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KINEMATICAL CONTROL FOR A MOBILE ROBOT

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Abstract: *Unlike the serial robots, in the kinematics and dynamics of the mobile robots the mathematical models are different, in the first time due to nonholonomic links. As a result, the kinematics and dynamics equations will be computed by means of the motion restrictions. In this paper will be applied matrix transformations and exponentials for define the forward kinematics model and control functions corresponding to the mobile robots.*

Key words: *mobile robots, kinematics, robot control, constraints, motion trajectories.*

1. INTRODUCTION

The mobile robots represent a different category of robots that appeared as a result of the necessity to build new transport systems. In the present days, the mobile robots become more wide spread as they are used in the military applications, demining, nuclear engineering, transport operations, manipulating, planetary and cosmic explorations. The kinematic control function of the mobile robot allows it to describe certain motion trajectories, to displace from one point, considered as the initial position, to a programmed (final) position. For achieve of this task, the kinematic control function are applied. They assure along trajectory a continuous control about the position and orientation, as well as velocities and accelerations. So, the geometrical and kinematic modeling of the mechanical structure for the mobile robot is fundamental.

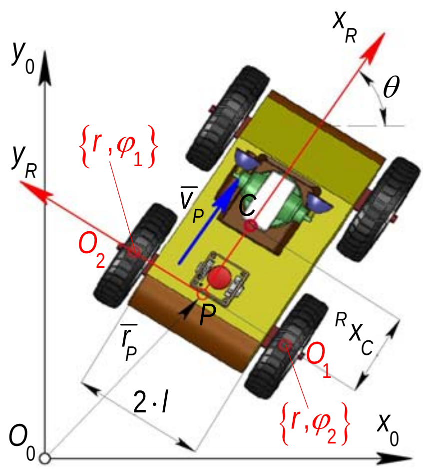

Fig. 1. The structure of the mobile robot

Kinematics of the mobile robots represents the study of the position and motion estimation which can be set or read from the internal sensors. The process of understanding robot motion is based on each wheel, but a few wheels impose constraints about the robot motion. According to [1], [2] the motion will be expressed in the two global frames, symbolized in Fig.1 as: $\{0\}$ the fixed frame; $\{R\}$ the robot's (moving) frame.

2. KINEMATIC MODELING

The robot position in the fixed frame or a position of an object into the robot path must be specified to a $\{R\}$ robot's frame, which has the origin in P, the center of wheel axis and the robot center. In this paper is analyzed the case of the two wheel axes. The moving frame is chosen as to be at half distance between them.

2.1 Kinematic Constraints

The motion law in the fixed frame (global frame) {0} is defined the expression:

$$\underset{(3\times1)}{\overline{X}(t)} = \begin{bmatrix} \overline{r}_p(t) \\ \theta(t) \end{bmatrix} = \begin{bmatrix} x_p(t) \\ y_p(t) \\ \theta(t) \end{bmatrix}. \quad (1)$$

In keeping with [1], [3] and [4], the orientation of the mobile robot in the $\{0\}$ fixed frame there is given by the rotation (orienting) matrix below written as:

$$R_{R0} \equiv R(\overline{z};\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

Its inverse expresses the orientation of $\{0\}$ frame with respect to moving frame $\{R\}$ as follows:

$$R_{0R} = R^T_{R_0} \equiv R^{-1}_{R_0} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

Using matrix exponentials [4], the above expressions are re- written as:

$$R_{R0} \equiv R\left(\overline{z};\, \theta\cdot\Delta_R\right) = \exp\left\{\left\{\overline{z}^{(0)}\times\right\}\theta\cdot\Delta_R\right\}; \quad (4)$$

$$R^T_{R0} \equiv R^T\left(\overline{z};\, \theta\cdot\Delta_R\right) = \exp\left\{-\left\{\overline{z}^{(0)}\times\right\}\theta\cdot\Delta_R\right\}; \quad (5)$$

$$\exp\left\{\left\{\overline{z}^{(0)}\times\right\}\theta\cdot\Delta_R\right\} = \left\{ \begin{matrix} I_3 + \left\{\overline{k}^{(0)}_R\times\right\}\sin\left(\theta\cdot\Delta_R\right) + \\ +\left\{\overline{k}^{(0)}_R\times\right\}^2\cdot\left[1-\cos\left(\theta\cdot\Delta_R\right)\right] \end{matrix} \right\}. (6)$$

