TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 60, Issue I, March, 2017

DYNAMIC MODEL OF 5R ARTICULATED INDUSTRIAL ROBOT USED IN WELDING PROCESSES

Florin BUGNAR, Claudiu Mihai NEDEZKI, Adrian TRIF

Abstract: *the paper presents the determination of the dynamic equations for the 5R-type articulated industrial robot used in welding processes. Newton-Euler method was used to perform this modeling and it works with the previously determined equations of geometric and kinematic model. The generalized driving forces are eventually determined, and they express the equation of the inverse dynamic model for the analyzed robot.*
Key words: *dynamic model, articulated robot, welding.*

1. INTRODUCTION

The articulated 5R industrial robot is presented as a kinematic diagram in fig.1, and the dynamic modeling will be accomplished using Newton-Euler's formulation, according to [1], [2] and [3].

In order to apply Newton-Euler's formulation, the geometric [4] and kinematic [5] models were previously determined.

2. MASS DISTRIBUTION PARAMETERS

Along with the equations of geometric and kinematic model, the mass distribution parameters are also required prior dynamic modeling, as well as stating of some simplifying hypotheses, as follows:

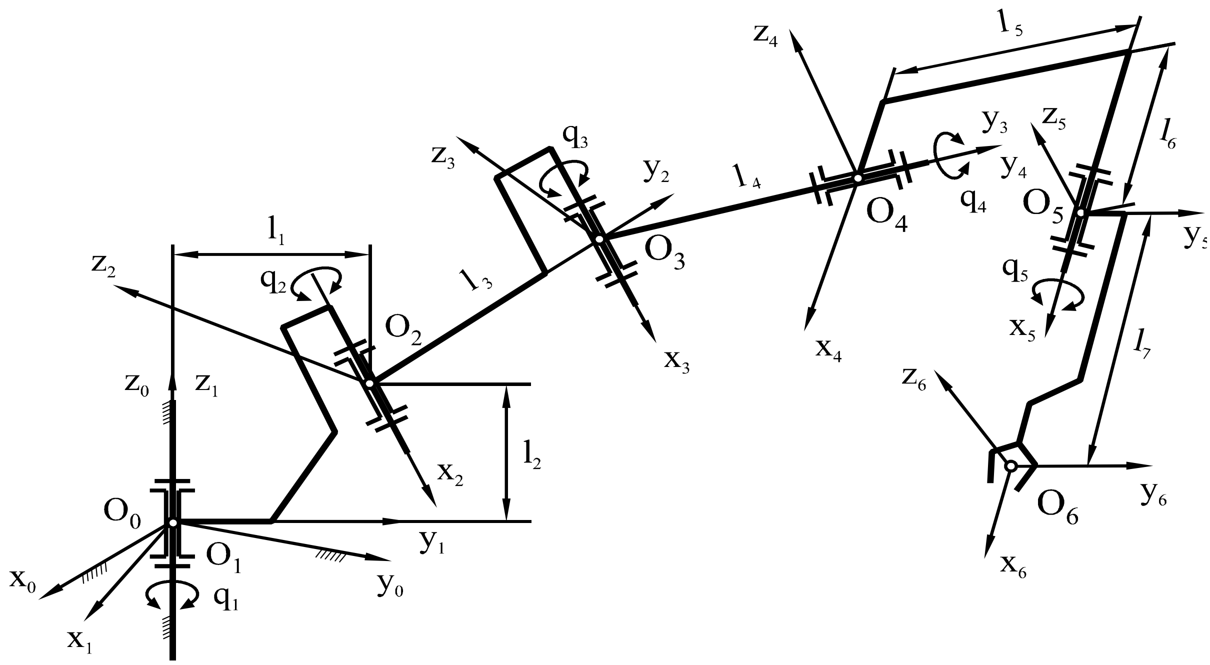

Fig. 1. The kinematic diagram of the 5R modular industrial robot

- the mass centers C_i will be chosen such as to coincide with the origins O_i of the Cartesian frames $O_ix_iy_iz_i$, $i = 1 \div 5$, leading to position vectors of the mass centers of zero modules;
- choosing the mobile frames in such a way that their axes will overlap the principal directions of inertia corresponding to the origins of these frames, resulting in the cancelling of the centrifugal mechanical moments of inertia.

