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THE PATH PLANNING OF TRR SMALL-SIZED ROBOT

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Abstract: *After a brief definition of the robot path, the paper establishes the interpolation polynomials that characterize the displacement of the TRR small-sized robot on a path defined by five successive configurations. The second part of the papers deals with the numerical and graphical simulation of the TRR robot using a script written in MATLAB, TRR_Simulation. The paper ends by representing the time variation of the generalized coordinates and the position of the gripper's characteristic point and its path, corresponding to the given task.*
Key words: *TRR robot, path planning, polynomial interpolation functions, MATLAB.*

1. INTRODUCTION

The path planning of a serial robot is an essential step both in the design stage and in the robot implementation into a technological process.

Unless the path or trajectory in the classical mechanics, the path of a robot is regarded as the union of the interpolation functions that describe the imposed displacement of the robot, on each of the robot joints [1]. These functions can be expressed either in the configuration space or in the Cartesian space [2].

The mechanical structure of TRR small-sized robot, having three degrees of freedom, was analyzed from the point of view of geometric, kinematic and dynamic modeling in [3], [4] and [5] and the numerical simulation [6] and the graphical simulation [7] were also performed, yielding the geometric, kinematic and dynamic parameters for a given task, as well as the dynamic functions, the graphical representations of the given configurations and the time variation of the generalized kinematic and dynamic parameters, as provided by *SimMecRob* [8].

The path representation of the TRR gripper's characteristic point for the given task, using a program written in MathCAD was also achieved in [7].

2. INTERPOLATION POLYNOMIALS

Based on coefficients determined in [6] with the help of *SimMecRob* program, the interpolation polynomials characterizing the displacement of the TRR small-sized robot on the path defined by five successive configurations are established.

The generalized coordinates of the robot in the five successive positions are known [6], denoted by q_{ij}, where $i = \overline{1,3}$ is the joint index and $j = \overline{0,4}$ is the current configuration index.

The numbers of conditions imposed on the path is 18, as follows:

- five position conditions (initial, final position and three intermediary positions);
- nine continuity conditions in position, velocity and acceleration in the intermediary points;
- two conditions of zero velocity in the initial and final points;
- two conditions of zero acceleration in the initial and final points.

The four path segments are described by interpolation polynomials of 4-3-4 kind:

- the first segment is modeled by a 4^{th} degree polynomial;
- the two intermediary segments are modeled by 3^{rd} degree polynomials;

