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AN EXHAUST SYSTEM LUMPED MODEL – IDENTIFICATION AND SIMULATION

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Abstract: *The dynamics of a car exhaust system is under observation, the vibration simulation being of particular interest. For rapid evaluation of the vertical vibrations of the system a lumped dynamic model is derived and integrated over time. Some of the system parameters are measured, others are estimated by using comparisons between measurements and finite elements analysis and some are estimated. The hanger rigidity coefficient is derived from the frequency response function measured by using the impulsive method on a system consisting of the hanger stretched by a gravity load. The system of differential equations is integrated and the natural frequencies are derived.*
Keywords: *exhaust system, rubber hanger, frequency response function, differential equations.*

1. INTRODUCTION

The automobile exhaust system is primarily used to silence the noise generated by the high pressure residual gases produced by the engine and to drive away from the habitacle these gases which are toxic and at a high temperature. One of the main exterior noise source of the autovehicle is the exhaust system. The residual gases due to the periodic pressure pulsation are producing vibrations in the exhaust system structure, which associated to the ones induced by the hanged structure are transmitted to the car body. On the other side the driveline system is the car main source of vibration and noise, which is connected to the vehicle body (frame) through bearings, isolators and mounts. Conventional rubber mounts together with hydraulic, semi active and active mounts are proper used in order to manage the complex vibration interaction between the driveline and the car frame.

Starting from the engine exhaust manifold the most important parts of a typical exhaust system are the flex decoupler, the catalyzer, the front silencer or resonator, rear silencer or muffler and the connecting pipe segments between them. The noise of the exhaust system is produced directly by the gasses at the tailpipe orifice and is radiated from the structural shells that build the components of the structure. At the end through the tailpipe the exhaust gasses are released out. 50% of the exhaust noise is generated at the tailpipe orifice. The mufflers are in charge with the noise reduction. The front one is a reactive silencer while the

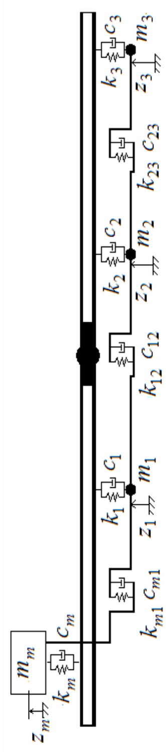

Fig. 1. The exhaust system discrete model

rear muffler can be as well dissipative in parallel to reactive. The catalytic converter converts exhaust gasses to a less harmful gas. The exhausted gas is analyzed and is participating to the combustion and emission control.

The exhaust system is hung from the chassis through elastomeric hungers placed in proper locations in the effort to manage the vibrations and to not transmit structural vibrations to the car body. The locations are influenced by the stiffness, the mass distribution of the exhaust system and the whole system dynamics. The range of the excitation frequencies coming from the engine and the idle frequency are also of great importance. The hunger mount points are important in order to minimize the transferred vibration from the exhaust system to the car frame.

For the estimation of the maximum excitation frequency the operating engine speed (for instance 4600 rpm/60) is multiplied by the engine firing order and the number of cylinders.

The dynamics in vertical plane of a simplified exhaust system model will be observed in the sequel.

2. THE DYNAMICAL MODEL

A simplified dynamical model of the whole exhaust system in vertical direction has been derived in a previous work [4]. The system of four differential equations has been written in compact matrix form:

$$M\ddot{Q} + D\dot{Q} + KQ = 0 \qquad (1)$$

Explicitly the inertial and symmetric matrix is:

$$M = \begin{bmatrix} m_m & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_3 \end{bmatrix} \qquad (2)$$

which is multiplied by the generalized acceleration vector:

$$\ddot{Q} = [\ddot{z}_m \quad \ddot{z}_1 \quad \ddot{z}_2 \quad \ddot{z}_3]^T$$

The viscous damping matrix D is of the form:

$$\begin{bmatrix} c_m + c_{m1} & -c_{m1} & 0 & 0 \\ -c_{m1} & c_{m1} + c_{12} + c_1 & -c_{12} & 0 \\ 0 & -c_{12} & c_2 + c_{12} + c_{23} & -c_{23} \\ 0 & 0 & -c_{23} & c_{23} + c_3 \end{bmatrix} \qquad (3)$$

and is multiplied by the generalized velocity vector:

$$\dot{Q} = [\dot{z}_m \quad \dot{z}_1 \quad \dot{z}_2 \quad \dot{z}_3]^T$$

The stiffness matrix K, is of the form:

$$\begin{bmatrix} k_{m1} + k_m & -k_{m1} & 0 & 0 \\ -k_{m1} & k_{m1} + k_{12} + k_1 & -k_{12} & 0 \\ 0 & -k_{12} & k_{12} + k_2 + k_{23} & -k_{23} \\ 0 & 0 & -k_{23} & k_{23} + k_3 \end{bmatrix} \qquad (4)$$

which is multiplied by the generalized coordinates vector:

$$Q = [z_m \quad z_1 \quad z_2 \quad z_3]^T \qquad (5)$$

In the sequel the parameters of the dynamical model are measured or estimated and the system is integrated.

3. HANGER PROPERTIES

3.1 The experimental set-up

The exhaust system is hung from the chassis by using exhaust mounts placed in proper locations in the effort to manage the vibrations and to not transmit structural vibrations to the car body.

A static determination has been performed by loading the exhaust mount with determined loads and measuring the displacement. In this article a dynamic procedure will be observed by measuring

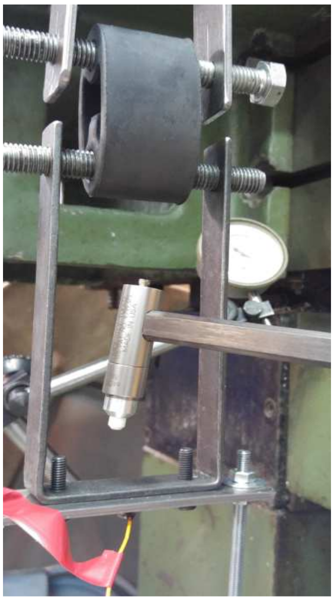

Fig. 2. The hanger measurement set up

frequency response functions (FRFs) in order to derive the elasticity coefficient of the hanger. The system shown in Figure 2 is assimilated to a one degree of freedom system in the lowest frequency band of interest. The exhaust mount is assimilated to a spring having a viscous absorber in parallel. By using a metallic frame a known mass is hung stretching the rubber of the hanger, similar to the real case in which the exhaust system is hanging under the car.

3.2. FRF measurement

A portable acquisition system based on a National Instruments USB, 24-bit, four acquisition channels board, has been used. Frequency response functions (FRFs) have been measured for different loads.

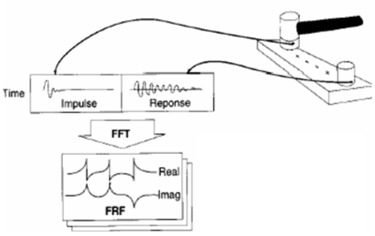

Fig. 3. FRF measurement

For the FRF measurement, the impulsive method has been used [6]. The impact hammer was connected to the first channel of the acquisition system and was used to excite the structure. A PCB mini-accelerometer of 1.5 g placed close to the impact location, as depicted in Figure 2, is connected to the second channel of the acquisition system in order to record the response of the structure. The schematic overview of the experimental test set-up and FRF derivation is shown in Figure 3. For the hung mass of 2, 4 and 8 kg, the FRFs [(m/s^2)/N] are presented in Figures 4, 5 and 6.

