TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue I, 2012

THE DYNAMIC STUDY OF THE SUSPENDED TTRT SERIAL ROBOT, GATEWAY TYPE

Nicusor Iosif URSA

Abstract:
From the dynamic studies conducted on a family of robots proposed to serve the radiators fluxing process, made at SC RAAL SA Bistrița, the author presents in this paper the dynamic study of the serial modular suspended robot TTRT. Using Lagrange's 2nd type equations, the robot's dynamic equations were derived.

The algorithm which leads to the dynamic equations includes: the kinetic energy of the robot, after, in advance, the kinetic energies of the component modules were determined and the generalized driving forces were established.

***The dynamic study of the robot allows the calculation of the actuators** and the determination of the optimal variant of arrangement of the modules in the robot structure, so that the energy consumption will be minimal.*

***Key words**:*
Lagrange equations, radiator, kinetic energy, forces, generalized motors, actuators, dynamic equations.

1. INTRODUCTION

To find the optimal variant of robotics in terms of construction and functioning, I conducted a dynamic study of robots proposed as an automation solution and upgrading brazing process for manufacturing aluminum radiators at SC RAAL S.A. Bistrita.
The study involves the following steps:

- The development of kinematical structural schemes for the proposed robots;
- Establishing the system of differential equations of motion for each robot;
- Establishing the optimal constructive variant of arranging the modules within the structure of the robot;
- Specific dynamic calculations on the choice of actuators.

Specialized industrial robots raise for manufacturer the problem of achieving a wide variety of types, from the simplest with reduced mobility and flexibility, to the intelligent robots.

An approach to solving this problem is modular building of robots. Modular construction involves making a number of standard modules common to the same family of robots, which judiciously combined, leads to a variety of constructions which are different in operation and complexity.

For a modular system to solve the problems of design, implementation and operation, it must be composed of a small number of modules, which have a unitary conception and multiple possibilities of functional interconnection.

Paper [1], presents a set of optimization criteria on the choice of kinematical structures of modular robots. By analysing the above mentioned optimization criteria there can be developed and designed several modules of robots proposed for study as outlined in Chapter 2, paragraph 2.3.3.2, paper [7]. In order to optimize the design of serial modular robots we use the energy optimization method, as proposed in [2].

Dynamic equations of the robots proposed for study can be obtained using Lagrange's equations in present case II, written according to [3] as:

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_k}\right)-\frac{\partial Ec}{\partial qk}=Q_k, \qquad k=1\div 4. \qquad (1)$$

In equations (1) the following should be clarrified:
q_k - represent the generalized coordinates;
k - represent the number of degrees of freedom;
$\dot{q}_k$ - represent the generalized velocities;
Ec - represents the kinetic energy of the robot;
Q_k - represents the generalized forces.

The kinetic energy of the system is equal to the amount of kinetic energy of all components. According to [8], the kinetic energy of an item "i", considered as rigid body can be determined as following:

$$Ec_i=\frac{1}{2}\left[\omega_x,\omega_y,\omega_z,v_x,v_y,v_z\right]_i\cdot$$
$$\cdot\begin{bmatrix} J_x & -J_{xy} & -J_{xz} & 0 & Mz_c & My_c \\ -J_{yx} & J_y & -J_{yz} & Mz_c & 0 & -Mx_c \\ -J_{zx} & -J_{zy} & J_z & -My_c & Mx_c & 0 \\ 0 & Mz_c & -My_c & M & 0 & 0 \\ -Mz_c & 0 & Mx_c & 0 & M & 0 \\ My_c & -Mx_c & 0 & 0 & 0 & M \end{bmatrix}_i \begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z\end{bmatrix}_i . \qquad (2)$$

The Mechanical quantities involved in equation (2) are
M - is the mass element (rigid);
x_c, y_c, z_c - are coordinates of center of gravity;
$J_x, J_y J_z$ - moments of mechanical axial inertia;
$J_{xy}, J_{yz} J_{zx}$ - are moments of mechanical centrifugal inertia;
$\omega_x, \omega_y \omega_z$ - represents the components of instantaneous angular velocity $\overline{\omega}$ on axis of a cartesian system $Oxyz$, jointly with rigid
v_x, v_y, v_z - are the Cartesian components of origin O of reference system $Oxyz$.