Table 1

Interpolation polynomial functions corresponding to the path of TRR robot

Joint	Path Segment	Polynomial functions
1	0-1	$q_{11}(t) = 129.832 \cdot t^4 - 199.832 \cdot t^3$ $\dot{q}_{11}(t) = 259.664 \cdot t^3 - 299.748 \cdot t^2$ $\ddot{q}_{11}(t) = 389.496 \cdot t^2 - 299.748 \cdot t$
	1-2	$q_{12}(t) = -168.616 \cdot t^3 + 403{,}867 \cdot t^2 - 120.251 \cdot t - 70$ $\dot{q}_{12}(t) = -168.616 \cdot t^2 + 269.244 \cdot t - 40.083$ $\ddot{q}_{12}(t) = -112.410 \cdot t + 89.748$
	2-3	$q_{13}(t) = 81.645 \cdot t^3 - 157.735 \cdot t^2 + 121.090 \cdot t + 45$ $\dot{q}_{13}(t) = 122.467 \cdot t^2 - 157.735 \cdot t + 60.545$ $\ddot{q}_{13}(t) = 122.467 \cdot t - 78.867$
	3-4	$q_{14}(t) = 44.166 \cdot t^4 + 84.166 \cdot t^3 - 517.494 \cdot t^2 + 605.826 \cdot t + 90$ $\dot{q}_{14}(t) = 58.888 \cdot t^3 + 84.166 \cdot t^2 - 344.996 \cdot t + 201.942$ $\ddot{q}_{14}(t) = 58.888 \cdot t^2 + 56.110 \cdot t - 114.998$
2	0-1	$q_{21}(t) = 4.814 \cdot t^4 - 7.956 \cdot t^3$ $\dot{q}_{21}(t) = 9.628 \cdot t^3 - 11.934 \cdot t^2$ $\ddot{q}_{21}(t) = 14.442 \cdot t^2 - 11.934 \cdot t$
	1-2	$q_{22}(t) = -2.805 \cdot t^3 + 11.291 \cdot t^2 - 6.915 \cdot t - 180$ $\dot{q}_{22}(t) = -2.805 \cdot t^2 + 7.527 \cdot t - 2.305$ $\ddot{q}_{22}(t) = -1.870 \cdot t + 2.509$
	2-3	$q_{23}(t) = -1.099 \cdot t^3 - 0.592 \cdot t^2 + 4.834 \cdot t - 90$ $\dot{q}_{23}(t) = -1.648 \cdot t^2 - 0.592 \cdot t + 2.417$ $\ddot{q}_{23}(t) = -1.648 \cdot t - 0.296$
	3-4	$q_{24}(t) = -2.879 \cdot t^4 - 3.665 \cdot t^3 + 28.269 \cdot t^2 - 34.027 \cdot t + 90$ $\dot{q}_{24}(t) = -3.838 \cdot t^3 - 3.665 \cdot t^2 + 18.846 \cdot t - 11.342$ $\ddot{q}_{24}(t) = 3.838 \cdot t^2 - 2.443 \cdot t + 6.282$
3	0-1	$q_{31}(t) = 5.051 \cdot t^4 - 8.193 \cdot t^3$ $\dot{q}_{31}(t) = 10.102 \cdot t^3 - 12.289 \cdot t^2$ $\ddot{q}_{31}(t) = 15.153 \cdot t^2 - 12.289 \cdot t$
	1-2	$q_{32}(t) = -4.761 \cdot t^3 + 12.892 \cdot t^2 - 6.559 \cdot t - 180$ $\dot{q}_{32}(t) = -4.761 \cdot t^2 + 8.594 \cdot t - 2.186$ $\ddot{q}_{32}(t) = -3.174 \cdot t + 2.864$
	2-3	$q_{33}(t) = 2.071 \cdot t^3 - 3.793 \cdot t^2 + 3.293 \cdot t - 90$ $\dot{q}_{33}(t) = 3.106 \cdot t^2 - 3.793 \cdot t + 1.646$ $\ddot{q}_{33}(t) = 3.106 \cdot t - 1.896$

	3-4	$q_{34}(t) = 1.832 \cdot t^4 + 3.403 \cdot t^3 - 21.201 \cdot t^2 + 24.865 \cdot t$ $\dot{q}_{34}(t) = 2.442 \cdot t^3 + 3.403 \cdot t^2 - 14.134 \cdot t + 8.288$ $\ddot{q}_{34}(t) = 2.442 \cdot t^2 + 2.286 \cdot t - 4.711$

- the last segment is modeled by a 4th degree polynomial.

In order to determine the path, the number of conditions imposed on the path must be equal to the number of the coefficients of the polynomial functions characterizing the path. In this case, the condition is fulfilled:

- the number of conditions imposed on path: $5 + 9 + 2 + 2 = 18$;
- the number of coefficients of the polynomial functions: $5 + 4 + 4 + 5 = 18$.

The time variation of positions, velocities and accelerations for each joint is presented in Table 1.

3. THE NUMERICAL AND GRAPHICAL SIMULATION USING *TRR_Simulation*

The application for simulating the parameters variation was developed in MATLAB and it begins with establishing of time variation laws of the generalized variables.

In order to simulate the variation of the geometrical parameters of TRR robot, the symbolic data of the geometric and kinematic model [4] has to be loaded from the data file *TRR_kin.mat*, into the script *TRR_geom_ graphs.m*.

The geometric elements of the robot are defined by numeric values. In order to establish a real robot task, a number of *ncfg* = 5 robot distinct configurations was taken, defined by joint displacements. A time moment was assigned to each configuration.

The time interval of the robot task was then divided, taking the time increment *inc_timp*, resulting the time vector *tt*. Based on this vector, the generalized variables were interpolated using the cubic spline interpolation function.

The symbolic data taken from the modeling process was processed by numeric substitutions, resulting the vectors representing the position of the characteristic point of the gripper on the three Cartesian axes.

The script used for studying and plotting the geometric parameters of the path is presented in Program 1.