Applying the time derivative about (1), the column vector of the operational velocities is obtained:

$$\dot{\overline{X}} = \begin{bmatrix} \dot{\overline{r}}_P \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \overline{v}_P \\ \omega \end{bmatrix} = \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{\theta} \end{bmatrix}. \quad (7)$$

The transfer on $\{R\}$ frame is:

$${}^R\dot{\overline{X}} = \begin{bmatrix} {}^R\dot{x}_p \\ {}^R\dot{y}_p \\ {}^R\omega \end{bmatrix} = R^{-1}_{R_0}\cdot\dot{\overline{X}} = \begin{bmatrix} \dot{x}_p\cdot\cos\theta + \dot{y}_p\cdot\sin\theta \\ -\dot{x}_p\cdot\sin\theta + \dot{y}_p\cdot\cos\theta \\ \omega \end{bmatrix}. (8)$$

It is taught that the motion is only along x_R axis:

$${}^R\dot{\overline{X}} = \begin{bmatrix} {}^R\overline{v}_P \\ {}^R\omega \end{bmatrix} = \begin{bmatrix} {}^R\dot{x}_p \\ 0 \\ {}^R\omega \end{bmatrix};\ \dot{x}_p\cdot\sin\theta + \dot{y}_p\cdot\cos\theta = 0. (9)$$

As a result, the equation (9) represents the sliding constraint along y_R, below written as:

$$\frac{dy_p}{dx_p} = \tan\theta\, ; -dx_p\cdot\sin\theta + dy_p\cdot\cos\theta + A\cdot d\theta = 0 \,(10)$$

where $P = -\sin\theta$; $Q = \cos\theta$; $R = A = 0$.

As a result, the Cauchy conditions are applied:

$$\frac{\partial P}{\partial y_p} = \frac{\partial Q}{\partial x_p};\ \frac{\partial Q}{\partial \theta} \neq \frac{\partial A}{\partial y_p};\ \frac{\partial A}{\partial x_p} \neq \frac{\partial P}{\partial \theta}. \,(11)$$

As the above conditions are not satisfied, the differential equation (10) becomes non-integral, and the robot is considered non-holonomic.

2.2. Forward Kinematic Model

The motion law of the robot with respect to fixed frame is given by the column vector:

$$\bar{X}'(t)=\left[x_p(t)\ \ y_p(t)\ \ \theta(t)\ \ \varphi_1(t)\ \ \varphi_2(t)\right]^T;\ (12)$$

where $\varphi_1(t)\neq\varphi_2(t)$ represent the rotation angles from the two driving wheels. It must determine the column vector of the operational velocities (7). Hence, every driving wheel will be analyzed.

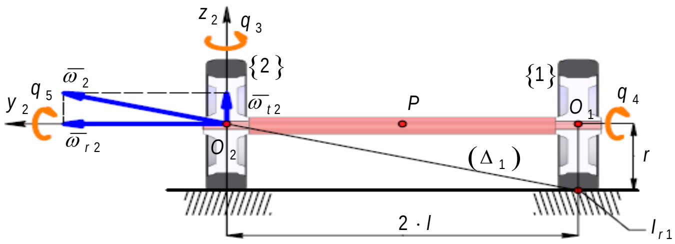

Fig.2 The angular rotation velocity for left wheel

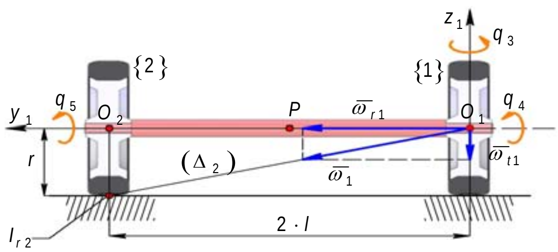

Fig.3 The angular rotation velocity for right wheel

Supposing that the wheel 2 is fixed, the wheel1performs a rotation around fixed point. The relative motion $\bar{\omega}_{r_1}$ is a pure rolling, while the transport motion $\bar{\omega}_{t_1}$ is a rotation around $\bar{z}_2$.

The resultant angular rotation velocity is defined with:

$$\bar{\omega}_1=\bar{\omega}_{r_1}+\bar{\omega}_{t_1}$$

aligned around Δ_1 axis.

