TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

# ACTA TECHNICA NAPOCENSIS

**Series: Applied Mathematics and Mechanics**
**Vol. 56, Issue II, June, 2013**

# DYNAMICS EQUATIONS OF TRTRR ROBOT USING NEWTON-EULER FORMALISM

**Ramona Maria GUI, Viorel ISPAS, Mariana ARGHIR, Virgil ISPAS**

***Abstract:*** *Dynamic modeling of the robot TRTRR, whose cinematic diagram is shown in this paper, and will be carried out in accordance with by applying formalism Newton-Euler, implemented in the program for symbolic modeling Robot Symbolic, Robot Dynamics module from within the program MatLab 7.1 .*

***Keywords:*** *TRTTR robot, dynamics equations, Newton-Euler formalism.*

## 1. INTRODUCTION

Dynamic modeling of the robot TRTRR, whose cinematic diagram is shown in figure 1 will be carried out in accordance with [Ghi04], [Mar93] and [Jen92], by applying formalism Newton-Euler, implemented in the program for symbolic modeling Robot_Symbolic, Robot_ Dynamics module from within the program MatLab 7.1.

For the application of Newton-Euler formalism should be make the geometric and cinematic modeling, as well as to determine the parameters for the distribution of masses. Also, certain hypotheses simplification proposals:

- Choose mass centers $C_i$ in the origins $O_i$ of reference Cartesian systems $O_ix_iy_iz_i$, i = 1÷5, and thus the position vectors of the mass centers are null;
- By choosing reference systems elements in such a way that their axes are aligned with the main directions of inertia corresponding to the origin of these systems, it appears that those mechanical moments of centrifugal inertia shall become null and void.

## 2. DINAMICS OF THE TRTRR ROBOT

The following parameters are presented for the distribution of masses, namely:

- masses: $M_1$, $M_2$, $M_3$, $M_4$, $M_5$;
- mass centers:

$$\bar{r}^1_{c_1} = [\,0\ \ 0\ \ 0]^T,\ \bar{r}^2_{c_2} = [\,0\ \ 0\ \ 0]^T,\ \bar{r}^3_{c_3} = [\,0\ \ 0\ \ 0]^T,\ \bar{r}^4_{c_4} = [\,0\ \ 0\ \ 0]^T,\ \bar{r}^5_{c_5} = [\,0\ \ 0\ \ 0]^T; \qquad (1)$$

- tensor of inertia:

$$J_1^{*1} = \begin{bmatrix} J_x^{*1} & 0 & 0 \\ 0 & J_y^{*1} & 0 \\ 0 & 0 & J_z^{*1} \end{bmatrix}, J_2^{*2} = \begin{bmatrix} J_x^{*2} & 0 & 0 \\ 0 & J_y^{*2} & 0 \\ 0 & 0 & J_z^{*2} \end{bmatrix},$$
$$J_3^{*3} = \begin{bmatrix} J_x^{*3} & 0 & 0 \\ 0 & J_y^{*3} & 0 \\ 0 & 0 & J_z^{*3} \end{bmatrix}, J_4^{*4} = \begin{bmatrix} J_x^{*4} & 0 & 0 \\ 0 & J_y^{*4} & 0 \\ 0 & 0 & J_z^{*4} \end{bmatrix},$$
$$J_5^{*5} = \begin{bmatrix} J_x^{*5} & 0 & 0 \\ 0 & J_y^{*5} & 0 \\ 0 & 0 & J_z^{*5} \end{bmatrix}. \qquad (2)$$

where: $J_x^{*i}$, $J_y^{*i}$, $J_z^{*i}$, i = 1, 2, 3, 4, 5, represents the axial mechanical moments of inertia in relation to the system i, with the origin at the mass center $C_i$ but also having the same guidance as the system attached with each component of the robot.

Determine, in accordance with [Isp04] and [Det07], the corresponding accelerations of mass centers, shall be carried out with the following relations:

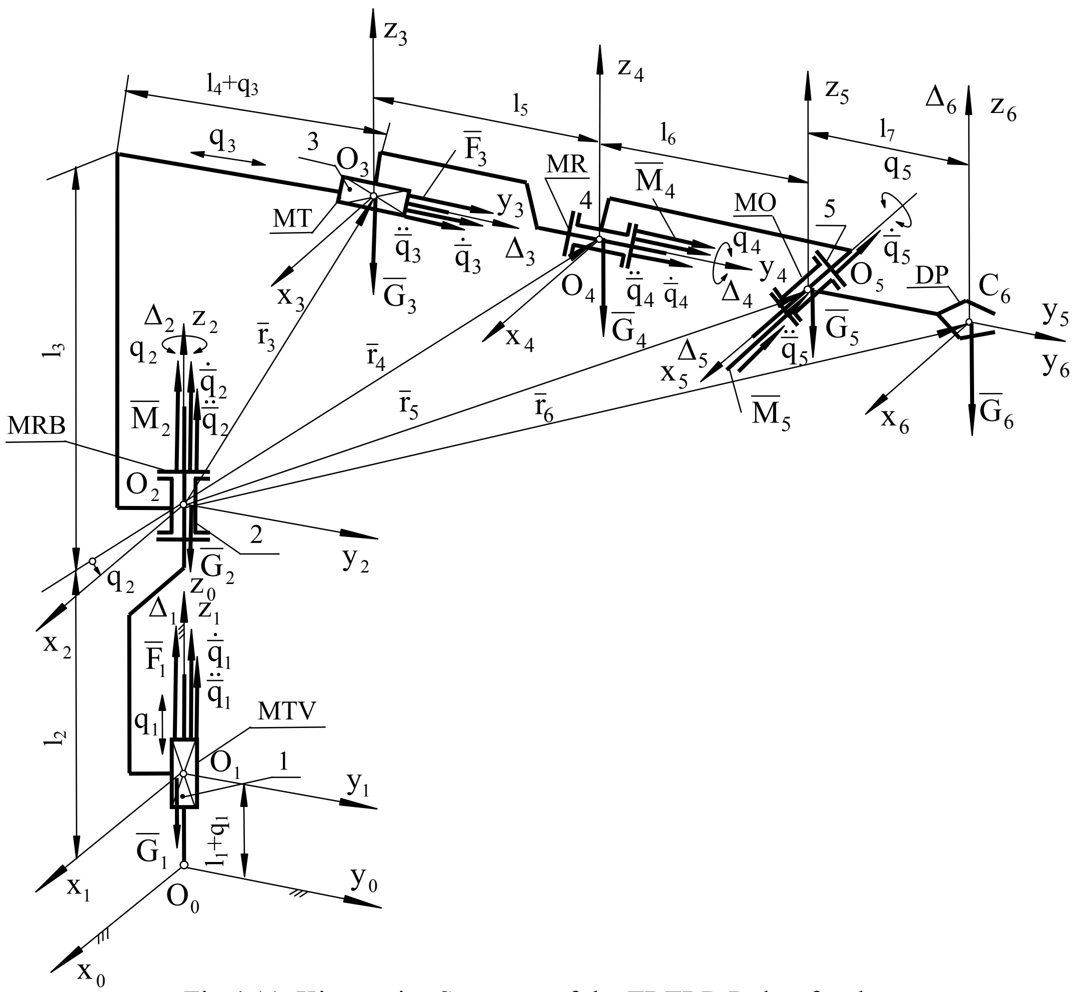


Fig.4.11. Kinematics Structure of the TRTRR Robot for the Dynamics Study

$$\bar{a}_{c_1}^1 = \bar{a}_1^1 + \bar{\varepsilon}_1^1 \times \bar{r}_{c_1}^1 + \bar{\omega}_1^1 \times \left(\bar{\omega}_1^1 \times \bar{r}_{c_1}^1\right), \left[\bar{a}_c\right]_1^1 = \begin{bmatrix} 0 \\ 0 \\ g + \ddot{q}_1 \end{bmatrix} \quad (3)$$

$$\bar{a}_{c_2}^2 = \bar{a}_2^2 + \bar{\varepsilon}_2^2 \times \bar{r}_{c_2}^2 + \bar{\omega}_2^2 \times \left(\bar{\omega}_2^2 \times \bar{r}_{c_2}^2\right), \left[\bar{a}_c\right]_2^2 = \begin{bmatrix} 0 \\ 0 \\ g + \ddot{q}_1 \end{bmatrix}; \quad (4)$$

$$\bar{a}_{c_3}^3 = \bar{a}_3^3 + \bar{\varepsilon}_3^3 \times \bar{r}_{c_3}^3 + \bar{\omega}_3^3 \times \left(\bar{\omega}_3^3 \times \bar{r}_{c_3}^3\right),$$

$$\left[\bar{a}_c\right]_3^3 = \begin{bmatrix} -\ddot{q}_2 l_4 - \ddot{q}_2 q_3 - 2\dot{q}_2 \dot{q}_3 \\ \ddot{q}_3 - \dot{q}_2^2 l_4 - \dot{q}_2^2 q_3 \\ g + \ddot{q}_1 \end{bmatrix}; \quad (5)$$

$$\bar{a}_{c_4}^4 = \bar{a}_4^4 + \bar{\varepsilon}_4^4 \times \bar{r}_{c_4}^4 + \bar{\omega}_4^4 \times \left(\bar{\omega}_4^4 \times \bar{r}_{c_4}^4\right), \quad (6)$$

$$\left[\bar{a}_c\right]_4^4 = \begin{bmatrix} -\ddot{q}_2 q_3 c q_4 - \ddot{q}_2 l_4 c q_4 - 2\dot{q}_2 \dot{q}_3 c q_4 - \ddot{q}_2 l_5 c q_4 - g s q_4 - \ddot{q}_1 s q_4 \\ -\dot{q}_2^2 q_3 - \dot{q}_2^2 l_4 + \ddot{q}_3 - \dot{q}_2^2 l_5 \\ -\ddot{q}_2 q_3 s q_4 - \ddot{q}_2 l_4 s q_4 - 2\dot{q}_2 \dot{q}_3 s q_4 - \ddot{q}_2 l_5 s q_4 + g c q_4 + \ddot{q}_1 c q_4 \end{bmatrix}; \quad (7)$$

$$\bar{a}_{c_5}^5 = \bar{a}_5^5 + \bar{\varepsilon}_5^5 \times \bar{r}_{c_5}^5 + \bar{\omega}_5^5 \times \left(\bar{\omega}_5^5 \times \bar{r}_{c_5}^5\right), \quad (8)$$

$$[\bar{a}_c]_5^5=\begin{bmatrix} -\ddot{q}_2q_3cq_4-\ddot{q}_2l_4cq_4-2\dot{q}_2\dot{q}_3cq_4-\ddot{q}_2l_5cq_4-gsq_4-\ddot{q}_1sq_4-\ddot{q}_2l_6cq_4 \\ \hline -\dot{q}_2^2q_3cq_5-\dot{q}_2^2l_4cq_5+\ddot{q}_3cq_5-\dot{q}_2^2l_5cq_5-\dot{q}_2^2l_6cq_5-\ddot{q}_2q_3sq_4sq_5-\ddot{q}_2l_4sq_4sq_5- \\ -2\dot{q}_2\dot{q}_3sq_4sq_5-\ddot{q}_2l_5sq_4sq_5+gcq_4sq_5+\ddot{q}_1cq_4sq_5-\ddot{q}_2l_6sq_4sq_5 \\ \hline \dot{q}_2^2q_3sq_5+\dot{q}_2^2l_4sq_5-\ddot{q}_3sq_5+\dot{q}_2^2l_5sq_5+\dot{q}_2^2l_6sq_5-\ddot{q}_2q_3sq_4cq_5-\ddot{q}_2l_4sq_4cq_5- \\ -2\dot{q}_2\dot{q}_3sq_4cq_5-\ddot{q}_2l_5sq_4cq_5+gcq_4cq_5+\ddot{q}_1cq_4cq_5-\ddot{q}_2l_6sq_4cq_5 \end{bmatrix} \quad (9)$$

For a start, mechanical structure is traveled through the iterative process toward the outside of the robot mechanical structure. In this way, it is determined the torsor of reduction for the system of environmental forces, giving the following relations:

- external forces:

$$[\bar{F}]_1^1=M_1[\bar{a}_c]_1^1,\quad [\bar{F}]_1^1=\begin{bmatrix}0\\0\\M_1(g+\ddot{q}_1)\end{bmatrix}; \quad (10)$$

$$[\bar{F}]_2^2=M_2[\bar{a}_c]_2^2,\quad [\bar{F}]_2^2=\begin{bmatrix}0\\0\\M_2(g+\ddot{q}_1)\end{bmatrix}; \quad (11)$$

$$[\bar{F}]_3^3=M_3[\bar{a}_c]_3^3,[\bar{F}]_3^3=\begin{bmatrix}-M_3(\ddot{q}_2l_4+\ddot{q}_2q_3+2\dot{q}_2\dot{q}_3)\\-M_3(-\ddot{q}_3+\dot{q}_2^2l_4+\dot{q}_2^2q_3)\\M_3(g+\ddot{q}_1)\end{bmatrix}; \quad (12)$$

$$[\bar{F}]_4^4=M_4[\bar{a}_c]_4^4,\quad [\bar{F}]_4^4=\begin{bmatrix}-M_4(\ddot{q}_2q_3cq_4+\ddot{q}_2l_4cq_4+2\dot{q}_2\dot{q}_3cq_4+\ddot{q}_2l_5cq_4+gsq_4+\ddot{q}_1sq_4)\\-M_4(\dot{q}_2^2q_3+\dot{q}_2^2l_4-\ddot{q}_3+\dot{q}_2^2l_5)\\-M_4(\ddot{q}_2q_3sq_4+\ddot{q}_2l_4sq_4+2\dot{q}_2\dot{q}_3sq_4+\ddot{q}_2l_5sq_4-gcq_4-\ddot{q}_1cq_4)\end{bmatrix}; \quad (13)$$

$$[\bar{F}]_5^5=M_5[\bar{a}_c]_5^5,$$

$$[\bar{F}]_5^5=\begin{bmatrix} -M_5(\ddot{q}_2q_3cq_4+\ddot{q}_2l_4cq_4+2\dot{q}_2\dot{q}_3cq_4+\ddot{q}_2l_5cq_4+gsq_4+\ddot{q}_1sq_4+\ddot{q}_2l_6cq_4) \\ \hline -M_5(\dot{q}_2^2q_3cq_5+\dot{q}_2^2l_4cq_5-\ddot{q}_3cq_5+\dot{q}_2^2l_5cq_5+\dot{q}_2^2l_6cq_5+\ddot{q}_2q_3sq_4sq_5+\ddot{q}_2l_4sq_4sq_5- \\ +2\dot{q}_2\dot{q}_3sq_4sq_5+\ddot{q}_2l_5sq_4sq_5-gcq_4sq_5-\ddot{q}_1cq_4sq_5+\ddot{q}_2l_6sq_4sq_5) \\ \hline -M_5(-\dot{q}_2^2q_3sq_5-\dot{q}_2^2l_4sq_5+\ddot{q}_3sq_5-\dot{q}_2^2l_5sq_5-\dot{q}_2^2l_6sq_5+\ddot{q}_2q_3sq_4cq_5+\ddot{q}_2l_4sq_4cq_5- \\ +2\dot{q}_2\dot{q}_3sq_4cq_5+\ddot{q}_2l_5sq_4cq_5-gcq_4cq_5-\ddot{q}_1cq_4cq_5+\ddot{q}_2l_6sq_4cq_5) \end{bmatrix}. \quad (14)$$

- moments of external forces:

$$\bar{M}_{c_1}^1=J_1^{*1}\bar{\varepsilon}_1^1+\bar{\omega}_1^1\times J_1^{*1}\bar{\omega}_1^1,\quad [\bar{M}_c]_1^1=[0\ \ 0\ \ 0]^T; \quad (15)$$

$$\bar{M}_{c_2}^2=J_2^{*2}\bar{\varepsilon}_2^2+\bar{\omega}_2^2\times J_2^{*2}\bar{\omega}_2^2,\quad [\bar{M}_c]_2^2=\begin{bmatrix}0\\0\\J_z^{*2}\ddot{q}_2\end{bmatrix}; \quad (16)$$

$$\bar{M}_{c_3}^3=J_3^{*3}\bar{\varepsilon}_3^3+\bar{\omega}_3^3\times J_3^{*3}\bar{\omega}_3^3,\quad [\bar{M}_c]_3^3=\begin{bmatrix}0\\0\\J_z^{*3}\ddot{q}_2\end{bmatrix}; \quad (17)$$

$$\bar{M}_{c_4}^4=J_4^{*4}\bar{\varepsilon}_4^4+\bar{\omega}_4^4\times J_4^{*4}\bar{\omega}_4^4,\quad [\bar{M}_c]_4^4=\begin{bmatrix}-J_x^{*4}\ddot{q}_2sq_4-J_x^{*4}\dot{q}_2\dot{q}_4cq_4-J_y^{*4}\dot{q}_2\dot{q}_4cq_4+J_z^{*4}\dot{q}_2\dot{q}_4cq_4\\J_y^{*4}\ddot{q}_4-J_x^{*4}\dot{q}_2^2sq_4cq_4+J_z^{*4}\dot{q}_2^2sq_4cq_4\\J_z^{*4}\ddot{q}_2cq_4-J_z^{*4}\dot{q}_2\dot{q}_4sq_4+J_x^{*4}\dot{q}_2\dot{q}_4sq_4-J_y^{*4}\dot{q}_2\dot{q}_4sq_4\end{bmatrix}; \quad (18)$$

$$\bar{M}_{c_5}^5=J_5^{*5}\bar{\varepsilon}_5^5+\bar{\omega}_5^5\times J_5^{*5}\bar{\omega}_5^5,$$