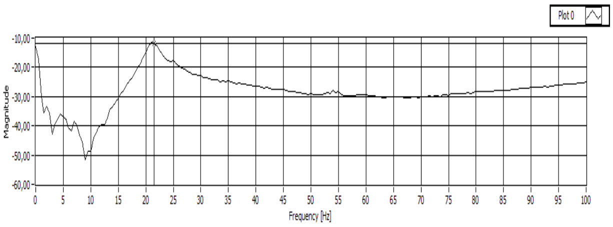

Fig. 4. FRF 2 kg, peak at 21,5 Hz

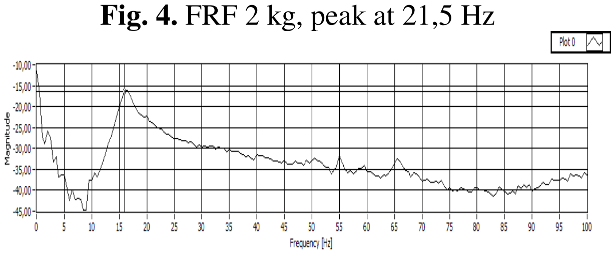

Fig. 5. FRF 4 kg, peak at 16 Hz

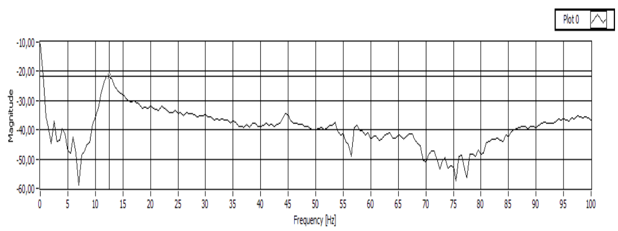

Fig. 6. FRF 8 kg, peak at 12,5 Hz

The frequencies of interest are low, hence acquisition sampling rate: 2500, the block size 5000, resulting the acquisition time of 2s. The FRF averaging is 4. By using relations (1), the resulted k values are presented in Table 1.

$$\omega_0 = \sqrt{k/m}\,, \quad k = m\omega_0^2 = 4m\pi^2 f^2 \qquad (1)$$

Table 1

Hung mass [kg]	FRF: first peak frequency [Hz]	k [N/m]
2	21.5	36 498
3	18	38 373
4	16	40 426
5	15	44 413
6	14	46 427
7	13	46 703
8	12.5	49 348

3.3. Finite element validation

The hanger geometry is mesh by using solid finite elements, as can be seen in Figure 7. The upper hole is immobile and to the lower bolt a force in vertical direction is applied. The applied force equals the gravity force like in the experimental approach.

Changing the Young modulus of the rubber in finite element simulation in order to have similar results for the experiment and the simulation the following value is resulting:

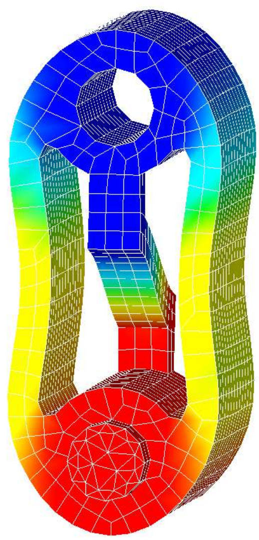

Fig. 7. The hanger FEA

E=5,2 MPa.

Other rubber properties used in the simulation were 0.48 for the Poisson coefficient and 1.1e-9 Tons/mm^3 for the density.

4. THE SIMULATION OF THE EXHAUST SYSTEM LUMPED MODEL

The total mass of the system is 12.5kg which can be divided in four lumped mass: m0=0.5 kg the mass of the pipe close to the engine, m1=4kg expressing mainly the mass of the catalyzer, m2=4kg representing mainly the mass of the intermediate mufflers and m3=4kg mainly the mass of the final muffler. m0 will be attached to the engine mass m_m=80kg resulting m_{m+}=80.5kg.

