## TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

## ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering Vol. 59, Issue III, September, 2016

# CONTRIBUTIONS TO THE STUDY OF LOWER LIMB BIOMECHANICS OF A HUMAN SUBJECT UNDO TO VIBRATION. PART II: SYSTEM OF DIFFERENTIAL EQUATIONS 

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#### Abstract

In the paper makes a study on the behavior of complex biomechanical limb under the action of vibration. The request is applied through the sole of the foot and it propagates along the leg requested through thank and the femur. The second leg is placed on a surface considered fixed, therefore the linkage of given leg and the second leg, it may be considered a cylindrical joint in plane. Mechanical System (part I) is analyzed through integration into Matlab Simulink, and it has the system of differential equations (part II), which characterizes the dynamics of it. The integrated results obtained are comparable to those existing in the literature (part III).


Key words: biomechanics study, lower limb, human subject, vibrations.

## 1. MATHEMATICAL FOUNDATION OF BIOMECHANICAL SYSTEM

It is considered a mechanical model noted 5LBGGF [Arg 16], which is presented in the first part of this paper, and it is the simplified biomechanical model of a foot human operator, who is supported on a vibrating platform, and it is subjected to vertical vibrations of this deep.

The operator has one foot on the platform, and the second is located on the ground, but about it not acts the vibrating forces.

The system is considered to be made up of five distinct elements, as concentrated masses, positioned in the centre of the masses of each constituent parts. Connections between consentrate masses is achieved by elastic or/and damping elements.

The five components are formed from: foot, shank - that is studied as being formed from the bone (tibia and fibula) and from muscle, finally adding knee and femur.

The first four parts are considered to be positioned vertically, but the fifth part - femur are required inclined on the vertical direction, and it does the linkage between the requested leg and the cilindrical henge in plane, given by
second leg of the body. Due to this position the movement was considered to be propagated along it.

The symbol adopted for this mechanical system is 5LBGGF, that has 5 degrees of freedom situated along the specified elements, which consist of: foot, shank (with two parts: bones and muscle), knee and femur. The symbol has the meaning in Romanian language, because they are: "LaBa piciorului, Gamba, Genunchi şi Femur".

## 2. STUCTURE OF BIOMECHANICAL MODEL 5LBGGF

For the purpose of determining the biomechanical system 5LBGGF structure it starts from Figure 1, the early works [Arg 16].

Mechanical 5LBGGF system is composed of 5 distinct parts, which is isolated individually. On each side part are inserted: initial direction of motion (as in Figure 1 from [Arg 16]), external forces, internal forces directly applied - they are the interaction between components of the system. The principle of action and reaction is aplyed through the transition from one part to another part of the system [Fod 15b].

For each component part applies the two principle of mechanics (principle of force action). Figures 1, 2, 3, 4, and 5 contain mechanical 5LBGGF system diagrams, on each component.


Fig. 1. Precision mechanics foot diagram of the system 5LBGGF

Notations in Figure 1 are the:
z - vertical vibratory platform movement, which represents the human operator's leg excitation applied;
$\mathrm{c}_{1}$ - the damping constant of the foot, about the vibrating platform;
$\mathrm{k}_{1}$ - elasticity constant of the foot, about the vibrating platform;
$\mathrm{m}_{1}$ - foot mass;
$\mathrm{Z}_{1}$ - vertical displacement of the foot;
$\mathrm{c}_{2}$ - damping constant of the calf (shank) bone relative to the foot;
$\mathrm{k}_{2}$ - elasticity constant of the calf (shank) bone relative to the foot;
$\mathrm{c}_{3}$ - damping constant of the muscle of the calf (shank) related to the foot;
$\mathrm{k}_{3}$ - elasticity constant of the muscle of the calf (shank) related to the foot.


Fig. 2. Precision mechanics bone scheme from shank of 5LBGGF system

Additional notations in Figure 2 are the: $\mathrm{m}_{2}$ - mass of calf (shank) bone (tibia and fibula);
$\mathrm{Z}_{2}$ - vertical displacement of calf (shank) bone;
$\mathrm{c}_{41}$ - damping constant of the bone of the shank, reported to the knees;
$\mathrm{k}_{41}$ - elasticity constant of the bone of the shank, reported to the knees.