$$
\left[\overline{M}_c\right]_5^5=\begin{bmatrix}
\begin{array}{l}
-J_x^{*5}\ddot{q}_2 sq_4 - J_x^{*5}\dot{q}_2\dot{q}_4 cq_4 + J_x^{*5}\ddot{q}_5 - 2J_y^{*5}\dot{q}_2\dot{q}_4 c^2 q_5 cq_4 - J_y^{*5}\dot{q}_2^2 c^2 q_4 sq_5 cq_5 + \\
\quad + J_y^{*5}\dot{q}_4^2 sq_5 cq_5 + J_y^{*5}\dot{q}_2\dot{q}_4 cq_4 + 2J_z^{*5}\dot{q}_2\dot{q}_4 c^2 q_5 cq_4 - J_z^{*5}\dot{q}_4^2 sq_5 cq_5 + \\
\quad + J_z^{*5}\dot{q}_2^2 c^2 q_4 sq_5 cq_5 - J_z^{*5}\dot{q}_2\dot{q}_4 cq_4
\end{array} \\ \hdashline
\begin{array}{l}
J_y^{*5}\ddot{q}_4 cq_5 + J_y^{*5}\ddot{q}_2 sq_5 cq_4 - J_y^{*5}\dot{q}_2\dot{q}_4 sq_5 sq_4 + J_y^{*5}\dot{q}_2\dot{q}_5 cq_5 cq_4 - J_y^{*5}\dot{q}_4\dot{q}_5 sq_5 - \\
\quad - J_x^{*5}\dot{q}_2^2 cq_5 sq_4 cq_4 + J_x^{*5}\dot{q}_2\dot{q}_5 cq_4 cq_5 + J_x^{*5}\dot{q}_2\dot{q}_4 sq_4 sq_5 - J_x^{*5}\dot{q}_4\dot{q}_5 sq_5 + \\
\quad + J_z^{*5}\dot{q}_2^2 cq_4 sq_4 cq_5 - J_z^{*5}\dot{q}_2\dot{q}_4 sq_5 sq_4 - J_z^{*5}\dot{q}_2\dot{q}_5 cq_4 cq_5 + J_z^{*5}\dot{q}_4\dot{q}_5 sq_5
\end{array} \\ \hdashline
\begin{array}{l}
-J_z^{*5}\ddot{q}_4 sq_5 + J_z^{*5}\ddot{q}_2 cq_5 cq_4 - J_z^{*5}\dot{q}_2\dot{q}_4 cq_5 sq_4 - J_z^{*5}\dot{q}_2\dot{q}_5 cq_4 sq_5 - J_z^{*5}\dot{q}_4\dot{q}_5 cq_5 + \\
\quad + J_x^{*5}\dot{q}_2^2 sq_5 sq_4 cq_4 - J_x^{*5}\dot{q}_2\dot{q}_5 cq_4 sq_5 + J_x^{*5}\dot{q}_2\dot{q}_4 sq_4 cq_5 - J_x^{*5}\dot{q}_4\dot{q}_5 cq_5 - \\
\quad - J_y^{*5}\dot{q}_2\dot{q}_4 cq_5 sq_4 - J_y^{*5}\dot{q}_2^2 sq_5 cq_4 sq_4 + J_y^{*5}\dot{q}_4\dot{q}_5 cq_5 + J_y^{*5}\dot{q}_2\dot{q}_5 sq_5 cq_4
\end{array}
\end{bmatrix}. \qquad (19)
$$

In second part of the Newton-Euler method, the mechanical structure is traveled through the iterative process toward the inside of the robot mechanical structure. So, determine the torsor of connecting forces moments of the components and their generalized forces that drive couplers from robot.

The torsor of reduction useful to manipulate are expressed by relations:

$$
F_6^6=\begin{bmatrix} F_x^6 \\ F_y^6 \\ F_z^6 \end{bmatrix}; \qquad M_{O_6}^6=\begin{bmatrix} M_x^6 \\ M_y^6 \\ M_z^6 \end{bmatrix}. \qquad (20)
$$

The linkage forces have the following relations:

$$\overline{F}_{l_5}^5=\left[R\right]_6^5\cdot\overline{F}_{l_6}^6+\overline{F}_5^5, \qquad (21)$$

$$\overline{F}_{l_4}^4=\left[R\right]_5^4\cdot\overline{F}_{l_5}^5+\overline{F}_4^4, \qquad (22)$$

$$\overline{F}_{l_3}^3=\left[R\right]_4^3\cdot\overline{F}_{l_4}^4+\overline{F}_3^3, \qquad (23)$$

$$\overline{F}_{l_2}^2=\left[R\right]_3^2\cdot\overline{F}_{l_3}^3+\overline{F}_2^2, \qquad (24)$$

$$\overline{F}_{l_1}^1=\left[R\right]_2^1\cdot\overline{F}_{l_2}^2+\overline{F}_1^1, \qquad (25)$$

The relations (21), (22), (23), (24), (25) can be transformed into:

$$
\left[\overline{F}_l\right]_5^5=\begin{bmatrix}
\begin{array}{l}
F_{l_x}^6 - M_5\ddot{q}_2 q_3 cq_4 - M_5\ddot{q}_2 l_4 cq_4 - 2M_5\dot{q}_2\dot{q}_3 cq_4 - M_5\ddot{q}_2 l_5 cq_4 - M_5 g sq_4 - M_5\ddot{q}_1 sq_4 - \\
\quad - M_5\ddot{q}_2 l_6 cq_4
\end{array} \\ \hdashline
\begin{array}{l}
F_{l_y}^6 - M_5\dot{q}_2^2 q_3 cq_5 - M_5\dot{q}_2^2 l_4 cq_5 + M_5\ddot{q}_3 cq_5 - M_5\dot{q}_2^2 l_5 cq_5 - M_5\dot{q}_2^2 l_6 cq_5 - M_5\ddot{q}_2 q_3 sq_5 sq_4 - \\
\quad - M_5\ddot{q}_2 l_4 sq_5 sq_4 - 2M_5\dot{q}_2\dot{q}_3 sq_5 sq_4 - M_5\ddot{q}_2 l_5 sq_5 sq_4 + M_5 g cq_4 sq_5 + M_5\ddot{q}_1 sq_5 cq_4 - \\
\quad - M_5\ddot{q}_2 l_6 sq_4 sq_5
\end{array} \\ \hdashline
\begin{array}{l}
F_{l_z}^6 + M_5\dot{q}_2^2 q_3 sq_5 + M_5\dot{q}_2^2 l_4 sq_5 - M_5\ddot{q}_3 sq_5 + M_5\dot{q}_2^2 l_5 sq_5 + M_5\dot{q}_2^2 l_6 sq_5 - M_5\ddot{q}_2 q_3 cq_5 sq_4 - \\
\quad - M_5\ddot{q}_2 l_4 cq_5 sq_4 - 2M_5\dot{q}_2\dot{q}_3 cq_5 sq_4 - M_5\ddot{q}_2 l_5 cq_5 sq_4 + M_5 g cq_4 cq_5 + M_5\ddot{q}_1 cq_5 cq_4 - \\
\quad - M_5\ddot{q}_2 l_6 sq_4 cq_5
\end{array}
\end{bmatrix}; \qquad (26)
$$