On the whole, it is considered that the elements of the system are the modules specific to the robot and for each module the reference system is chosen with originating from the center of gravity. Thus the coordinates are zero for the center of gravity:

$$x_c=y_c=z_c=0.$$

Also the supportive referential systems for the robot modules are chosen so that the coordinate axes coincide with the main directions of inertia corresponding to the origin of these systems, so that:

$$J_{xy}=J_{yz}=J_{zx}=0.$$

Thus, expression (2) for the kinetic energy becomes:

$$Ec_i=\frac{1}{2}M_i\left(v_x^2+v_y^2+v_z^2\right)+\frac{1}{2}\left(J_x\omega_x^2+J_y\omega_y^2+J_z\omega_z^2\right)_i \qquad (3)$$

2. THE DYNAMIC EQUATIONS FOR THE TTRT ROBOT

If the translation and rotation modules on the vertical axis of the robot TTTR are interchangeable, then the robot can be reconfigured, having as a result the suspended TTRT robot. The structural kinematical scheme of the robot is shown in Figure 1.

The TTRT robot consists in the following modules: two identical modules for horizontal translation on the direction of Δ_1 axis, their assembly with center of gravity in O_1, one translation module 2 in horizontal plane, in direction of axis $\Delta_2 \perp \Delta_1$, one rotation module 4 around the vertical axis Δ_4 and a translation module 3 along the vertical axis Δ_3 of the entire assembly radiator R – sustaining support RSS to be introduced in fluxing bath. Also for the TTRT robot, each module has a single degree of freedom, the movement being accomplished through a drive, control and independent positioning.

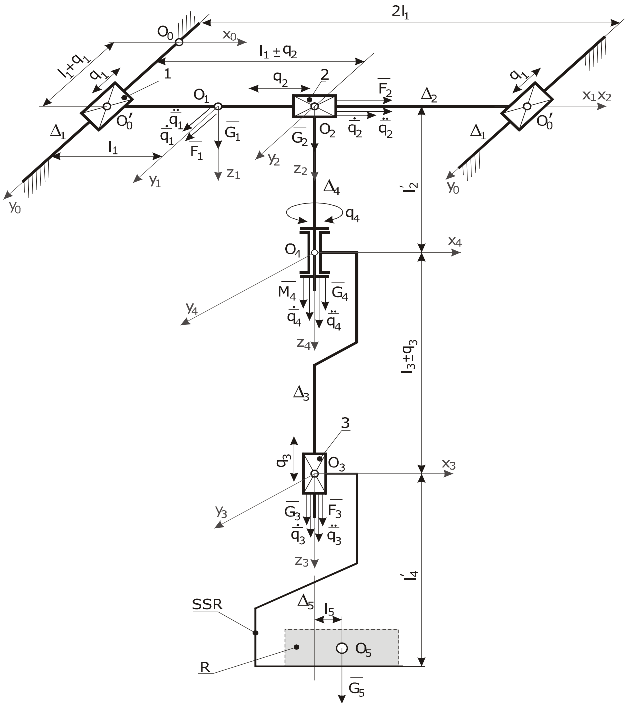

Fig. 1 Kinematic scheme of the TTRT suspended robot

The notations introduced in Figure 1 are:

l_0, l_1, l_2', l_3, l_4', l_5 - are constructive parameters of the robot;

$k = 1 \div 4$ - the number of degrees of freedom;

$\overline{G}_i$, $i = 1 \div 5$ - represents the forces of gravity of the modules, respectively of the mobile assembly radiator R- sustaining support of RSS radiator;

m_i, $i = 1 \div 5$ - masses of the constructive elements of the robot;

O_0 - is marking the zero point;

O_i, $i = 1 \div 4$ - are the origins of mobile cartesian reference systems $O_i x_i y_i z_i$ in solidarity with the mobile part of robot modules, which coincide with the centers of gravity of these modules;

Δ_i, $i = 1 \div 4$ - the axis of motion;

Δ_5 - the axis passing through the center of gravity O_5 of the assembly radiator R - radiator sustaining support RSS and is parallel to the axis Δ_3;

$\overline{F}_i$, $i = 1 \div 4$,- are the driving forces;

$\overline{M}_3$ - is the torque;

$J_{\Delta_4}^{(4)}$ - is the mechanical moment of inertia of the mobile equipment of rotation module 4, determined in relation to the axis Δ_4;

$J_{\Delta_4}^{(3)}$ - is the mechanical moment of inertia of the translation module 3 in relation to the axis Δ_4;

$J_{\Delta_4}^{(5)}$ - is the mechanical moment of inertia in relation to axis Δ_4 of assembly R-RSS.