The linear velocity for O_1 is:

$$v_{O_1}=r\cdot\dot{\varphi}_1\equiv 2\cdot l\cdot\omega_{t_1};\ where\ \ \dot{\varphi}_1=\omega_{r_1}.\quad(13)$$

The angular transport velocity is obtained as:

$$\omega_{t_1}=\frac{v_{O_1}}{2\cdot l}=\frac{r\cdot\dot{\varphi}_1}{2\cdot l}.\qquad(14)$$

The velocity of the point $P\in O_1O_2$ is expressed:

$$v_{p_1}=\omega_{t_1}\cdot l=\frac{r\cdot\dot{\varphi}_1}{2}.\qquad(15)$$

The resultant angular rotation velocity is obtained:

$$\bar{\omega}_1=\dot{\varphi}_1\cdot\bar{x}_2+\left(\frac{r\cdot\dot{\varphi}_1}{2\cdot l}\right)\cdot\bar{z}_2.\qquad(16)$$

Considering the wheel 1 a fixed one, in the following the wheel 2 is analyzed. The results concerning the velocities are below presented:

$$\bar{\omega}_2=\bar{\omega}_{r_2}+\bar{\omega}_{t_2};\qquad(17)$$

$$\omega_{r_2}=\dot{\varphi}_2;\ \omega_{t_2}=\frac{r\cdot\dot{\varphi}_2}{2\cdot l};\ v_{p_2}=\frac{\dot{r}\cdot\varphi_2}{2};\qquad(18)$$

$$\bar{\omega}_2=\dot{\varphi}_2\cdot\bar{x}_1-\left(\frac{r\cdot\dot{\varphi}_2}{2\cdot l}\right)\cdot\bar{z}_1.\qquad(19)$$

Because the robot spinning after wheel's axes is forbidden, the angular spinning velocity shows as:

$$\bar{\omega}_R=\bar{\omega}_{t_1}+\bar{\omega}_{t_2}=\left(\frac{r\cdot\dot{\varphi}_1}{2\cdot l}-\frac{r\cdot\dot{\varphi}_2}{2\cdot l}\right)\cdot\bar{z}_R;\ (20)$$

$\bar{z}_R\in\{R\}$ is the unit vector of the robot frame.

Considering (15) and (18), the velocity of the point P is a resultant vector as follows:

$$\dot{\bar{x}}_R=\bar{v}_p=\bar{v}_{p_1}+\bar{v}_{p_2}=\left(\frac{r\cdot\dot{\varphi}_1}{2}+\frac{r\cdot\dot{\varphi}_2}{2}\right)\cdot\bar{x}_R.\ (21)$$

The expressions (20) and (21) are substituted into (7). The operational velocities are defined as:

$$\dot{\bar{X}}=\begin{bmatrix}\dot{x}_p\\ \dot{y}_p\\ \omega=\dot{\theta}\end{bmatrix}=R_{R0}\cdot{}^{R}\dot{\bar{X}}=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\dot{x}_R\\ 0\\ \omega_R\end{bmatrix};$$

$$\dot{\bar{X}}=\left\{\exp\left\{\left\{\bar{z}^{(0)}\times\right\}\theta\cdot\Delta_R\right\}\right\}\cdot{}^{R}\dot{\bar{X}}.\qquad(22)$$

The same expression is also matrix written thus:

$$\dot{\bar{X}} = \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \omega = \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{r}{2}\left(\dot{\varphi}_1 + \dot{\varphi}_2\right) \\ 0 \\ \frac{r}{2 \cdot l}\left(\dot{\varphi}_1 - \dot{\varphi}_2\right) \end{bmatrix}. \quad (23)$$

This expression characterizes the direct kinematic model *(DKM)* in Cartesian coordinates.

2.3 Forward Model in Polar Coordinates

Considering the notations from Fig.4, the matrix equation (23) can be written as below:

$$\dot{\bar{X}} = \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \omega = \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v \\ \omega \end{bmatrix}. \quad (24)$$

The polar coordinates are symbolized in Fig 4 by: ρ -the polar radius and α -the polar angle. In the same figure β is the angle with the goal frame.