The needed mass distribution parameters are the following:

- the masses defining the structure of the articulated robot: M_1, M_2, M_3, M_4, M_5;
- the mass centers of the 5R articulated robot:

$$\overline{r}_{c_1}^{1} = \begin{bmatrix} r_{c_{1x}}^{1} \\ r_{c_{1y}}^{1} \\ r_{c_{1z}}^{1} \end{bmatrix},\ \overline{r}_{c_2}^{2} = \begin{bmatrix} r_{c_{2x}}^{2} \\ r_{c_{2y}}^{2} \\ r_{c_{2z}}^{2} \end{bmatrix},\ \overline{r}_{c_3}^{3} = \begin{bmatrix} r_{c_{3x}}^{3} \\ r_{c_{3y}}^{3} \\ r_{c_{3z}}^{3} \end{bmatrix}, \quad (1)$$

$$\overline{r}_{c_4}^{4} = \begin{bmatrix} r_{c_{4x}}^{4} \\ r_{c_{4y}}^{4} \\ r_{c_{4z}}^{4} \end{bmatrix},\ \overline{r}_{c_5}^{5} = \begin{bmatrix} r_{c_{5x}}^{5} \\ r_{c_{5y}}^{5} \\ r_{c_{5z}}^{5} \end{bmatrix}; \quad (2)$$

- the inertial tensors:

$$J_1^{*1} = \begin{bmatrix} J_x^{*1} & -J_{xy}^{*1} & -J_{xz}^{*1} \\ -J_{yx}^{*1} & J_y^{*1} & -J_{yz}^{*1} \\ -J_{zx}^{*1} & -J_{zy}^{*1} & J_z^{*1} \end{bmatrix}, \quad (3)$$

$$J_2^{*2} = \begin{bmatrix} J_x^{*2} & -J_{xy}^{*2} & -J_{xz}^{*2} \\ -J_{yx}^{*2} & J_y^{*2} & -J_{yz}^{*2} \\ -J_{zx}^{*2} & -J_{zy}^{*2} & J_z^{*2} \end{bmatrix}, \quad (4)$$

$$J_3^{*3} = \begin{bmatrix} J_x^{*3} & -J_{xy}^{*3} & -J_{xz}^{*3} \\ -J_{yx}^{*3} & J_y^{*3} & -J_{yz}^{*3} \\ -J_{zx}^{*3} & -J_{zy}^{*3} & J_z^{*3} \end{bmatrix}, \quad (5)$$

$$J_4^{*4} = \begin{bmatrix} J_x^{*4} & -J_{xy}^{*4} & -J_{xz}^{*4} \\ -J_{yx}^{*4} & J_y^{*4} & -J_{yz}^{*4} \\ -J_{zx}^{*4} & -J_{zy}^{*4} & J_z^{*4} \end{bmatrix}, \quad (6)$$

$$J_5^{*5} = \begin{bmatrix} J_x^{*5} & -J_{xy}^{*5} & -J_{xz}^{*5} \\ -J_{yx}^{*5} & J_y^{*5} & -J_{yz}^{*5} \\ -J_{zx}^{*5} & -J_{zy}^{*5} & J_z^{*5} \end{bmatrix}. \quad (7)$$

The components of the inertial tensors situated on the main diagonal J_x^{*i}, J_y^{*i}, J_z^{*i}, i = 1, 2, 3, 4, 5 are the axial mechanical moments of inertia, expressed with respect to the frame i, with the origin into the mass center C_i and having the same orientation with the frame attached to each link of the robot.

The accelerations corresponding to the mass centers are determined, according to [6], [7] and [8], with the following relation:

$$\overline{a}_{c_i}^{i} = \overline{a}_i^{i} + \overline{\varepsilon}_i^{i} \times \overline{r}_{c_i}^{i} + \overline{\omega}_i^{i} \times \left(\overline{\omega}_i^{i} \times \overline{r}_{c_i}^{i}\right). \quad (8)$$