$$
\left[\overline{F}_l\right]_4^4=\left[\begin{array}{l}
F^{6}_{l_x}-M_5\ddot{q}_2q_3cq_4-M_5\ddot{q}_2l_4cq_4-2M_5\dot{q}_2\dot{q}_3cq_4-M_5\ddot{q}_2l_5cq_4-M_5gsq_4-M_5\ddot{q}_1sq_4-\\
\quad -M_5\ddot{q}_2l_6cq_4-M_4\ddot{q}_2q_3cq_4-M_4\ddot{q}_2l_4cq_4-2M_4\dot{q}_2\dot{q}_3cq_4-M_4\ddot{q}_2l_5cq_4-\\
\quad -M_4gsq_4-M_4\ddot{q}_1sq_4\\
\hdashline
-M_4\dot{q}_2^2l_5-M_4\dot{q}_2^2l_4-M_4\dot{q}_2^2q_3-M_5\ddot{q}_3-M_4\ddot{q}_3-F^{6}_{l_z}sq_5+F^{6}_{l_y}cq_5-M_5\dot{q}_2^2l_4-M_5\dot{q}_2^2l_5-\\
\quad -M_5\dot{q}_2^2l_6-M_5\dot{q}_2^2q_3\\
\hdashline
F^{6}_{l_z}cq_5+F^{6}_{l_y}sq_5+M_4\ddot{q}_1cq_4+M_4gcq_4-M_5\ddot{q}_2q_3sq_4-M_5\ddot{q}_2l_4sq_4-2M_5\dot{q}_2\dot{q}_3sq_4-\\
\quad -M_5\ddot{q}_2l_5sq_4-M_5\ddot{q}_2l_6sq_4-M_4\ddot{q}_2q_3sq_4-M_4\ddot{q}_2l_4sq_4-2M_4\dot{q}_2\dot{q}_3sq_4-\\
\quad -M_4\ddot{q}_2l_5sq_4+M_5gcq_4+M_5\ddot{q}_1cq_4
\end{array}\right]; \quad (27)
$$

$$
\left[\overline{F}_l\right]_3^3=\left[\begin{array}{l}
F^{6}_{l_x}cq_4-M_5\ddot{q}_2q_3-M_5\ddot{q}_2l_4-2M_5\dot{q}_2\dot{q}_3-M_5\ddot{q}_2l_5-M_5\ddot{q}_2l_6-M_4\ddot{q}_2q_3-M_4\ddot{q}_2l_4-\\
\quad -2M_4\dot{q}_2\dot{q}_3-M_4\ddot{q}_2l_5+F^{6}_{l_z}sq_4cq_5+F^{6}_{l_y}sq_4sq_5-M_3\ddot{q}_2q_3-M_3\ddot{q}_2l_4-\\
\quad -2M_3\dot{q}_2\dot{q}_3\\
\hdashline
-M_4\dot{q}_2^2l_5-M_4\dot{q}_2^2l_4-M_4\dot{q}_2^2q_3+M_5\ddot{q}_3+M_4\ddot{q}_3-F^{6}_{l_z}sq_5+F^{6}_{l_y}cq_5-M_5\dot{q}_2^2l_4-M_5\dot{q}_2^2l_5-\\
\quad -M_5\dot{q}_2^2l_6-M_5\dot{q}_2^2q_3-M_3\dot{q}_2^2q_3-M_3\dot{q}_2^2l_4+M_3\ddot{q}_3\\
\hdashline
M_5g+M_5\ddot{q}_1+M_4\ddot{q}_1+M_4g+F^{6}_{l_z}cq_4cq_5+F^{6}_{l_y}cq_4sq_5-F^{6}_{l_x}sq_4+M_3g+M_3\ddot{q}_1
\end{array}\right]; \quad (28)
$$

$$
\left[\overline{F}_l\right]_2^2=\left[\begin{array}{l}
F^{6}_{l_x}cq_4-M_5\ddot{q}_2q_3-M_5\ddot{q}_2l_4-2M_5\dot{q}_2\dot{q}_3-M_5\ddot{q}_2l_5-M_5\ddot{q}_2l_6-M_4\ddot{q}_2q_3-M_4\ddot{q}_2l_4-\\
\quad -2M_4\dot{q}_2\dot{q}_3-M_4\ddot{q}_2l_5+F^{6}_{l_z}sq_4cq_5+F^{6}_{l_y}sq_4sq_5-M_3\ddot{q}_2q_3-M_3\ddot{q}_2l_4-\\
\quad -2M_3\dot{q}_2\dot{q}_3\\
\hdashline
-M_4\dot{q}_2^2l_5-M_4\dot{q}_2^2l_4-M_4\dot{q}_2^2q_3+M_5\ddot{q}_3+M_4\ddot{q}_3-F^{6}_{l_z}sq_5+F^{6}_{l_y}cq_5-M_5\dot{q}_2^2l_4-M_5\dot{q}_2^2l_5-\\
\quad -M_5\dot{q}_2^2l_6-M_5\dot{q}_2^2q_3-M_3\dot{q}_2^2q_3-M_3\dot{q}_2^2l_4+M_3\ddot{q}_3\\
\hdashline
M_5g+M_5\ddot{q}_1+M_4\ddot{q}_1+M_4g+F^{6}_{l_z}cq_4cq_5+F^{6}_{l_y}cq_4sq_5-F^{6}_{l_x}sq_4+M_3g+M_3\ddot{q}_1+\\
\quad +M_2\ddot{q}_1+M_2g
\end{array}\right]; \quad (29)
$$