The dynamic equations of the industrial suspended TTRT robot are determined by using Lagrange's equations in present case II, written as:

$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_k}\right) - \frac{\partial Ec}{\partial qk} = Q_k, \quad k = 1 \div 4 .$$

Following Figure 1, we can make the following comments:

- Origins O_i of mobile reference systems were chosen in the centers of gravity of the modules, so that $x_c = y_c = z_c = 0$ in expression (2) of kinetic energy;
- Mobile reference systems are considered the main systems of inertia, so that mechanical centrifugal moments of inertia are zero, $J_{xy} = J_{yz} = J_{zx} = 0$.

The kinetic energies of the components of the robot (modules and radiator support supporting) are determined by applying successively the relation (3) and following Figure 1. Thus:

- for module 1 of translation along the axis Δ_1:

$$Ec_1 = \frac{1}{2} m_1 \dot{q}_1^2 \tag{4}$$

because the kinematical parameters that characterize this movement are:

$$\omega_{x_1} = \omega_{y_1} = \omega_{z_1} = 0;$$

$$v_{x_1} = v_{z_1} = 0; \; v_{y_1} = \dot{q}_1; \tag{5}$$

- for module 2 of translation along the axis Δ_2:

$$Ec_2 = \frac{1}{2} m_2 \left(\dot{q}_1^2 + \dot{q}_2^2\right) \tag{6}$$

for characterizing the movement kinematical parameters are:

$$\omega_{x_2} = \omega_{y_2} = \omega_{z_2} = 0 ;\ v_{x_2} = \dot{q}_2 ;$$
$$v_{y_2} = \dot{q}_1 ;\ v_{z_2} = 0 ; \tag{7}$$

- for module 4 of rotation around the vertical axis Δ_4 can specify the kinematical parameters of movement, as:

$$\omega_{x_4} = \omega_{y_4} = 0 ;\ \omega_{z_4} = \dot{q}_4 ;\ \bar{v}_{0_4} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 ;$$
$$v_{x_4} = \dot{q}_2 ;\ v_{y_4} = \dot{q}_1 ;\ v_{z_4} = 0 . \tag{8}$$

According to (3) the kinetic energy of this module is:

$$Ec_4 = \frac{1}{2} m_4 \left(\dot{q}_1^2 + \dot{q}_2^2\right) + \frac{1}{2} J_{\Delta_4}^{(4)} \dot{q}_4^2 \tag{9}$$

- for translation module 3 along the vertical axis, kinematical parameters of movement are:

$$\omega_{x_3} = \omega_{y_3} = 0 ;\ \omega_{z_3} = \dot{q}_4 ;\ \bar{v}_{0_3} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 + \dot{\bar{q}}_3 ;$$
$$v_{x_3} = \dot{q}_2 ;\ v_{y_3} = \dot{q}_1 ;\ v_{z_3} = \dot{q}_3 . \tag{10}$$

so that the kinetic energy corresponding to this module is, according to (3):

$$Ec_3 = \frac{1}{2} m_3 \left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2\right) + \frac{1}{2} J_{\Delta_4}^{(3)} \dot{q}_4^2 , \tag{11}$$

- for assembly radiator R - radiator sustaining support RSS that can be specified in kinematical parameters of movement, namely:

$$\bar{v}_{0_5} = \dot{\bar{q}}_1 + \dot{\bar{q}}_2 + \dot{\bar{q}}_3 + \dot{\bar{q}}_4 \times \bar{r}_5 ;$$
$$\bar{r}_5 = \overline{O_4 O_5} = l_5 \bar{i}_4 + \left(l_3 + l_4^{'} \pm q_3\right) \bar{k}_4 ;$$
$$v_{0_5}^2 = \dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2 + \dot{q}_4^2 l_5^2 ;\ \omega_{x_4} = \omega_{y_4} = 0 ;$$
$$\omega_{z_4} = \dot{q}_4 . \tag{12}$$

Introducing (12) in (3), we obtain the kinetic energy of the assembly R-RSS, whose expression is:

$$Ec_5 = \frac{1}{2} m_5 \left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2 + \dot{q}_4^2 l_5^2\right) + \frac{1}{2} J_{\Delta_4}^{(5)} \dot{q}_4^2 . \tag{13}$$