The following accelerations yield:

$$\overline{a}_{c_1}^{1} = \overline{a}_1^{1} + \overline{\varepsilon}_1^{1} \times \overline{r}_{c_1}^{1} + \overline{\omega}_1^{1} \times \left(\overline{\omega}_1^{1} \times \overline{r}_{c_1}^{1}\right), \quad (9)$$

$$[\overline{a}_c]_1^1 = \begin{bmatrix} -\dot{q}_1^2 r_{c_{1x}}^1 - \ddot{q}_1 r_{c_{1y}}^1 \\ \ddot{q}_1 r_{c_{1x}}^1 - \dot{q}_1^2 r_{c_{1y}}^1 \\ g \end{bmatrix}, \quad (10)$$

$$\overline{a}_{c_2}^{2} = \overline{a}_2^{2} + \overline{\varepsilon}_2^{2} \times \overline{r}_{c_2}^{2} + \overline{\omega}_2^{2} \times \left(\overline{\omega}_2^{2} \times \overline{r}_{c_2}^{2}\right), \quad (11)$$

$$[\overline{a}_c]_2^2 = \begin{bmatrix} \ddot{q}_1 s q_2 r_{c_{2z}}^2 - \dot{q}_1^2 r_{c_{2x}}^2 - \ddot{q}_1 c q_2 r_{c_{2y}}^2 - \\ - \ddot{q}_1 l_1 + 2\dot{q}_1 \dot{q}_2 c q_2 r_{c_{2z}}^2 + 2\dot{q}_{12} \dot{q}_2 s q_2 r_{c_{2y}}^2 \\ \hdashline g s q_2 - \dot{q}_2^2 r_{c_{2y}}^2 - \ddot{q}_2 r_{c_{2z}}^2 + \ddot{q}_1 c q_2 r_{c_{2x}}^2 + \\ + \dot{q}_1^2 l_1 c q_2 - \dot{q}_1^2 c q_2^2 r_{c_{2y}}^2 + \dot{q}_1^2 c q_2 s q_2 r_{c_{2z}}^2 \\ \hdashline \ddot{q}_2 r_{c_{2y}}^2 + g c q_2 - \dot{q}_2^2 r_{c_{2z}}^2 - \ddot{q}_1 s q_2 r_{c_{2x}}^2 + \\ + \dot{q}_1^2 l_1 s q_2 - \dot{q}_1^2 s q_2^2 r_{c_{2z}}^2 + \dot{q}_1^2 c q_2 s q_2 r_{c_{2y}}^2 \end{bmatrix}. \quad (12)$$

Because the complexity of the mass centers accelerations equations of the links 3, 4, 5, they cannot be presented in this paper, but they can be sent upon request, via email to the main author. The computation relations are the following:

$$\overline{a}_{c_3}^{3} = \overline{a}_3^{3} + \overline{\varepsilon}_3^{3} \times \overline{r}_{c_3}^{3} + \overline{\omega}_3^{3} \times \left(\overline{\omega}_3^{3} \times \overline{r}_{c_3}^{3}\right), \quad (13)$$

$$\overline{a}_{c_4}^{4} = \overline{a}_4^{4} + \overline{\varepsilon}_4^{4} \times \overline{r}_{c_4}^{4} + \overline{\omega}_4^{4} \times \left(\overline{\omega}_4^{4} \times \overline{r}_{c_4}^{4}\right), \quad (14)$$

$$\overline{a}^5_{c_5} = \overline{a}^5_5 + \overline{\varepsilon}^5_5 \times \overline{r}^5_{c_5} + \overline{\omega}^5_5 \times \left(\overline{\omega}^5_5 \times \overline{r}^5_{c_5}\right). \quad (15)$$

3. ITERATIONS OUTWARDS THE MECHANICAL STRUCTURE

Using Newton-Euler's formulations, the robot mechanical structure is passed by both outward and inward iterations. In the first computation stage, iterations outwards the mechanical structure, the system of external forces and their moments are determined, according to [9], [10] and [11]:

$$\begin{aligned} \overline{R}^i_i &= M_i \overline{a}^i_{c_i} \\ \overline{M}^i_{c_i} &= J^{*i}_i \overline{\varepsilon}^i_i + \overline{\omega}^i_i \times J^{*i}_i \overline{\omega}^i_i \end{aligned} . \quad (16)$$