Program 1. *TRR_geom_ graphs.m* - MATLAB script used for studying the robot path for the given task

```
%% plots generalized coordinates,
%% position, path for TRR robot
clear all
clc
% symbolic data loading
load TRR_kin.mat
% degree definition
deg=pi/180;
% geometric elements
l0n=100;
l1n=200;
l2n=130;
l3n=130;
l4n=150;
gn=9.81;
% configurations definition
%(relative displacements)
ncfg=5;
q(:,1)=[0;0;0];
q(:,2)=[-70;-180*deg;-180*deg];
q(:,3)=[45;-90*deg;-90*deg];
q(:,4)=[90;90*deg;0*deg];
q(:,5)=[130;45*deg;90*deg];
% absolute positions
qa(:,1)=q(:,1);
qa(:,2)=q(:,1)+q(:,2);
qa(:,3)=qa(:,2)+q(:,3);
qa(:,4)=qa(:,3)+q(:,4);
qa(:,5)=qa(:,4)+q(:,5);
% absolute time vector
t(1)=0;
t(2)=2;
t(3)=5;
t(4)=7;
t(5)=10;
% computations for graphs
i=1;
inc_timp=0.25;
m_pi=pi;
tt=0:inc_timp:t(5);
ltt=length(tt);
time=tt;
```

```
% generalized coordinates
q1=interp1(t,qa(1,:),tt,'spline');
q2=interp1(t,qa(2,:),tt,'spline');
q3=interp1(t,qa(3,:),tt,'spline');
for i=1:length(tt)
    l0(i)=l0n;
    l1(i)=l1n;
    l2(i)=l2n;
    l3(i)=l3n;
    l4(i)=l4n;
    pi(i)=m_pi;
end
% end-efector position
p4=subs(p4);
p4x=p4(1,:);
p4y=p4(2,:);
p4z=p4(3,:);
% graphs generalized coordinates
figure('Name','Generalized coord');
subplot(3,1,1);
plot(time,q1,'LineWidth',2);
ylabel('q1 [mm]')
title('Position in joint 1')
grid on
subplot(3,1,2);
plot(time,q2/deg,'r','LineWidth',2);
ylabel('q2 [deg]')
title('Position in joint 2')
grid on
subplot(3,1,3);
plot(time,q3/deg,'g','LineWidth',2);
xlabel('Time [s]')
ylabel('q3 [deg]')
title('Position in joint 3')
grid on
% graphs position end-efector
figure('Name','End-efector position');
subplot(2,2,1);
plot(time,p4x,'LineWidth',2);
xlabel('Time [s]')
ylabel('p4x [mm]')
title('x-component')
grid on
subplot(2,2,2);
plot(time,p4y,'LineWidth',2);
xlabel('Time [s]')
ylabel('p4y [mm]')
title('y-component')
grid on
subplot(2,2,3);
plot(time,p4z,'LineWidth',2);
xlabel('Time [s]')
ylabel('p4z [mm]')
title('z-component')
grid on
subplot(2,2,4);
plot3(p4x,p4y,p4z,'r','LineWidth',2);
xlabel('x [mm]')
ylabel('y [mm]')
zlabel('z [mm]')
title('Robot path')
grid on
axis tight
save TRR_geom_graphs
```