By adding the kinetic energy expressed by equations (4), (6), (9), (11) and (13), we obtain the expression of kinetic energy of the robot TTRT, written as:

$$Ec = \frac{1}{2}\left(\sum_{i=1}^{5} m_i\right)\dot{q}_1^2 + \frac{1}{2}\left(\sum_{i=2}^{5} m_i\right)\dot{q}_2^2 + \frac{1}{2}\left(m_3 + m_5\right)\dot{q}_3^2 +$$
$$+ \frac{1}{2}\left[m_5 l_5^2 + J_{\Delta_4}^{(3)} + J_{\Delta_4}^{(4)} + J_{\Delta_4}^{(5)}\right]\dot{q}_4^2 . \tag{14}$$

As a result of relations (1) and (14) we can write the following expressions:

$$\frac{\partial Ec_1}{\partial q_1} = 0 ;\ \frac{\partial Ec_2}{\partial q_2} = 0 ;\ \frac{\partial Ec_3}{\partial q_3} = 0 ;\ \frac{\partial Ec_4}{\partial q_4} = 0 ;$$
$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_1}\right) = \left(\sum_{i=}^{5} m_i\right)\ddot{q}_1 ;\quad \frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_2}\right) = \left(\sum_{i=2}^{5} m_i\right)\ddot{q}_2 ;$$
$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_3}\right) = \left(m_3 + m_5\right)\ddot{q}_3 ;$$
$$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{q}_4}\right) = \left[m_5 l_5^2 + J_{\Delta_4}^{(3)} + J_{\Delta_4}^{(4)} + J_{\Delta_4}^{(5)}\right]\ddot{q}_4 . \tag{15}$$

Generalized forces Q_k are derived under the assumption that the robot links are olonome, which means that these links do not depend explicitly on generalized speeds and therefore do not depend on displacements. So virtual elementary displacements are independent, so that one sequence can be considered different of zero, the others being zero.

According to [1], generalized forces can be determined by the following:

$$Q_k = \frac{\partial L}{\partial q_k} ;\ k = 1 \div 4 . \tag{16}$$

The elementary virtual mechanical work of the external forces (driving forces $\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$, and forces of weight $\overline{G}_i$, $i = 1 \div 5$) and torque M_4, corresponding to elementary virtual displacements compatible with the robot links, is determined following Figure 1. The expression of this mechanical work is:

$$\delta L = F_1 \delta q_1 + F_2 \delta q_2 + \left(F_3 + G_3 + G_5\right)\delta q_3 + M_4 \delta q_4 \quad (17)$$

considering that
$\overline{G}_1 \perp \delta\overline{q}_1$, $\overline{G}_2 \perp \delta\overline{q}_1$, $\overline{G}_2 \perp \delta\overline{q}_2$, $\overline{G}_3 \perp \delta\overline{q}_1$, $\overline{G}_3 \perp \delta\overline{q}_2$, $\overline{G}_3 \perp \lambda\delta\overline{q}_3$, $\overline{G}_4 \perp \delta\overline{q}_1$, $\overline{G}_4 \perp \delta\overline{q}_2$, $\overline{G}_5 \perp \delta\overline{q}_1$, $\overline{G}_5 \perp \delta\overline{q}_2$, $\overline{G}_5 \perp \lambda_1\delta\overline{q}_3$, $\overline{F}_1 \perp \lambda_2\delta\overline{q}_1$, $\overline{F}_2 \perp \lambda_3\delta\overline{q}_2$, $\overline{F}_3 \perp \lambda_4\delta\overline{q}_3$, $\overline{M}_4 \perp \lambda_5\delta\overline{q}_4$.

According to (16) and (17), the generalized forces expressions are:

$$Q_1 = F_1,\ Q_2 = F_2,$$
$$Q_3 = F_3 + G_3 + G_5,\ Q_4 = M_4. \quad (18)$$

The differential equations of motion of the robot are obtained from (1) by introducing (15) and (18). These equations are:

$$\left(\sum_{i=1}^{5} m_i\right)\ddot{q}_1 = F_1,\ \left(\sum_{i=2}^{5} m_i\right)\ddot{q}_2 = F_2,$$
$$\left(m_3 + m_5\right)\ddot{q}_3 = F_3 + G_3 + G_5$$
$$\left[m_5 l_5^2 + J_{\Delta_4}^{(3)} + J_{\Delta_4}^{(4)} + J_{\Delta_4}^{(5)}\right]\ddot{q}_4 = M_4. \quad (19)$$

The differential equations (19) of the robot have been established on the assumption that all movements take place simultaneously.