The following equation can be written for the first link:

$$\left[\overline{F}\right]^1_1 = M_1 \left[\overline{a}_c\right]^1_1, \quad (17)$$

which, according to (10), leads to:

$$\overline{F}^1_1 = \begin{bmatrix} -M_1\left(\dot{q}^2_1 r^1_{c_{1x}} + \ddot{q}_1 r^1_{c_{1y}}\right) \\ M_1\left(\ddot{q}_1 r^1_{c_{1x}} - \dot{q}^2_1 r^1_{c_{1y}}\right) \\ M_1 g \end{bmatrix}. \quad (18)$$

For the second link:

$$\overline{F}^2_2 = M_2 \left[\overline{a}_c\right]^2_2, \quad (19)$$

where, introducing (12), it generates:

$$\left[\overline{F}\right]^2_2 = \begin{bmatrix} -M_2(\ddot{q}_1 l_1 + \dot{q}^2_1 r^2_{c_{2x}} + \ddot{q}_1 cq_2 r^2_{c_{2y}} - \\ -\ddot{q}_1 sq_2 r^2_{c_{2z}} - 2\dot{q}_1 \dot{q}_2 cq_2 r^2_{c_{2z}} - 2\dot{q}_1 \dot{q}_2 sq_2 r^2_{c_{2y}}) \\ -M_2(\ddot{q}_2 r^2_{c_{2z}} + \dot{q}^2_2 r^2_{c_{2y}} - gsq_2 - \ddot{q}_1 cq_2 r^2_{c_{2x}} + \\ + \dot{q}^2_1 l_1 cq_2 + \dot{q}^2_1 cq^2_2 r^2_{c_{2y}} - \dot{q}^2_1 cq_2 sq_2 r^2_{c_{2z}}) \\ M_2(\ddot{q}_2 r^2_{c_{2y}} + gcq_2 - \dot{q}^2_2 r^2_{c_{2z}} - \ddot{q}_1 sq_2 r^2_{c_{2x}} + \\ + \dot{q}^2_1 l_1 sq_2 - \dot{q}^2_1 sq^2_2 r^2_{c_{2z}} + \dot{q}^2_1 cq_2 sq_2 r^2_{c_{2y}}) \end{bmatrix}. \quad (20)$$

The expressions of the external forces for the other three links are very complex, difficult to display and they can be sent by email at the reader's request. They are determined using the following equations:

$$\left[\overline{F}\right]^3_3 = M_3 \left[\overline{a}_c\right]^3_3, \quad (21)$$

$$\left[\overline{F}\right]^4_4 = M_4 \left[\overline{a}_c\right]^4_4, \quad (22)$$

$$\left[\overline{F}\right]^5_5 = M_5 \left[\overline{a}_c\right]^5_5, \quad (23)$$

where the computed expressions of the mass acceleration centers given by (13), (14) and (15) where replaced.

The moments of the external forces are obtained using the second equation from (16) and for the first link it has the following shape:

$$\overline{M}^1_{c_1} = J^{*1}_1 \overline{\varepsilon}^1_1 + \overline{\omega}^1_1 \times J^{*1}_1 \overline{\omega}^1_1, \quad (24)$$

where, by replacing the inertial tensor, expressed by (3), the equation leads to:

$$\left[\overline{M}_c\right]^1_1 = \begin{bmatrix} \dot{q}^2_1 J^{*1}_{yz} - \ddot{q}_1 J^{*1}_{xz} \\ -\dot{q}^2_1 J^{*1}_{xz} - \ddot{q}_1 J^{*1}_{yz} \\ \ddot{q}_1 J^{*1}_z \end{bmatrix}. \quad (25)$$

For the second link the following equation applies:

$$\overline{M}^2_{c_2} = J^{*2}_2 \overline{\varepsilon}^2_2 + \overline{\omega}^2_2 \times J^{*2}_2 \overline{\omega}^2_2, \quad (26)$$

leading to:

$$\left[\overline{M}_c\right]^2_2 = \begin{bmatrix} \ddot{q}_2 J^{*2}_x + \dot{q}_2\left(\dot{q}_1 cq_2 J^{*2}_{yx} - \dot{q}_1 sq_2 J^{*2}_{zx}\right) - J^{*2}_{xy}\left(\ddot{q}_1 sq_2 + \dot{q}_1 \dot{q}_2 cq_2\right) - J^{*2}_{xz}\left(\ddot{q}_1 cq_2 - \dot{q}_1 \dot{q}_2 sq_2\right) - \\ -\dot{q}_1 sq_2\left(J^{*2}_y \dot{q}_1 cq_2 + J^{*2}_{zy} \dot{q}_1 sq_2\right) + \dot{q}_1 cq_2\left(J^{*2}_{yz} \dot{q}_1 cq_2 + J^{*2}_z \dot{q}_1 sq_2\right) \\ \dot{q}_2\left(\dot{q}_2 J^{*2}_{zx} + \dot{q}_1 cq_2 J^{*2}_x\right) - \ddot{q}_2 J^{*2}_{yx} + J^{*2}_y\left(\ddot{q}_1 sq_2 + \dot{q}_1 \dot{q}_2 cq_2\right) - J^{*2}_{yz}\left(\ddot{q}_1 cq_2 - \dot{q}_1 \dot{q}_2 sq_2\right) - \\ -\dot{q}_1 cq_2\left(\dot{q}_2 J^{*2}_z + \dot{q}_1 cq_2 J^{*2}_{xz}\right) + \dot{q}_1 sq_2\left(\dot{q}_2 J^{*2}_{zy} - \dot{q}_1 cq_2 J^{*2}_{xy}\right) \\ J^{*2}_z\left(\ddot{q}_1 cq_2 - \dot{q}_1 \dot{q}_2 sq_2\right) - \dot{q}_2\left(\dot{q}_2 J^{*2}_{yx} + \dot{q}_1 sq_2 J^{*2}_x\right) - \ddot{q}_2 J^{*2}_{zx} - J^{*2}_{zy}\left(\ddot{q}_1 sq_2 + \dot{q}_1 \dot{q}_2 cq_2\right) - \\ -\dot{q}_1 cq_2\left(J^{*2}_{yz} \dot{q}_2 - J^{*2}_{xz} \dot{q}_1 sq_2\right) + \dot{q}_1 sq_2\left(\dot{q}_2 J^{*2}_y + J^{*2}_{xy} \dot{q}_1 sq_2\right) \end{bmatrix}. \quad (27)$$

The equations for the last three links are very complex as well and they could be sent by request, in the same way as the previous complex results.

4. ITERATIONS INWARDS THE MECHANICAL STRUCTURE

The torsor of the connection forces and their moments and the generalized driving forces from the robot joints are determined with the second algorithm of Newton-Euler's formulation, consisting in iterations inwards the robot's mechanical structure.

The connection forces and their moments have, according to [12] and [13], the following general computation relations:

$$\begin{aligned}&\overline{F}_{l_i}^{i} = M_i \overline{a}_{c_i} - \overline{F}_i^{i} - [R]_{i+1}^{i} \overline{F}_{l_{i+1}}^{i+1}\\ &\overline{M}_{l_{O_i}}^{i} = \overline{r}_{c_i} \times M_i \overline{a}_{c_i} + J_i^{*} \overline{\varepsilon}_i + \overline{\omega}_i \times J_i^{*} \overline{\omega}_i -\\ &- \overline{M}_{c_i}^{i} - \overline{r}_{c_i} \times \overline{F}_i^{i} - [R]_{i+1}^{i} \overline{M}_{l_{O_{i+1}}}^{i+1} -\\ &- \overline{r}_{i+1}^{i} \times [R]_{i+1}^{i} \overline{F}_{l_{i+1}}^{i}.\end{aligned} \quad (28)$$

For the 5R articulated robot, the connection forces are computed with:

$$\overline{F}_{l_5}^{5} = [R]_6^5 \cdot \overline{F}_{l_6}^{6} + \overline{F}_5^{5}, \quad (29)$$

$$\overline{F}_{l_4}^{4} = [R]_5^4 \cdot \overline{F}_{l_5}^{5} + \overline{F}_4^{4}, \quad (30)$$

$$\overline{F}_{l_3}^{3} = [R]_4^3 \cdot \overline{F}_{l_4}^{4} + \overline{F}_3^{3}, \quad (31)$$

$$\overline{F}_{l_2}^{2} = [R]_3^2 \cdot \overline{F}_{l_3}^{3} + \overline{F}_2^{2}, \quad (32)$$

$$\overline{F}_{l_1}^{1} = [R]_2^1 \cdot \overline{F}_{l_2}^{2} + \overline{F}_1^{1}, \quad (33)$$

where the external forces will be replaced by (23), (22), (21), (20) and (18).