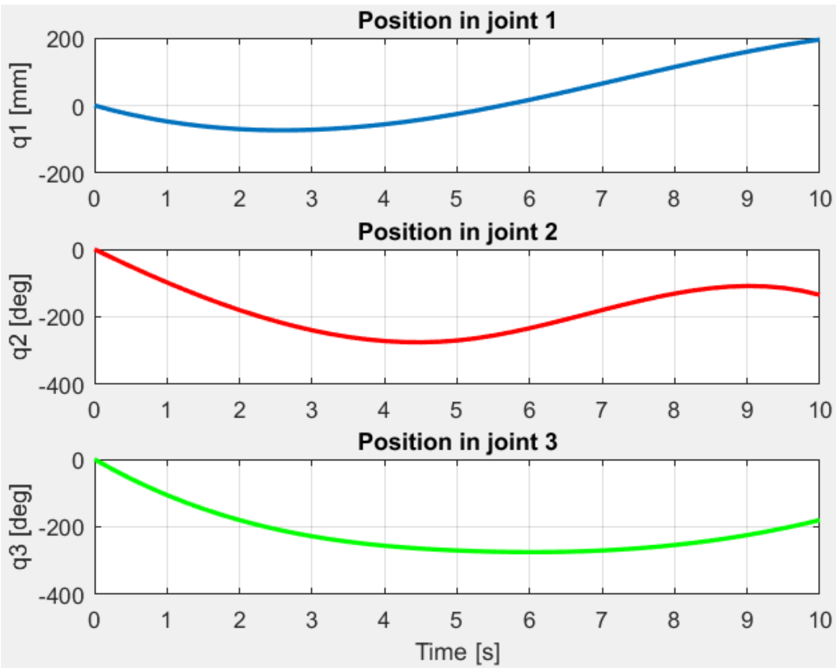

Fig. 1 Time variation of the generalized coordinates of TRR robot

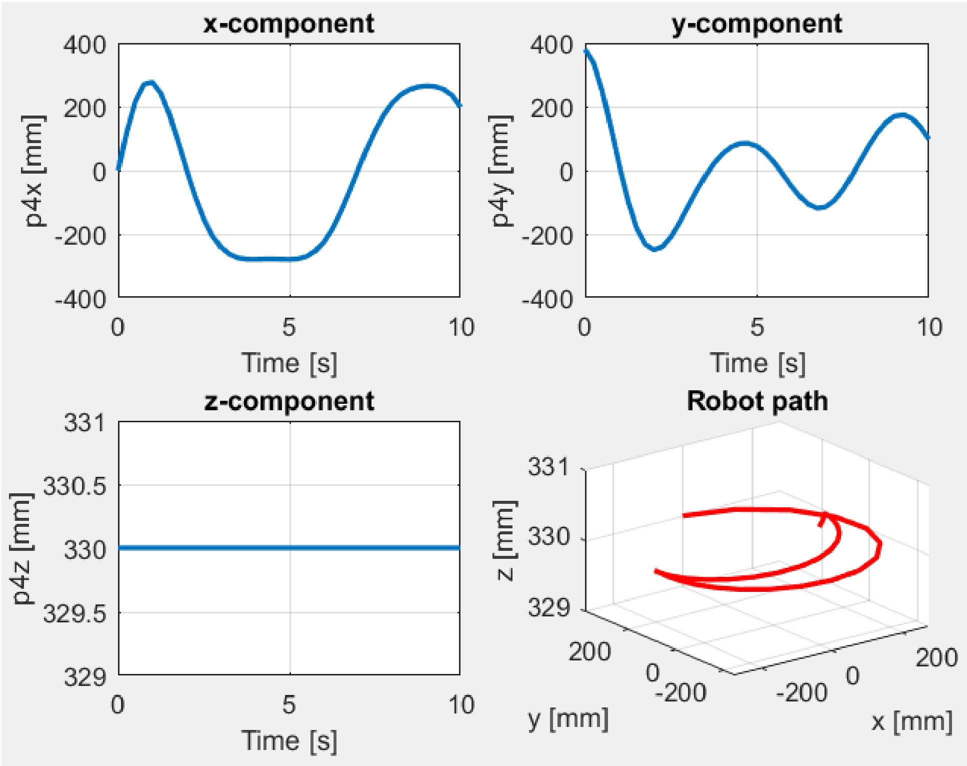

Fig. 2 The position of the gripper's characteristic point and its trajectory, corresponding to the given task

The following parameters were represented with respect to time:

- the generalized coordinates (fig.1);
- the position of the characteristic point of the gripper on the three Cartesian axes and its space trajectory (fig. 2).

4. CONCLUSIONS

The constant value (330 mm) of the z-component in figure 2 is explained by the fact that the characteristic point of TRR robot gripper moves in a plane parallel to *xOy*, situated at the elevation $z = 330$ mm.

The obtained graphs, along with the numerical data obtained and presented in [6] are useful in the study of the behavior of the TRR small-sized robot, ensuring the parameters of the technological process the robot is implemented in.

5. REFERENCES

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Planificarea triectoriei de mișcare a minirobotului TRR

Rezumat: După o scurtă definire a traiectoriei roboților, lucrarea stabilește funcțiile polinomiale de interpolare care caracterizează deplasarea minirobotului TRR pe o traiectorie definită prin cinci configurații successive. Partea a doua a lucrării se ocupă de simularea numerică și grafică a robotului TRR, utilizând un script scris în MATLAB, având numele de *TRR_Simulation*. Lucrarea se încheie prin reprezentarea funcțiilor de variație în timp a coordonatelor generalizate și a poziției punctului caracteristic al efectorului final și reprezentarea traiectoriei acestuia, corespunzătoare sarcinii impuse.

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