3. CONCLUSIONS

* With the obtained system of differential equations, we may solve the two fundamental and reciprocal aspects of the dynamic of the robot: the problems of direct and inverse dynamics. In the case of the direct problem, the robot's movement is determined if the forces and moments acting upon it are known.

* In the case of the inverse problem (which is preferred), the robot's movement is supposed to be known and it is required that we find out the variation laws of forces and torques.

* Knowing the variation laws of forces and moments, actuators can be chosen, taking into consideration each module's organology and the robot's dynamics.

* For future consideration, chosing the laws of motion on each axis and finding a proper variant of module arrangement within the mechanical structure of the robot may represent two staple elements in energy consumption analysis so that this one becomes minimal. [4], [5], [2], [3].

REFERENCES

[1] Ispas, V., *Manipulatoare şi roboţi industriali.* Editura didactică şi pedagogică, Bucureşti, 2004, 600 p.

[2] Ispas, V., *Elemente de calcul* şi *construcţie a manipulatoarelor şi roboţilor,* Editura UT PRES, Cluj-Napoca, 2003, 350 p.

[3] Ispas, V., Ursa, N.I., *Modelarea dinamică a roboţilor propuşi spre implementare la S.C. RAAL S.A. Bistriţa*, În: Ştiinţă şi Inginerie, vol.16, pag.157-164, Ed. AGIR, Bucureşti 2009, ISBN 9738130-82-4.

[4] Ispas, Vrg., Petrişor, S.M., Arghir, M., Ispas, V., *Aspects regarding the conception, modeling and implementation of a articulated robot in spaces with noises and vibrations*, 79th Annual Meeting of International Association of Applied Mathematics and Mechanics – March 31st – April 4th – University Bremen, GAMM Mitteilungen, Pamm Gesellschaft fur Angewandte-Mathematik und Mechanik, ISSN 1617-7061, 30, No. 8, 2008.

[5] Petrişor, S.M., Simion, M., Ispas, V., *Contributions to calculus, designing, modeling and constructive optimization of a rotation module from the base of an articulated robot,* Sesiunea Anuală de Comunicări Ştiinţifice „IMT Oradea – 2008", Băile Felix, 29-30 Mai, Secţiunea Mecatronică, Annals of the Oradea University, Fascicle of Management and Technological Engineering, CD-ROM EDITION, vol. VII, 2008, pp. 1071-1076, ISSN 1583-0691, CNCSIS, B+.
[6] Ripianu, A., *Mecanică Cinematică şi Dinamică*, Lito IPCN, Cluj – Napoca, 1977.
[7] Ursa, N.I., *Contribuţii la calculul şi construcţia structurii mecanice a roboţilor industriali seriali utilizaţi la fabricarea radiatoarelor,* Teză de doctorat, Cluj-Napoca, 2011.
[8] Voinea, R., Voiculescu, D., Ceauşu, V., *Mecanica*, E.D.P., 1983.

Studiul dinamic al robotului serial suspendat TTRT de tip portal

Rezumat:
Din studiile dinamice realizate pe o familie de roboţi propuşi să servească procesul de fluxare (pastare) a radiatoarelor în S.C. RAAL S.A. Bistriţa, autorul prezintă în această lucrare studiul dinamic al robotului serial suspendat TTRT de tip portal, construit din module interschimbabile. Ecuaţiile dinamice ale robotului au fost determinate utilizând ecuaţiile lui Lagrange de speţa a doua.

Algoritmul care conduce la ecuatiile dinamice include: energia cinetica a robotului, dupa ce in prealabil au fost determinate energiile cinetice ale modulelor componente si dupa ce s-au stabilit fortele conductoare generalizate.

Studiul dinamic al robotului permite calculul motoarelor de acţionare şi determinarea variantei optime de aranjare a modulelor în structura robotului, astfel încât consumurile energetice să fie minime.

Nicusor Josif URSA, Dr. Eng. Department of Engineering-Programming and Robotics, Faculty of Mechanical Engineering, Technical University of Cluj Napoca, e-mail: ursaregina@yahoo.co.uk