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CONTRIBUTIONS TO THE STUDY OF THE PLASTIC COLLISIONS BETWEEN A BODY AND A VIBRATING PLATE

Nicolae URSU-FISCHER, Iuliana MOHOLEA, Ioan RADU

Abstract: *The vertical movement of a body which is thrown repeatedly from a vibrating plate that has sinusoidal movements is studied. Between the vibrating plate and the body a plastic collision occurs. The theoretical background is presented and also some numerical results, calculated with C software, for different values of mechanical system parameters and initial conditions.*

Key words: *plastic impact, dynamics of material point, vibrating plate, programming in C*

1. INTRODUCTION

The vertical movement for a small body thrown from a horizontal plate, movement that follows a harmonic law, is described by the bouncing ball phenomenon in [2], [3], [4]. In this paper, we consider the mechanical system formed by a vibrating plate and a small body.

The body is thrown upwards, after a plastic impact, by a horizontal plate which has vertical movements that follow a harmonic law. When the body falls back on the plate so that it might remain on it or be thrown up again [7].

Periodic movements were revealed in this study, the movement laws depending not only on the kinematic and dynamic parameters, such as the amplitude and frequency of the plate's movement, but also on the initial conditions like the body's position.

Based on the mathematical model was developed a C program, which simulates the mechanical system movements and plots the diagrams of the vibrating plate and body displacements.

2. THEORETICAL BACKGROUND

The studied mechanical system is composed of a vibrating plate, which moves according to a sinusoidal law and a body which is free to execute a plastic collision with the plate.

A horizontal plate with vertical harmonic movements, is considered

$$s_{table}(t) = -a\cos\omega t \qquad (1)$$

at the moment t = 0 the plate being on its lowest point, in this case the movement will be $s_{plate\ min} = -a$.

After two derivatives with respect to the time its speed and acceleration result,

$$\begin{aligned} v_{table} &= a\,\omega \sin\omega t \\ a_{table} &= a\,\omega^2 \cos\omega t \end{aligned} \qquad (2)$$

To obtain the separation of the body from the vibrating plate, the transport inertial force have to be greater than the body weight, thus

$$-m\,a\,\omega^2 \cos\omega t \geq m\,g \qquad (3)$$

After solving the equation

$$\cos(\omega t) = -\frac{g}{a\,\omega^2} \qquad (4)$$

the solutions result

$$t_1 = \frac{1}{\omega} \arccos\left(-\frac{g}{a\,\omega^2}\right),$$
$$t_2 = \frac{1}{\omega}\left[2\pi - \arccos\left(-\frac{g}{a\,\omega^2}\right)\right] \quad (5)$$

If the body stands initially on the vibrating plate its separation will occur at the moment t_1. If the body is falling back in the time interval $[t_1; t_2]$, then it will be thrown upwards back again.

In the time interval $[0; t_1]$ the body is on the vibrating plate and from the time t_1 will perform a vertical motion with an initial velocity and acceleration equal to –g, according to relations [1], [6], [8]:

$$s_{body} = s_0 + v_0(t - t_1) - \frac{g(t - t_1)^2}{2}, \quad t \geq t_1 \quad (6)$$

where the displacement and the initial speed of body are: $s_0 = -a\cos\omega t_1$ and $v_0 = a\,\omega\sin\omega t_1$.

If the body falls back on the vibrating plate at the moment $t = t_0$ and $t_1 \leq t_0 \leq t_2$, considering a plastic impact it will be thrown back upwards after the law

$$s_{body} = s_0 + v_0(t - t_0) - \frac{g(t - t_0)^2}{2}, \quad t \geq t_0$$
$$s_0 = -a\cos\omega t_0 \quad (7)$$
$$v_0 = a\,\omega\sin\omega t_0$$

The next moment when the body falls on the vibrating plate is determined by solving the equation obtained equating the two expressions:

$$s_{body}(t) = -a\cos\omega t_0 + a\omega\sin\omega t_0(t - t_0) - \frac{g(t - t_0)^2}{2} \quad (8)$$
$$s_{table}(t) = -a\cos\omega t$$

and after determining the closest solution to t_0.

Depending on time, the harmonic curve representing the oscillating plate displacement will intersect the parabolic curve representing the body movement, both with respect of time.

In some cases, which depend on the amplitude and frequency of movements of the vibrating plate, the body will not be thrown up, it remains permanently on plate.

3. NUMERICAL RESULTS

With C program [5] the movements of the two elements, according to established laws of motion, was simulated and the diagrams based on initial conditions have been determined.

Depending on the amplitude and frequency were obtained the diagrams graphs in figures 1 – 6. The frequencies for the harmonic movement law of the considered vibrating plate were 16, 20, 24 and 26 [Hz], two different amplitudes being considered 0.00125 and 0.00175 [m].

