	
15.123-0.259	36.940+0.259	-78.679+0.259		**-0.242740**	
15.123+0.259	36.940-0.259	-78.679-0.259		**-0.257242**	
15.123+0.259	36.940-0.259	-78.679+0.259		**-0.257492**	
15.123+0.259	36.940+0.259	-78.679-0.259		**-0.256051**	
15.123+0.259	36.940+0.259	-78.679+0.259		**-0.256285**	

Table 2

φ1	**φ2**	**φ3**	Min. $\mathbf{y_{P0}}$	$\mathbf{y_{P0}}$ (obtained)	Max. $\mathbf{y_{P0}}$
15.123-0.518	36.940-0. 518	-78.679-0. 518	**1.4925**	**1.5055**	**1.5075**
15.123-0. 518	36.940-0. 518	-78.679+0. 518		**1.5074**	
15.123-0. 518	36.940+0. 518	-78.679-0. 518		**1.4969**	

15.123-0. 518	36.940+0. 518	-78.679+0. 518		**1.4985**	
15.123+0. 518	36.940-0. 518	-78.679-0. 518		**1.5010**	
15.123+0. 518	36.940-0. 518	-78.679+0. 518		**1.5028**	
15.123+0. 518	36.940+0. 518	-78.679-0. 518		**1.4925**	
15.123+0. 518	36.940+0. 518	-78.679+0. 518		**1.4940**	

Table 3

φ1	**φ2**	**φ3**	Min. **ZP0**	**ZP0** (obtained)	Max. **ZP0**
15.123-0.322	36.940-0.322	-78.679-0.322	**1.4925**	*1.4874*	**1.5125**
15.123-0.322	36.940-0.322	-78.679+0.322		**1.4955**	
15.123-0.322	36.940+0.322	-78.679-0.322		**1.5044**	
15.123-0.322	36.940+0.322	-78.679+0.322		**1.5125**	
15.123+0.322	36.940-0.322	-78.679-0.322		*1.4874*	
15.123+0.322	36.940-0.322	-78.679+0.322		**1.4955**	
15.123+0.322	36.940+0.322	-78.679-0.322		**1.5044**	
15.123+0.322	36.940+0.322	-78.679+0.322		**1.5125**	

In Table no. 3 we can notice that exists two situations (noted with italic) when the method fails. The explanations are the following: only two terms were considered from the Taylor series and all the errors of generalized coordinates were considered equals.

The numerical results contained in Tables 1, 2 and 3 show the maximal possible values of robot coordinates that allow us to obtain a point P accurate position [-0.25, 1.5, 1.5], the maximal deviations from these coordinates being as follows,

$|\Delta_{xP0}| =$ 0.0075[m], $|\Delta_{yP0}| =$ 0.0075[m],

$|\Delta_{zP0}| = 0.0125$ [m].

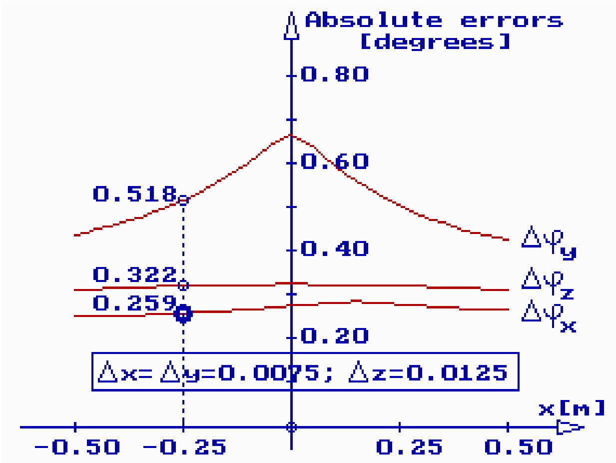

Fig. 8 The maximal allowed absolute errors of robot coordinates during the displacement along the AB direction