The moments of the connection forces, according to (28) are:

$$\overline{M}_{l_{O_5}}^{5} = [R]_6^5 \cdot \overline{M}_{l_{O_6}}^{6} + \overline{r}_{c_5}^{5} \times \overline{F}_5^{5} + \overline{r}_6^{5} \times [R]_6^5 \cdot \overline{F}_{l_6}^{6} + \overline{M}_{c_5}^{5}, \quad (34)$$

$$\overline{M}_{l_{O_4}}^{4} = [R]_5^4 \cdot \overline{M}_{l_{O_5}}^{5} + \overline{r}_{c_4}^{4} \times \overline{F}_4^{4} + \overline{r}_5^{4} \times [R]_5^4 \cdot \overline{F}_{l_5}^{5} + \overline{M}_{c_4}^{4}, \quad (35)$$

$$\overline{M}_{l_{O_3}}^{3} = [R]_4^3 \cdot \overline{M}_{l_{O_4}}^{4} + \overline{r}_{c_3}^{3} \times \overline{F}_3^{3} + \overline{r}_4^{3} \times [R]_4^3 \cdot \overline{F}_{l_4}^{4} + \overline{M}_{c_3}^{3}, \quad (36)$$

$$\overline{M}_{l_{O_2}}^{2} = [R]_3^2 \cdot \overline{M}_{l_{O_3}}^{3} + \overline{r}_{c_2}^{2} \times \overline{F}_2^{2} + \overline{r}_3^{2} \times [R]_3^2 \cdot \overline{F}_{l_3}^{3} + \overline{M}_{c_2}^{2}, \quad (37)$$

$$\overline{M}_{l_{O_1}}^{1} = [R]_2^1 \cdot \overline{M}_{l_{O_2}}^{2} + \overline{r}_{c_1}^{1} \times \overline{F}_1^{1} + \overline{r}_2^{1} \times [R]_2^1 \cdot \overline{F}_{l_2}^{2} + \overline{M}_{c_1}^{1}. \quad (38)$$

Because of the high complexity of the expressions of connection forces and of their moments, they will be presented in a future paper. The generalized driving forces Q_m^i are determined according to [14], [15] and [16] this way:

$$Q_m^i = \Delta_i \left[M_{l_{O_i}}^{i}\right]^T \cdot \overline{k}_i^{i} + (1-\Delta_i)\left[\overline{F}_{l_i}^{i}\right]^T \cdot \overline{k}_i^{i} + Q_f^i. \quad (39)$$

In this approach, Q_f^i is the generalized force due to the frictions ad it is assumed to be zero. The expressions of these forces represent the dynamic differential equation system, characterizing the dynamic behavior of the considered 5R articulated robot.

Having the expressions of the axes versors deducted from the kinematic diagram:

$$\overline{k}_1^{1} = \begin{bmatrix}0\\0\\1\end{bmatrix},\ \overline{i}_2^{2} = \begin{bmatrix}1\\0\\0\end{bmatrix},\ \overline{i}_3^{3} = \begin{bmatrix}1\\0\\0\end{bmatrix},\ \overline{j}_4^{4} = \begin{bmatrix}0\\1\\0\end{bmatrix},\ \overline{i}_5^{5} = \begin{bmatrix}1\\0\\0\end{bmatrix}, \quad (40)$$

the generalized driving forces in the case of the 5R articulated robot can be expressed as:

$$Q_m^1 = \left[\overline{M}_{l_{O_1}}^{1}\right]^T \cdot \overline{k}_1^{1} = \left[M_{l_x}^{1} \quad M_{l_y}^{1} \quad M_{l_z}^{1}\right] \cdot \begin{bmatrix}0\\0\\1\end{bmatrix} = M_{l_z}^{1}, \quad (41)$$

$$Q_m^2 = \left[\overline{M}_{l_{O_2}}^{2}\right]^T \cdot \overline{i}_2^{2} = \left[M_{l_x}^{2} \quad M_{l_y}^{2} \quad M_{l_z}^{2}\right] \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} = M_{l_x}^{2}, \quad (42)$$