Fig. 3. Precision mechanics muscle scheme from shank of 5LBGGF system

Extra notations in the Figure 3 are the:
$\mathrm{m}_{3}$ - mass of the muscle of the calf (shank);
$\mathrm{z}_{3}$ - vertical displacement of the muscle of the calf (shank);
$\mathrm{c}_{42}$ - damping constant of the muscle of the shank, reported to the knees;
$\mathrm{k}_{42}$ - elasticity constant of the muscle of the shank, reported to the knees.


Fig. 4. Precision mechanics knee scheme of 5LBGGF system

The new notations in the Figure 4 are the: $\mathrm{m}_{4}$ - the mass of the knee;
$\mathrm{Z}_{4}$ - vertical displacement of the knee;
$\mathrm{c}_{5}$ - longitudinal damping constant of the femur (bone + muscle), about the knee;
$\mathrm{k}_{5}$ - longitudinal elasticity constant of the femur (bone + muscle), about the knee;
$\varphi$ - inclination angle of the femur to the horizontal direction.


Fig. 5. Precision mechanics femur scheme of 5LBGGF system

The notations in the Figure 5 regarding the last four figures are the:
$\mathrm{c}_{6}$ - longitudinal damping constant of the femur (bone + muscle), about the cylindrical henge in plane (considered fixed) of basin;
$\mathrm{k}_{6}$ - longitudinal elasticity constant of the femur (bone + muscle), about the cylindrical henge in plane (considered fixed) of basin;
$\mathrm{m}_{5}$ - mass of the femur;
$\mathrm{d}_{5}$ - longitudinal displacement of the femur (bone + muscle) slanted approach towards the horizontal direction.
Biomechanical system 5LBGGF is subject to the laws of mechanics, as any existing system in nature is composed of three distinct elements: mass, damper and arc.

In biomechanical system, in addition to the above issues, intervenes and the subject will be subjected to the action of vibration, but the position request occurs, it may be considered willful human operator intervention, is negligible and should not be taken into account.

## 3. THE DIFFERENTIAL EQUATIONS WHAT CHARACTERIZES THE DYNAMICS OF 5LBGGF SYSTEM

The representations in figures 1-5 contain the mechanical schemas related to the concentrate masses of lower limb from an operator, who has a lower member placed on a vibrating platform, and the second member shall be placed on the ground.

In this situation your foot positioned on the vibrating platform is requested through the sole of the foot, as a vibratory force, with harmonic variation.

It is considered, that require only the foot vibration posted on the platform, and the connection with the second leg and the body is accomplished through a cylindrical henge in plane, which picks up the vibration and displacement occur only within the limits of the joint.

Vibration requires only the body placed on the platform and the femur with angle $\varphi$ slant, which calculates the relationship with $\varphi=$ $\operatorname{arcsine}(\mathrm{h} / \mathrm{lf}$ ) (fig. 1 given in the paper [Arg 16]) makes the request to be much decreased axial force reported at the request of the vertical movement.

The differential equations system, which characterized the mechanical system, is given in the system (1), and it will be transformed in the system (2) having the structure after the order of unknowns ordered derivation. The new form is necessary for the integration of the differential equations systems.

The (2) system is a system of secondorder differential equations, homogeneous, with constant coefficients in the unknowns: z1, z2, $\mathrm{z} 3, \mathrm{z} 4, \mathrm{~d} 5$, which give the coordinates of the center of masses of each element in the longitudinal direction. The system first-order derivatives occurs each generalized coordinates that constitutes their speeds and second-order derivatives, which belong to the accelerations of each element individually.

$$
\left\{\begin{align*}
m_{1} \ddot{z}_{1}= & c_{1}\left(\dot{z}-\dot{z}_{1}\right)+k_{1}\left(z-z_{1}\right)-c_{2}\left(\dot{z}_{1}-\dot{z}_{2}\right)-c_{3}\left(\dot{z}_{1}-\dot{z}_{3}\right)-k_{2}\left(z_{1}-z_{2}\right)-k_{3}\left(z_{1}-z_{3}\right)  \tag{1}\\
m_{2} z_{2} & =c_{2}\left(\dot{z}_{1}-\dot{z}_{2}\right)+k_{2}\left(z_{1}-z_{2}\right)-c_{41}\left(\dot{z}_{2}-\dot{z}_{4}\right)-k_{41}\left(z_{2}-z_{4}\right) \\
m_{3} \ddot{z}_{3}= & +c_{3}\left(\dot{z}_{1}-\dot{z}_{3}\right)+k_{3}\left(z_{1}-z_{3}\right)-c_{42}\left(\dot{z}_{3}-\dot{z}_{4}\right)-k_{42}\left(z_{3}-z_{4}\right) \\
m_{4} \ddot{z}_{4}= & c_{41}\left(\dot{z}_{2}-\dot{z}_{4}\right)+k_{41}\left(z_{2}-z_{4}\right)+c_{42}\left(\dot{z}_{3}-\dot{z}_{4}\right)+k_{42}\left(z_{3}-z_{4}\right)- \\
& -c_{5}\left(\dot{z}_{4}-\dot{d}_{5} \cos (\pi / 2-\varphi)\right\}-k_{5}\left(z_{4}-d_{5} \cos (\pi / 2-\varphi)\right\} \\
m_{5} \ddot{d}_{5}= & c_{5}\left(\dot{z}_{4}-\dot{d}_{5} \cos (\pi / 2-\varphi)\right\}+k_{5}\left(z_{4}-d_{5} \cos (\pi / 2-\varphi)\right\}-c_{6} \dot{d}_{5}-k_{6} d_{5}
\end{align*}\right.
$$