$$\left[\overline{F}_1\right]_1^1=\begin{bmatrix}
\begin{aligned}
&F_{l_x}^6 cq_2 cq_4 - M_5\ddot{q}_3 sq_2 - M_4\ddot{q}_3 sq_2 + F_{l_z}^6 sq_2 sq_5 - F_{l_y}^6 sq_2 cq_5 - M_3\ddot{q}_3 sq_2 - M_5\ddot{q}_2 q_3 cq_2 -\\
&\quad - M_5\ddot{q}_2 l_4 cq_2 - 2M_5\dot{q}_2\dot{q}_3 cq_2 - M_5\ddot{q}_2 l_5 cq_2 - M_5\ddot{q}_2 l_6 cq_2 - M_4\ddot{q}_2 q_3 cq_2 - M_4\ddot{q}_2 l_4 cq_2 -\\
&\quad - 2M_4\dot{q}_2\dot{q}_3 cq_2 - M_4\ddot{q}_2 l_5 cq_2 + F_{l_z}^6 cq_2 sq_4 cq_5 + F_{l_y}^6 cq_2 sq_4 sq_5 - M_3\ddot{q}_2 q_3 cq_2 -\\
&\quad - M_3\ddot{q}_2 l_4 cq_2 - 2M_3\dot{q}_2\dot{q}_3 cq_2 + M_4\dot{q}_2^2 l_5 sq_2 + M_4\dot{q}_2^2 l_4 sq_2 + M_4\dot{q}_2^2 q_3 sq_2 + M_5\dot{q}_2^2 l_4 sq_2 +\\
&\quad + M_5\dot{q}_2^2 l_5 sq_2 + M_5\dot{q}_2^2 l_6 sq_2 + M_5\dot{q}_2^2 q_3 sq_2 + M_3\dot{q}_2^2 q_3 sq_2 + M_3\dot{q}_2^2 l_4 sq_2
\end{aligned}\\
\begin{aligned}
&M_5\ddot{q}_3 cq_2 + F_{l_x}^6 sq_2 cq_4 + M_4\ddot{q}_3 cq_2 - F_{l_z}^6 cq_2 sq_5 + F_{l_y}^6 cq_2 cq_5 + M_3\ddot{q}_3 cq_2 - M_5\ddot{q}_2 q_3 sq_2 -\\
&\quad - M_5\ddot{q}_2 l_4 sq_2 - 2M_5\dot{q}_2\dot{q}_3 sq_2 - M_5\ddot{q}_2 l_5 sq_2 - M_5\ddot{q}_2 l_6 sq_2 - M_4\ddot{q}_2 q_3 sq_2 - M_4\ddot{q}_2 l_4 sq_2 -\\
&\quad - 2M_4\dot{q}_2\dot{q}_3 sq_2 - M_4\ddot{q}_2 l_5 sq_2 + F_{l_z}^6 sq_2 sq_4 cq_5 + F_{l_y}^6 sq_2 sq_4 sq_5 - M_3\ddot{q}_2 q_3 sq_2 -\\
&\quad - M_3\ddot{q}_2 l_4 sq_2 - 2M_3\dot{q}_2\dot{q}_3 sq_2 - M_4\dot{q}_2^2 l_5 cq_2 - M_4\dot{q}_2^2 l_4 cq_2 - M_4\dot{q}_2^2 q_3 cq_2 - M_5\dot{q}_2^2 l_4 cq_2 -\\
&\quad - M_5\dot{q}_2^2 l_5 cq_2 - M_5\dot{q}_2^2 l_6 cq_2 - M_5\dot{q}_2^2 q_3 cq_2 - M_3\dot{q}_2^2 q_3 cq_2 + M_3\dot{q}_2^2 l_4 cq_2
\end{aligned}\\
\begin{aligned}
&M_5 g + M_5\ddot{q}_1 + M_4\ddot{q}_1 + M_4 g + F_{l_z}^6 cq_4 cq_5 + F_{l_y}^6 cq_4 sq_5 - F_{l_x}^6 sq_4 + M_3 g + M_3\ddot{q}_1 +\\
&\quad + M_2\ddot{q}_1 + M_2 g + M_1 g + M_1\ddot{q}_1
\end{aligned}
\end{bmatrix}. \quad (30)$$

Determine the generalized motors forces, for which general expressions are:

$$Q_m^1 = \left[\overline{F}_{l_1}^1\right]^T \cdot \overline{k}_1^1 = \left[F_{l_x}^1 \quad F_{l_y}^1 \quad F_{l_z}^1\right] \cdot \begin{bmatrix}0\\0\\1\end{bmatrix} = F_{l_z}^1, \qquad (31)$$

$$Q_m^2 = \left[\overline{M}_{l_{O_2}}^2\right]^T \cdot \overline{k}_2^2 = \left[M_{l_x}^2 \quad M_{l_y}^2 \quad M_{l_z}^2\right] \cdot \begin{bmatrix}0\\0\\1\end{bmatrix} = M_{l_z}^2, \qquad (32)$$

$$Q_m^3 = \left[\overline{F}_{l_3}^3\right]^T \cdot \overline{j}_3^3 = \left[F_{l_x}^3 \quad F_{l_y}^3 \quad F_{l_z}^3\right] \cdot \begin{bmatrix}0\\1\\0\end{bmatrix} = F_{l_y}^3, \qquad (33)$$

$$Q_m^4 = \left[\overline{M}_{l_{O_4}}^4\right]^T \cdot \overline{j}_4^4 = \left[M_{l_x}^4 \quad M_{l_y}^4 \quad M_{l_z}^4\right] \cdot \begin{bmatrix}0\\1\\0\end{bmatrix} = M_{l_y}^4, \qquad (34)$$

$$Q_m^5 = \left[\overline{M}_{l_{O_5}}^5\right]^T \cdot \overline{i}_5^5 = \left[M_{l_x}^5 \quad M_{l_y}^5 \quad M_{l_z}^5\right] \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} = M_{l_x}^4, \qquad (35)$$