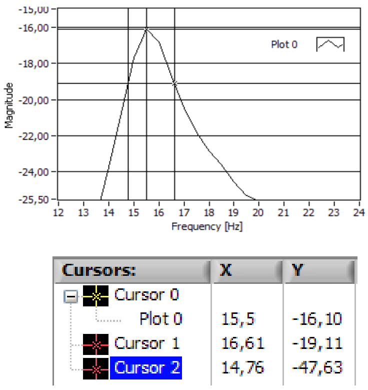

Fig. 8. Half power method

For the determination of the damping ratio (ζ) associated to the three hangers (k1, k2 and k3) an experimental approach has been used. A hung mass of 3,9 kg is attached, stretching the rubber support. From the frequency response function the resonant peak at 15,5Hz which is well separated, can be used for the estimation of the damping ratio value ζ of the hanger.

$$\zeta \approx \frac{\Delta\omega}{2\omega_0}, \quad \zeta \approx \frac{1{,}85}{2 \cdot 15{,}5} = 0.06 \qquad (2)$$

The damping ratio ζ associated to the three hangers can be determined as well from the logarithmic decrement δ of the damped vibration recorded on a vibrogram of the same system:

$$\delta = \ln\frac{x(t)}{x(t+T_d)} \qquad \zeta \approx \frac{\delta}{2\pi}$$

km1, k12, k23 are to be determined from the first eigenvalue tuning of the lumped system and the modal analysis of the finite element model.

For the vertical stiffness k_m of the engine support some aspects of the isolation design are to be mentioned. The power plant is placed typically by three mounts on a subframe which is connected to the vehicle frame. The mounts number varies between two to five to different power plants. When two mounts are used a free-free roll axis is present and one to two restrictors of the rotations of the power plant about the roll axis [9]. The mount system sustain the weight of the power plant (several hundred kilograms) and isolate the effects of the structural vibrations, forces from accelerations/ decelerations, cornering, impact excitations etc. Mounts are made of rubber and can be assimilated to a linear spring in parallel with a linear viscous damper. Let us take a total stiffness of 800 N/mm for the mount in the vertical direction.

The Matlab application for the system integration is as follows.

```
t0=0;tf=2; %int. time
   %initial conditions: m, m/s
X0=[1.5*10^-3 5*10^-3  -10^-3  0  0 0 0 0]';
global K M D invM;
mm=80.5;m1=4;m2=4;m3=4;
k1=40426; k2=k1; k3=k1;%N/m;
km=800*10^3; %N/m;
km1=10*k1; k12=km1; k23=km1;

 M=[mm 0 0 0;0 m1 0 0;0 0 m2 0; 0 0 0 m3];

 K=[km1+km  -km1        0        0;
    -km1  km1+k12+k2    -k12       0;
    0       -k12     k12+k2+k23  -k23;
    0         0      -k23      k23+k3];
c1=5; c2=c1; c3=c1;
cm=100 % Ns/m;
cm1=0,5*c1; c12=cm1; c23=cm1;

 D=[cm1+cm  -cm1        0        0;
    -cm1  cm1+c12+c2    -c12       0;
    0       -c12     c12+c2+c23  -c23;
    0         0      -c23      c23+c3];
 invM=M^-1;

[t,X]=ode45('s_prim2', [t0 tf], X0);

subplot(211);
plot(t,X(:,1:4),'linewidth',1.6); grid on;
```

```
subplot(212);
plot(t,X(:,5:8),'linewidth',1.6); grid on;
```

The first parameter of the function ode45 is the name of a called function which definition is not presented here.

The displacement of the four masses in time are depicted in Figure 9.