$$
\left\{\begin{array}{l}
m_{1} \ddot{z}_{1}+\left(c_{1}+c_{2}+c_{3}\right) \dot{z}_{1}+\left(k_{1}+k_{2}+k_{3}\right) z_{1}-c_{2} \dot{z}_{2}-c_{3} \dot{z}_{3}-k_{2} z_{2}-k_{3} z_{3}=c_{1} \dot{z}+k_{1} z  \tag{2}\\
m_{2} \ddot{z}_{2}-c_{2} \dot{z}_{1}+\left(c_{2}+c_{41}\right) \dot{z}_{2}-k_{2} z_{1}+\left(k_{2}+k_{41}\right) z_{2}-c_{41} \dot{z}_{4}-k_{41} z_{4}=0 \\
m_{3} \ddot{z}_{3}-c_{3} \dot{z}_{1}+\left(c_{3}+c_{42}\right) \dot{z}_{3}-k_{3} z_{1}+\left(k_{3}+k_{42}\right) z_{3}-c_{42} \dot{z}_{4}-k_{42} z_{4}=0 \\
m_{4} \ddot{z}_{4}-c_{41} \dot{z}_{2}+\left(c_{41}+c_{42}+c_{5}\right) \dot{z}_{4}-k_{41} z_{2}+\left(k_{41}+k_{42}+k_{5}\right) z_{4}-c_{42} \dot{z}_{3}-k_{42} z_{3}- \\
\quad-c_{5} \dot{d}_{5} \cos (\pi / 2-\varphi)-k_{5} d_{5} \cos (\pi / 2-\varphi)=0 \\
m_{5} \ddot{d}_{5}-c_{5} \dot{z}_{4}+c_{5} \dot{d}_{5} \cos (\pi / 2-\varphi)+c_{6} \dot{d}_{5}-k_{5} z_{4}+k_{5} d_{5} \cos (\pi / 2-\varphi)+k_{6} d_{5}=0
\end{array}\right.
$$

## 4. CONCLUSIONS

This work is a continuation of a natural first part, which is found in [Arg 16], and which realizes:

- Mechanics schema of components parts (5 parts) of the lower limb of a human operator, subjected to vibrations;
- Explication of figures notations;
- Drawing up the initial system differential equations;
- Preparing the final system of differential equations, for integration with the RungeKutta method, the order of 4.5 with variable step.

The idea of this work, to take a leg on a vibrating platform, represents the original contribution of authors, which eliminates the influence of vibration upon the action of the second leg, which is placed on the ground.

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Contribuţii la studiul biomecanicii membrului inferior al unui subiect uman supus la vibraţii.
Partea II-a: Sistemul ecuaţiilor diferenţiale
Rezumat: In lucrare se face un studiu complex, biomecanic asupra comportarii membrului inferior sub actiunea vibratiilor. Solicitarea se aplica prin talpa labei piciorului si se propagă de-a lungul piciorului solicitat prin gambă şi femur. Cel de al doilea picior se considera asezat pe o suprafata fixa, de aceea legǎtura dintre piciorul solicitat şi cel de al doilea picior, se poate considera o articulaţie cilindricǎ planǎ. Sistemul mecanic (Partea I-a) este analizat prin integrarea in MatLab Simulinc a sistemului de ecuatii diferentiale (Partea II-a), ce caracterizeaza dinamica acestuia. Rezultatele obtinute sunt comparabile cu cele existente in literatura de specialitate (Partea III-a).

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