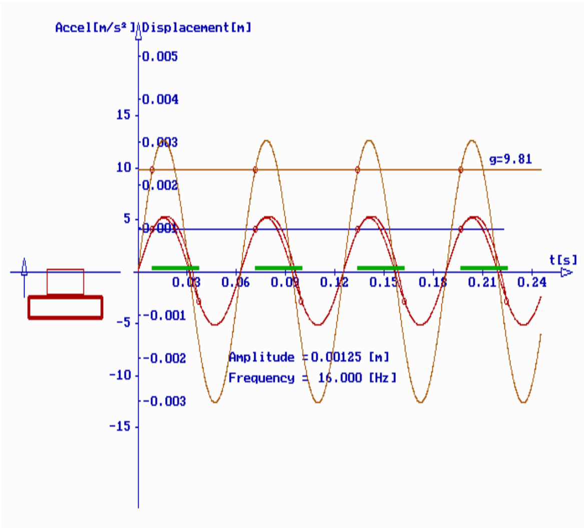

Fig.1 The movement of the mechanical system with amplitude of plate, a = 0.00125 [m], F = 16 [Hz]

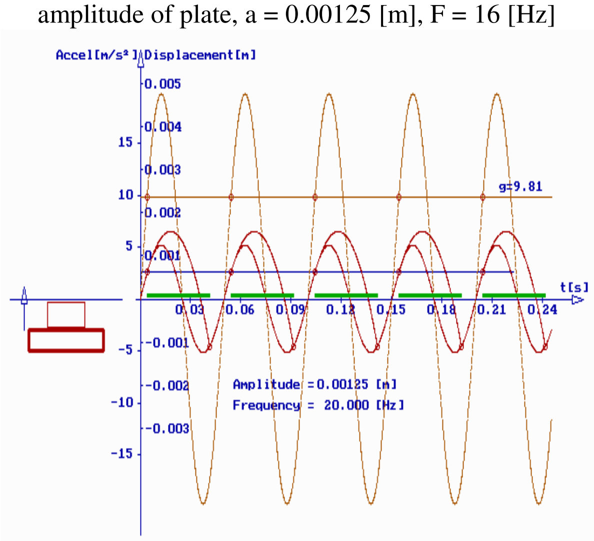

Fig. 2 The movement of the mechanical system with amplitude of plate, a = 0.00125 [m], F = 20 [Hz]

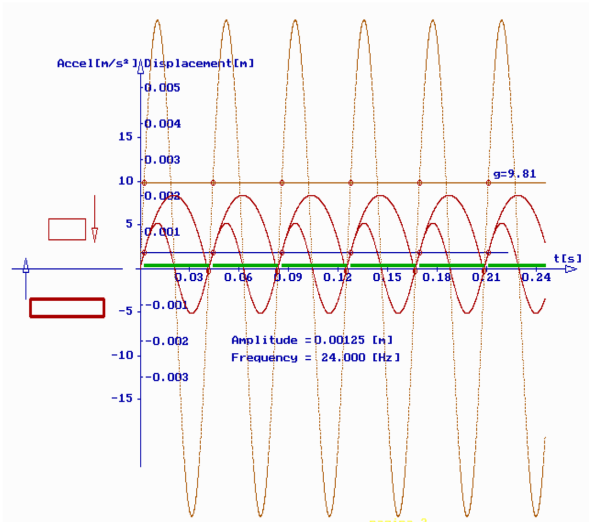

Fig. 3 The movement of the mechanical system with amplitude of plate, a = 0.00125 [m], F = 24 [Hz]

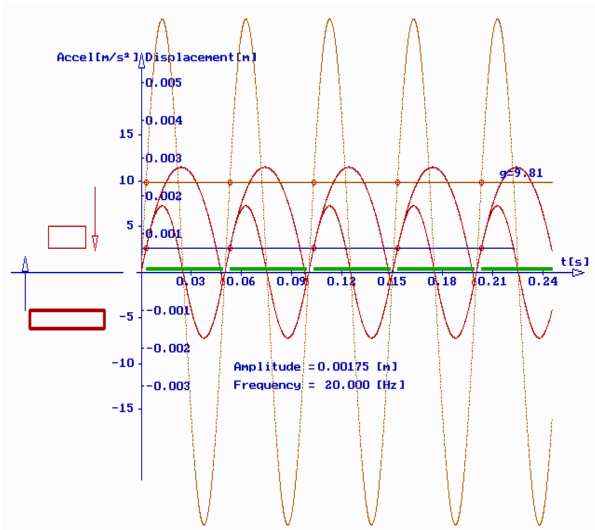

Fig. 4 The movement of the mechanical system with amplitude of plate, a = 0.00175 [m], F = 20 [Hz]