With the introduction of forces and moments components corresponding relations as demonstrated in this work should be obtained from relations (31), (32), (33), (34), (35) the expressions of the generalized motor forces. They are:

$$\begin{aligned}
Q_m^1 &= M_5 g + M_5\ddot{q}_1 + M_4\ddot{q}_1 + M_4 g + F_{l_z}^6 cq_4 cq_5 + F_{l_y}^6 cq_4 sq_5 - F_{l_x}^6 sq_4 + M_3 g + M_3\ddot{q}_1 +\\
&\quad + M_2\ddot{q}_1 + M_2 g + M_1 g + M_1\ddot{q}_1;
\end{aligned} \qquad (36)$$

$$\begin{aligned} Q_m^2 = {} & -J_x^{*5}\ddot{q}_2 c^2 q_4 + J_x^{*5}\ddot{q}_2 + J_x^{*4}\ddot{q}_2 + 2M_5\dot{q}_2\dot{q}_3 l_6 + 2M_5\ddot{q}_2 l_5 l_6 + 2M_4\dot{q}_2\dot{q}_3 l_5 - J_x^{*4}\ddot{q}_2 c^2 q_4 + \\ & + M_5\ddot{q}_2 l_6^2 + M_4\ddot{q}_2 l_5^2 + 2M_4\ddot{q}_2 l_5 l_4 + 2M_5\ddot{q}_2 q_3 l_6 + M_5\ddot{q}_2 l_5^2 + 2M_5\ddot{q}_2 q_3 l_4 + 2M_5\dot{q}_2\dot{q}_3 q_3 + \\ & + 2M_4\ddot{q}_2 q_3 l_4 + 2M_4\dot{q}_2\dot{q}_3 q_3 + 2M_3\ddot{q}_2 q_3 l_4 + 2M_3\dot{q}_2\dot{q}_3 q_3 + 2M_5\dot{q}_2\dot{q}_3 l_4 + 2M_4\dot{q}_2\dot{q}_3 l_4 + \\ & + 2M_3\dot{q}_2\dot{q}_3 l_4 + J_z^{*2}\ddot{q}_2 - J_x^{*5}\ddot{q}_2 sq_4 - F_{l_z}^5 l_7 sq_4 + J_z^{*4}\ddot{q}_2 c^2 q_4 + J_y^{*5}\ddot{q}_2 c^2 q_4 + M_{l_y}^6 cq_4 sq_5 + \\ & + M_{l_z}^6 cq_4 cq_5 - F_{l_x}^6 l_6 cq_4 - F_{l_x}^6 l_5 cq_4 + 2M_5\ddot{q}_2 l_4 l_6 + 2M_5\ddot{q}_2 q_3 l_5 + 2M_5\ddot{q}_2 l_4 l_5 + \\ & + 2M_4\ddot{q}_2 q_3 l_5 - M_{l_x}^6 sq_4 + 2M_5\dot{q}_2\dot{q}_3 l_5 - F_{l_z}^6 q_3 sq_4 cq_5 - F_{l_y}^6 q_3 sq_4 sq_5 - F_{l_z}^6 l_4 sq_4 cq_5 - \\ & - F_{l_y}^6 l_4 sq_4 sq_5 - F_{l_z}^6 l_6 sq_4 cq_5 - F_{l_y}^6 l_6 sq_4 sq_5 + J_z^{*5}\ddot{q}_2 c^2 q_4 c^2 q_5 - J_y^{*5}\ddot{q}_2 c^2 q_4 c^2 q_5 + \\ & + J_z^{*5}\dot{q}_4\dot{q}_5 cq_4 - F_{l_x}^6 l_7 cq_4 cq_5 - J_y^{*5}\dot{q}_4\dot{q}_5 cq_4 - J_x^{*5}\dot{q}_4\dot{q}_5 cq_4 - F_{l_z}^6 l_5 sq_4 cq_5 - F_{l_y}^6 l_5 sq_4 sq_5 + \\ & + J_z^{*3}\ddot{q}_2 - 2J_z^{*4}\dot{q}_2\dot{q}_4 sq_4 cq_4 + 2J_x^{*4}\dot{q}_2\dot{q}_4 sq_4 cq_4 + 2J_y^{*5}\dot{q}_2\dot{q}_4 c^2 q_5 sq_4 cq_4 - \\ & - 2J_z^{*5}\dot{q}_2\dot{q}_4 c^2 q_5 sq_4 cq_4 + 2J_x^{*5}\dot{q}_2\dot{q}_4 sq_4 cq_4 + J_z^{*5}\dot{q}_4^2 cq_5 sq_4 sq_5 - J_y^{*5}\dot{q}_4^2 sq_4 sq_5 cq_5 - \\ & - 2J_y^{*5}\dot{q}_2\dot{q}_4 sq_4 cq_4 + 2J_y^{*5}\dot{q}_2\dot{q}_5 c^2 q_4 sq_5 cq_5 - 2J_z^{*5}\dot{q}_2\dot{q}_5 c^2 q_4 sq_5 cq_5 + J_y^{*5}\ddot{q}_4 cq_4 sq_5 cq_5 - \\ & - 2J_z^{*5}\dot{q}_4\dot{q}_5 c^2 q_5 cq_4 + 2J_y^{*5}\dot{q}_2\dot{q}_5 c^2 q_5 cq_4 - J_z^{*5}\ddot{q}_4 cq_4 sq_5 cq_5 + M_5\ddot{q}_2 q_3^2 + M_4\ddot{q}_2 q_3^2 + \\ & + M_3\ddot{q}_2 q_3^2 + M_5\ddot{q}_2 l_4^2 + M_4\ddot{q}_2 l_4^2 + M_3\ddot{q}_2 l_4^2 - F_{l_x}^6 q_3 cq_4 - F_{l_x}^6 l_4 cq_4 \; ; \end{aligned} \tag{37}$$

$$\begin{aligned} Q_m^3 = {} & -M_4\dot{q}_2^2 l_5 - M_4\dot{q}_2^2 l_4 - M_4\dot{q}_2^2 q_3 + M_5\ddot{q}_3 + M_4\ddot{q}_3 - F_{l_z}^6 sq_5 + F_{l_y}^6 cq_5 - M_5\dot{q}_2^2 l_4 - M_5\dot{q}_2^2 l_5 - \\ & - M_5\dot{q}_2^2 l_6 - M_5\dot{q}_2^2 q_3 - M_3\dot{q}_2^2 q_3 - M_3\dot{q}_2^2 l_4 + M_3\ddot{q}_3; \end{aligned} \tag{38}$$