$$Q_m^3 = \left[\overline{M}_{l_{O_3}}^{3}\right]^T \cdot \overline{i}_3^{3} = \left[M_{l_x}^{3} \quad M_{l_y}^{3} \quad M_{l_z}^{3}\right] \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} = M_{l_x}^{3}, \quad (43)$$

$$Q_m^4 = \left[\overline{M}_{l_{O_4}}^{4}\right]^T \cdot \overline{j}_4^{4} = \left[M_{l_x}^{4} \quad M_{l_y}^{4} \quad M_{l_z}^{4}\right] \cdot \begin{bmatrix}0\\1\\0\end{bmatrix} = M_{l_y}^{4}, \quad (44)$$

$$Q_m^5 = \left[\overline{M}_{l_{O_5}}^{5}\right]^T \cdot \overline{i}_5^{5} = \left[M_{l_x}^{5} \quad M_{l_y}^{5} \quad M_{l_z}^{5}\right] \cdot \begin{bmatrix}1\\0\\0\end{bmatrix} = M_{l_x}^{5}. \quad (45)$$

Because all the joints of the robot are joints of rotation, the generalized driving forces are in fact moments (torques). The complex system of dynamic equations (41)-(45) represent the equations of the inverse dynamic model of the 5R articulated robot.

5. CONCLUSIONS

By means of the dynamic study performed on the 5R articulated industrial robot, the actuators of the robot can be chosen in order to achieve the planned task, as in [17]. The dynamic results obtained in this paper were possible using the Symbolic Computation Toolbox in a MATLAB script. They can be compared for exactness with the results from Robotic Toolbox [15], for the same mechanical structure, presented in figure 1.

6. REFERENCES

[1] Negrean, I., Duca, A., Negrean, C., Kacso, K., *Mecanică avansată în robotică*, Editura U.T. PRESS, Cluj-Napoca, 2008, ISBN 978-973-662-420-9.

[2] Deteşan, O.A., *Cercetări privind modelarea, simularea şi construcţia miniroboţilor*, Ph.D. Thesis, Technical University of Cluj-Napoca, 2007.

[3] Deteşan, O., Ispas, V., Jucan, G., *The Dynamic Study of VIPAS 2 Industrial Robot Using Newton-Euler's Formulation*, The 9th International Conference of Mechanisms and Mechanical Transmissions, MTM 2004, Acta Technica Napocensis, Series: Applied Mathematics and Mechanics vol. 47, no.I, pp.643-648, Cluj-Napoca, 2004, ISSN 1221-5872.

[4] Bugnar, F., Deteşan, O.A., Ispas, V., *Contributions to the Geometric Modeling of Industrial Modular, Articulated, Serial Robots with Five Degrees of Freedom, Using Symbolic Computation*, Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, Vol. 55, issue III, pp. 719-724, Cluj-Napoca, 2012, ISSN 1221-5872.

[5] Bugnar, F., Trif, A., Nedezki, C.M., *The Kinematic Model of 5R Articulated Industrial Robot Used in Welding Processes*, Academic Journal of Manufacturing Engineering, Vol. 14, issue IV, 6 pag., Cluj-Napoca, 2016, ISSN 1583-7904.

[6] Costin, I.O., Deteşan, O., Zdroba, A.D., *Dynamic Model of the TRR-type Modular Industrial Robot Using Newton-Euler's Formulation,* Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, No.42, Cluj-Napoca, 1999, pp. 33-40, ISSN 1221-5872.

[7] Ispas, V., Ispas, Vrg., Simion, M., Deteşan, O.A., *Contributions of the Dynamic Study of a Modular Serial Industrial Robot of TRTRR Type*, published in: *SYROM 2009. Proceedings of the 10th IFToMM International Symposium on Science of Mechanisms and Machines, 2009*, Ed. Springer Netherlands, pp.299-311, ISBN 978-90-481-3521-9.

[8] Deteşan, O., Ispas, Srg., *Algorithm for Dynamic Study of Industrial Robots with Three Degrees of Freedom,* Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, Vol.42, Cluj-Napoca, 1999, pp. 19-26, ISSN 1221-5872.