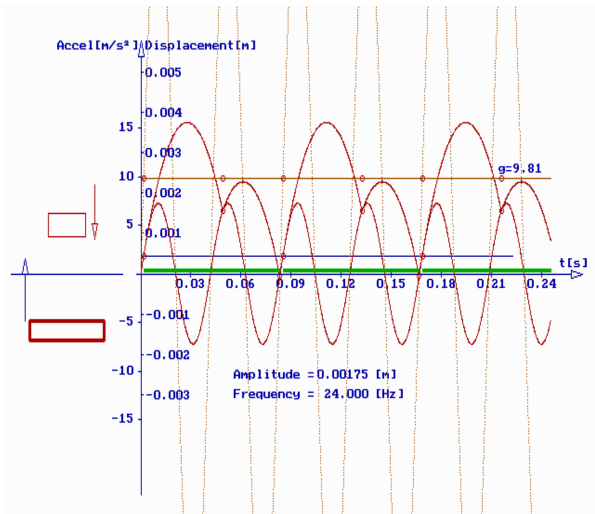

Fig. 5 The movement of the mechanical system with amplitude of plate, a = 0.00175 [m], F = 24 [Hz]

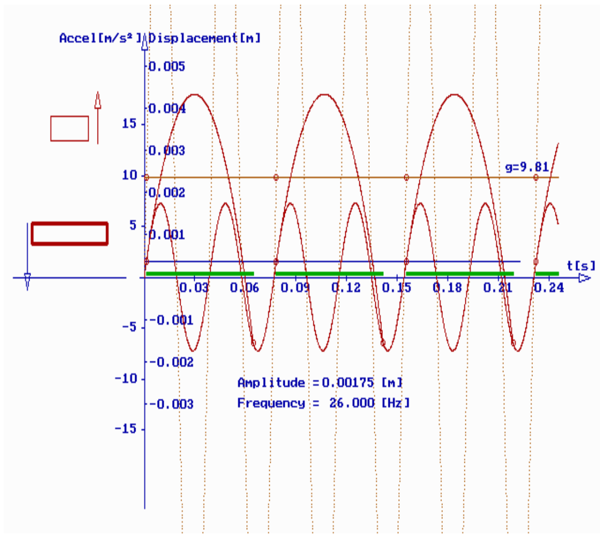

Fig. 6 The movement of the mechanical system with amplitude of plate, a = 0.00175 [m], F = 26 [Hz]

In the left side of the diagrams are represented the movements of the vibrating plate and of the thrown body, the sense and length of the arrows being linked to the movement direction and speed.

In the right side are illustrated the movement laws of the two elements, a sinusoid during several periods for the vibrating plate, and a few parabolas which correspond to the movement laws of the thrown body.

The moments at which the body is thrown from the vibrating plate are marked by circles in all figures, as well as the ones belonging to the intervals where the body is above the vibrating plate. The solid line segments represent the time intervals when the body is above the vibrating plate.

When acceleration diagram intersects the horizontal line (which represents the value of gravitational acceleration) then the body is thrown up.

4. CONCLUSIONS

In contrast to the bouncing ball system, when between the ball and the vibrating plate is elastic or semi elastic collision and will result a chaotic behavior, in this case a plastic collision occurs and the mechanical system behavior is periodic.

The periodic character of the movements of the body depends on parameters such as amplitudes and frequents of the vibrating plate.

We can notice the different situations when the body is thrown up and how long the body is in the air.

We can notice the periodicity of the movements of the body at one, two or more vibrating plate period of movement.

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Contributii la studiul ciocnirilor plastice dintre un corp şi o placă vibrantă

Abstract: *Lucrarea prezintă un studiu al mişcărilor verticale ale unui corp aruncat repetat de pe o placă orizontală care efectuează mişcări armonice. Impactul dintre corp şi suprafaţa vibrantă este de tip plastic. În lucrare este prezentat modelul matematic al acestor mişcări precum şi o serie de rezultate numerice obţinute cu ajutorul unui program C, considerând diferite valori ale amplitudinii şi frecvenţei oscilaţiilor plăcii.*

Nicolae URSU-FISCHER, Prof. dr. eng. math., Technical University of Cluj-Napoca, Faculty of Machine Building, Department of Mechanical Systems Engineering, e-mail: nic_ursu@yahoo.com, Office phone 0264-401656

Iuliana MOHOLEA, PhD student, Technical University of Cluj-Napoca, Faculty of Machine Building, Department of Mechanical Systems Engineering, e-mail: iuliana.stef@mep.utcluj.ro, Office phone 0264-401781

Ioan RADU, Prof. dr. eng., "Aurel Vlaicu" University of Arad, Faculty of Engineering, Department of Automatics, Industrial Engineering, Textiles and Transport, e-mail: raduioan@gmail.com, Office phone: 0257-283010