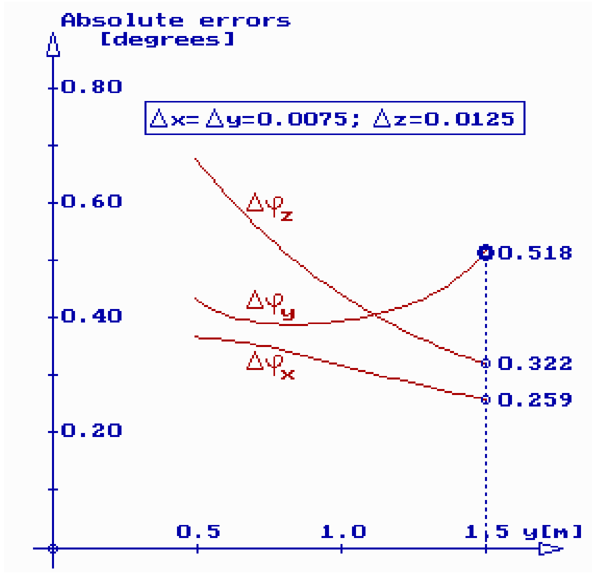

Fig. 9 The maximal allowed absolute errors of robot coordinates during the displacement along the CD direction

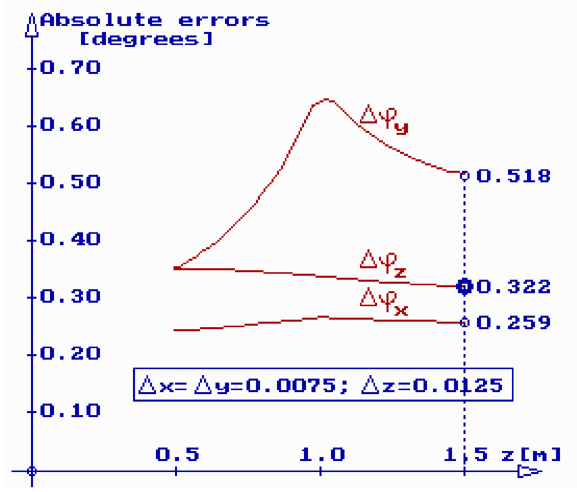

Fig. 10 The maximal allowed absolute errors of robot coordinates during the displacement along the DE direction

Using the same method one can solve this problem not only for a fixed established point but for all points belonging to some covered trajectories. For each point of the three selected trajectories having known positional deviations were determined the allowed absolute errors of generalized coordinates.

The numerical results are presented in figures 8, 9 and 10.

5. CONCLUSIONS

The proposed method successfully solves the imposed problem and was transposed in a C program.

It is obvious that the method works also in the case of other considered trajectories, other absolute errors values and we can obtain similar diagrams as in figures 5 ~ 10.

Based on the results contained in figures 8, 9 and 10 we can notice if the imposed problem is really possible – can the robot command guarantee the obtaining of the necessary accurate positions of the robot elements or such precision can't be attained.

Also, the proposed method is usefull for the study of kinematic accuracy, solving the inverse problem of error.

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O metodă de calcul a preciziei poziţionale a roboţilor seriali de tipul R-R-R

Abstract: *O caracteristică importantă a roboţilor utilizaţi în industrie şi în alte domenii este cea privind precizia. Rezoluţia, precizia, repetabilitatea sunt definitorii pentru roboţii industriali, fiind influenţaţi de o serie de factori cum ar fi actuatoarele utilizate, tipul comenzii şi al senzorilor, dimensiunea şi masa proprie a elementelor, viteza de lucru, greutatea sarcinilor manipulate. Se prezintă o metodă de determinare a posibilelor variaţii ale coordonatelor robot astfel încât să se obţină valori ale coordonatelor operaţionale între anumite limite impuse. Exemplul numeric analizat se referă la un robot de tip R-R-R, des întâlnit în aplicaţii industriale.*

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