[9] Deteşan, O., Ispas, V., *The Dynamic Study of VIPAS 2 Industrial Robot Using Newton-Euler's Formulation. Part I. Determination of the External Forces and Moments*, Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, vol. 47, no.II, Cluj-Napoca, 2005, ISSN 1221-5872.

[10] Ispas, V., Deteşan, O., *Kinematic and Dynamic Modelling of the RTTR Industrial Robot of Modular Construction,* Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, No.41, pp. 37-42, Cluj-Napoca, 1998, ISSN 1221-5872.

[11] Deteşan, O.A., Vahnovanu, A.D., *The Dynamic Model of RTTRR Serial Robot – Iterations Outwards and Inwards the Robot's Mechanical Structure,* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering, Vol. 59, Issue 1, pp. 25-32, Cluj-Napoca, 2016, ISSN 1221-5872.

[12] Deteşan, O., *Approach to the Kinematics and Dynamics of the Modular Industrial Robot of TRR Type,* Annals of DAAAM for 1998 & Proceedings of the 9th International

DAAAM Symposium, Vienna, Austria, 1998, pp. 163-164, ISBN 3-901509-08-9.
[13] Deteşan, O.A., Gui, R.M., Ispas, Vrg., Ispas, V., *The Dynamic Model of TRTTR1 Modular Serial Robot,* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering, vol. 57, issue. III, p. 367 - 378, Cluj-Napoca, 2014, ISSN 1221-5872.
[14] Gui, R.M., Deteşan, O.A., Ispas, V., *The Dynamic Equations of the TRTR Robot Using the Newton-Euler Method,* Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, vol. 53, no. III, p. 37 - 42, Cluj-Napoca, 2010, ISSN 1221-5872 .
[15] Deteşan, O.A., *The Dynamic Modelling of the Robot Mechanical Structure Using the Symbolic Computation in MATLAB,* Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, vol. 56, issue. IV, pp. 659-664, Cluj-Napoca, 2013, ISSN 1221-5872.
[16] Deteşan, O.A., *The Dynamic Model of RTTRR Serial Robot – The Determination of the Generalized Driving Forces,* Acta Technica Napocensis, Series: Applied Mathematics, Mechanics and Engineering, vol. 59, no. I, pp.33-38, Cluj-Napoca, 2016, ISSN 1221-5872.
[17] Gui, R.M., Ispas, V., Ispas, Vrg., Detesan, O., *Choosing the Actuators for the TRTTR1 Modular Serial Robot,* 3rd European Conference on Mechanism Science EUCOMES 2010, published in: *New Trends in Mechanism Science. Analysis and Design*, Editors: Pisla, D., Ceccarelli, M., Husty, M., Corves, B., Ed. Springer Netherlands, pp.665-672, ISBN 978-90-481-9688-3, DOI: 10.1007/978-90-481-9689-0_56.

Modelul dinamic al robotului industrial articulat 5R utilizat în procese de sudură

Rezumat: Lucrarea prezintă determinarea ecuațiilor dinamice pentru robotul industrial articulat de tipul 5R, utilizat în procese de sudură. În vederea modelării dinamice a fost folosită metoda Newton-Euler, aceasta preluând ecuațiile modelului geometric și cinematic, care au fost determinate în prealabil. În finalul lucrării sunt determinate forțele generalizate motoare, acestea constituind ecuațiile modelului dinamic invers al robotului analizat.

Florin BUGNAR, Ph.D. Eng., Associated Senior Lecturer**,** Technical University of Cluj-Napoca, B-dul Muncii, no. 103-105, e-mail: bugnarf@hotmail.com, Phone: +40743111693, Cluj-Napoca, Romania.
Claudiu Mihai NEDEZKI, Ph.D. Eng., Associate Professor, Technical University of Cluj-Napoca, Department of Engineering and Robotics, e-mail: claudiu_nedezki@yahoo.com, Office Phone +40264401639, Home Address: Calea Turzii street, no. 67, Home Phone +40264440639.
Adrian TRIF, Ph.D. Eng., Senior Lecturer, Technical University of Cluj-Napoca, Department of Manufacturing Engineering, e-mail: adrian.trif@tcm.utcluj.ro, Office Phone +40264401614; Home Address Răsăritului Street, no 102/11, 400587, Cluj-Napoca, Romania, Home Phone +40264419601.