$$\begin{aligned} Q_m^4 = {} & -J_x^{*4}\dot{q}_2^2 cq_4 sq_4 + J_z^{*4}\dot{q}_2^2 cq_4 sq_4 + M_{l_y}^6 cq_5 - M_{l_y}^6 sq_5 + F_{l_x}^6 l_7 sq_5 + J_z^{*5}\dot{q}_2\dot{q}_5 cq_4 - J_x^{*5}\dot{q}_2^2 cq_4 sq_4 - \\ & - J_z^{*5}\ddot{q}_4 c^2 q_5 - 2J_z^{*5}\dot{q}_2\dot{q}_5 c^2 q_5 cq_4 + J_y^{*5}\ddot{q}_2 cq_5 sq_5 cq_4 - 2J_y^{*5}\dot{q}_4\dot{q}_5 cq_5 sq_5 + 2J_z^{*5}\dot{q}_4\dot{q}_5 cq_5 sq_5 + \\ & + J_y^{*5}\ddot{q}_4 c^2 q_5 + 2J_y^{*5}\dot{q}_2\dot{q}_5 c^2 q_5 cq_4 - J_z^{*5}\ddot{q}_2 cq_5 sq_5 cq_4 + J_z^{*5}\dot{q}_2^2 c^2 q_5 cq_4 sq_4 + J_z^{*5}\ddot{q}_4 + \\ & + J_x^{*5}\dot{q}_2\dot{q}_5 cq_4 + J_y^{*5}\dot{q}_2^2 cq_4 sq_4 - J_y^{*5}\dot{q}_2\dot{q}_5 cq_4 - J_y^{*5}\dot{q}_2^2 c^2 q_5 cq_4 sq_4 + J_y^{*4}\ddot{q}_4 \; ; \end{aligned} \tag{39}$$

$$\begin{aligned} Q_m^5 = {} & M_{l_x}^6 + F_{l_x}^6 l_7 - J_x^{*5}\ddot{q}_4 sq_4 - J_x^{*5}\dot{q}_2\dot{q}_4 cq_4 + J_x^{*5}\ddot{q}_5 - 2J_y^{*5}\dot{q}_2\dot{q}_4 c^2 q_5 cq_4 - J_y^{*5}\dot{q}_2^2 c^2 q_4 cq_5 sq_5 + \\ & + J_y^{*5}\dot{q}_4^2 sq_5 cq_5 + J_y^{*5}\dot{q}_2\dot{q}_4 cq_4 + 2J_z^{*5}\dot{q}_2\dot{q}_4 c^2 q_5 cq_4 - J_z^{*5}\dot{q}_4^2 sq_5 cq_5 + J_z^{*5}\dot{q}_2^2 c^2 q_4 cq_5 sq_5 - \\ & - J_z^{*5}\dot{q}_2\dot{q}_4 cq_4 . \end{aligned} \tag{40}$$

The system composed of relations (36), (37), (38), (39), (40) together form the system of dynamic differential equations as characterizes the dynamic model of the TRTRR modular serial robot.

## 3. CONCLUSIONS

To carry out a study of the dynamic movement of a body in motion, apply motion Center of mass theorem, known as Newton's equation and theorem of angular momentum with respect to the applied, i.e. the dynamics of Euler's equation. According to this method, for determining the dynamic equations of the robots apply d'Alembert's principle for every element in the structure of the mechanics of the robot.

The Newton-Euler formalism requires use of iterative calculation method based on the method – Walter Luh Pol [Fu 87]. Calculation algorithm consists of two parts, namely: iterations out mechanical robot structure, after which is determined for each element “i” (i = 1, ..., n) of the robot and accelerations speeds angular and linear elements, respectively torsor for reduction of external forces; iterations towards the inside of the mechanical structure of the robot, which is determined for each item "i" of the robot torsor of reduction for forces in the generalized forces couplers, engines of kinematic axis of the robot.

In the work are presented in detail on these calculations, in the end, the generalized forces of the engines expressions $Q_m^{(i)}$, i = 1, ..., 5.

Comparing the methods of dynamic study of industrial robots (formalism of Lagrange, the principle of virtual displacements, the Newton-Euler formalism), are found by applying Newton-Euler formalism can be determined from the installed adapters, robot reactions without frictions, while by other methods determine the equations of robot dynamic without taking into consideration the frictions, while the top methods determine the equations of robot dynamic without taking into consideration the frictions and reactions of the installation.

Newton-Euler method disadvantage is complexity obtained for the member deformations and generalized forces, making it cumbersome to use.

## REFERENCES


[Det07]. Deteşan, O.A., *“Cercetări privind modelarea, simularea şi construcția miniroboților”*, Teză de doctorat, Universitatea Tehnică din Cluj-Napoca, 2007.

[Fu87]. Fu, K., Gonzales, R., Lee, C., „*Robotics Control, Sensing, Vision and Intelligence*”, McGraw-Hill International Editions, 1987.

[Ghi04]. Ghinea, M., Firețeanu, V., „*Matlab. Calcul numeric. Grafică. Aplicații*” Editura Teora, Cluj-Napoca, 2004.

[Isp04]. Ispas, V., „*Manipulatoare şi roboți industriali*”, Editura Didactică şi Pedagogică, Bucureşti, 2004, ISBN 973-30-1349-8.

[Jen92]. Jennings, A., Mckeown, J.J., “*Matrix Computation*” Wiley and Son. Inc., New York, 1992.

[Mar93]. Marcus, M., “*Matrices and Matlab: a Tutorial*”, Prentince Hall, New Jersey, 1993.


## ECUAȚIILE DINAMICE ALE ROBOTULUI TRTRR UTILIZÂND FORMALISMUL NEWTON-EULER


***Rezumat:*** Modelarea dinamică a robotului TRTRR, a cărui schemă cinematică este prezentată în acest articol va fi realizată în conformitate cu literatura de specialitate prin aplicarea formalismului Newton-Euler, implementat în programul de modelare simbolică *Robot_Symbolic*, modulul *Robot_Dynamics* din cadrul programului MatLab 7.1.


.


**Ramona Maria GUI,** Dr. Eng., Cluj-Napoca;

**Viorel ISPAS,** Prof. Dr. Eng., Technical University of Cluj-Napoca, Department of Mechanical Engineering Systems, no. 103-105 B-dul Muncii, Cluj-Napoca, ROMANIA, e-mail: ispasviorel@yahoo.com, Office Tel: (+) 40 264 401 657;

**Mariana ARGHIR**, Prof. Dr. Eng., Technical University of Cluj-Napoca, Department of Mechanical Engineering Systems, no. 103-105 B-dul Muncii, Cluj-Napoca, ROMANIA, e-mail: marianaarghir@yahoo.com; Mariana.Arghir@mep.utcluj.ro.;

**Virgil ISPAS,** Conf. Dr. Eng., Dept. IPR, Technical University of Cluj-Napoca, ROMANIA,