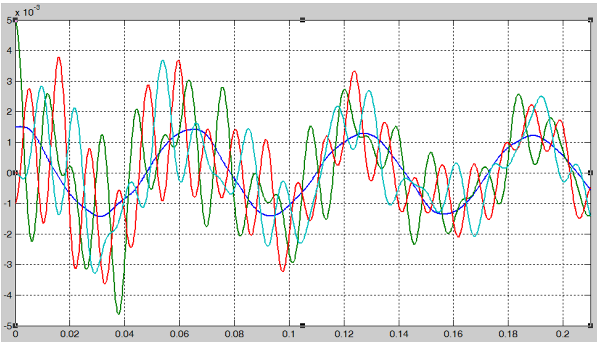

Fig. 9. Lumped masses system vibration Matlab simulation

The eigenvalues and the natural frequencies of the simplified system can be obtained by solving the eigenvalue problem. By replacing a global harmonic proposed solution in the system of differential equations, results:

$$Ku - \omega^2 Mu = 0 \ or \ (K - \omega^2 M) \cdot u = 0 \quad (3)$$

The set of the homogeneous algebraic equations (3) has the unknown vector *u*. Considering $\lambda = \omega^2$ as a parameter, one gets:

$$Ku = \lambda Mu \quad (4)$$

known as the eigenvalue problem when trying to determine λ values for which the system (3) has nontrivial solutions. By solving for λ and nontrivial solution ($u \neq 0$), the following characteristic equation (5) is obtained:

$$\det(K - \lambda M) = 0 \quad (5)$$

where λ_r ($r=1,2\ldots,n$) values are the eigenvalues (or characteristic values) of the system.

For the exhaust system chosen parameters the four natural frequencies ($\omega/2/\pi$) calculated by using Matlab are:

f_1=15.88 Hz, f_2=28.82 Hz,

f_3=65.43 Hz, f_4=92.64 Hz.

For dummy initial conditions when all modes of vibrations are acting, FFT can be applied to the displacement values associated to the displacement of each of the system lumped mass. For each FFT graph the same frequency peaks are observed but with different amplitudes indicating a particular participation of each vibration mode to the resulted system motion.

4. CONCLUSIONS

An exhaust system and the engine are modelled by using lumped masses. The degrees of freedom are observing the masses displacements in vertical direction. Some of the system parameters are measured, others are estimated by using comparisons between measurements and finite elements analysis and the rest are estimated. Further effort in tuning the model and the real system is necessary. A Matlab application is written in order to numerical integrate the system of differential equations and to calculate the natural frequencies of the model. The simplified models can be utilized for a rapid estimation of the system vibrations in vertical direction.

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Modelul unui sistem de evacuare cu mase concentrate. Identificare si simulare.

Rezumat: *Articolul abordează modelarea dinamică prin mase concentrate a unui sistem de evacuare, inclusiv motorul automobilului. Este observat sistemul celor patru ecuaţii diferenţiale de ordinul doi, mişcarea maselor fiind pe direcţie verticală. Sunt estimaţi parametrii inerţiali, de rigiditate şi de amortizare vâscoasă. Se acordă importanţă crescută suporţilor de cauciuc prin care sistemul de evacuare este atârnat de şasiu. Suportul este întins cu diferite forţe măsurându-se funcţia de răspuns în frecvenţă prin excitaţie impulsivă pe direcţie verticală. Este folosit un ciocan de impact cu senzor de forţă, un miniaccelerometru pentru măsurarea răspunsului, un sistem de echiziţie cu eşantionare simultană pe cele două canale de măsurare şi o aplicaţie Labview. Din primul vârf al modulului FRF este estimat coeficientul de rigiditate şi raportul de amortizare vâscoasă al suportului elastic. Suportul este de asemenea modelat cu elemente finite de volum, este simulată o întindere statică iar prin comparaţie cu întinderea experimentală se estimează modulul lui Young al cauciucului din care este confecţionat suportul. Sistemul de ecuaţii diferenţiale este integrat printr-o aplicaţie Matlab pentru condiţii iniţiale impuse. Sunt determinate numeric de asemenea frecvenţele naturale ale sistemului analizat.*

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