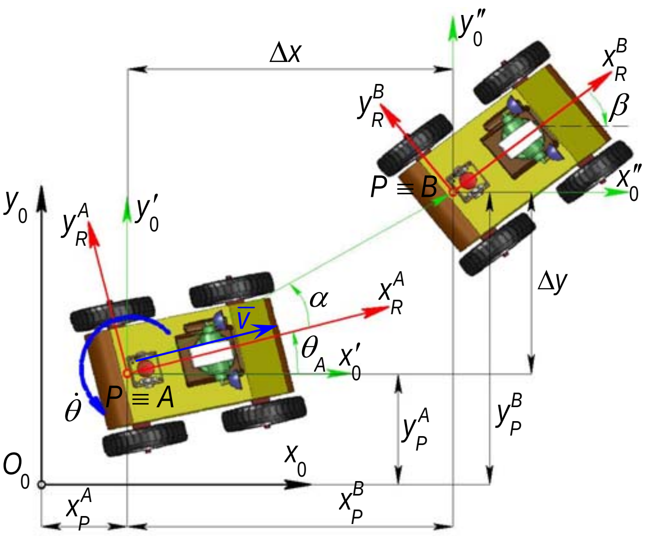

Fig.4. The polar coordinates for mobile robot

According to Figure 4 the following expressions are obtained:

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}; \quad tg\left(\alpha + \theta\right) = \frac{\Delta y}{\Delta x}; \quad (25)$$

$$\alpha = -\theta + \tan 2\left(\Delta y; \Delta x\right); \ \beta = -\theta - \alpha. \quad (26)$$

Applying the time derivative about previous expressions are obtained:

$$\rho^2 = \Delta x^2 + \Delta y^2;$$

$$2 \cdot \rho \cdot \dot{\rho} = 2 \cdot \Delta x \cdot \Delta \dot{x} + 2 \cdot \Delta y \cdot \Delta \dot{y} \quad (27)$$

From the above expression is made explicit $\dot{\rho}$:

$$\dot{\rho} = \frac{1}{\rho}\left(\Delta x \cdot \Delta \dot{x} + \Delta y \cdot \Delta \dot{y}\right); \quad (28)$$

$$\Delta \dot{x} = v \cdot \cos\theta; \ \Delta \dot{y} = v \cdot \sin\theta; \quad (29)$$

$$\dot{\rho} = \frac{1}{\rho} \cdot v \cdot \left(\Delta x \cdot \cos\theta + \Delta y \cdot \sin\theta\right). \quad (30)$$

From the same Fig. 4 it yields the following:

$$\Delta x \cdot \cos\theta + \Delta y \cdot \sin\theta = \rho \cdot \cos\alpha; \ \dot{\rho} = -v \cdot \cos\alpha.$$

The equation (26) is time derived as follows:

$$\left(\dot{\alpha} + \dot{\theta}\right) = \begin{cases} \cos^2\left(\alpha + \theta\right) \cdot \dfrac{\Delta \dot{y} \cdot \Delta x - \Delta y \cdot \Delta \dot{x}}{\Delta x^2} \\ \dfrac{\Delta x^2}{\Delta x^2 + \Delta y} \cdot \dfrac{\Delta \dot{y} \cdot \Delta x - \Delta y \cdot \Delta \dot{x}}{\Delta x^2} \end{cases}; \quad (31)$$

$$\dot{\alpha} + \dot{\theta} = \frac{1}{\rho^2} \cdot \left[\Delta x \cdot v \cdot \sin\theta - \Delta y \cdot v \cdot \cos\theta\right]; \quad (32)$$

$$\dot{\alpha} + \dot{\theta} = \frac{v}{\rho^2} \cdot \left[\Delta x \cdot \sin\theta - \Delta y \cdot \cos\theta\right] = \frac{v}{\rho^2} \cdot \rho \cdot \sin\alpha$$

$$\dot{\alpha} = \frac{v \cdot \sin\alpha}{\rho} - \dot{\theta} = \frac{v \cdot \sin\alpha}{\rho} - \omega. \quad (33)$$

Finally, the motion in polar coordinates is:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \dfrac{\sin\alpha}{\rho} & -1 \\ -\dfrac{\sin\alpha}{\rho} & 0 \end{bmatrix} \cdot \begin{bmatrix} v \\ \omega \end{bmatrix}. \quad (34)$$

The direct kinematics equations in the Cartesian or polar coordinates stay at the basis of the determination of the kinematic control functions.

2.4 Kinematic Control Functions

There are given the two positions for the mobile robot, according to next expression:

$$\overline{X} = \left\{\overline{X}_A ;\ \overline{X}_B\right\} = \left\{ \begin{pmatrix} x_P^A \\ y_B^A \\ \theta_A \end{pmatrix} \begin{pmatrix} x_P^B \\ y_B^B \\ 0 \end{pmatrix} \right\}. \quad (35)$$

The input data are completed with the time between the above two points: $\Delta t = t_B - t_A$.

From Fig. 4 the following are determined:

$$\Delta x = x_B - x_A\ ;\ \Delta y = y_B - y_A ; \quad (36)$$

$$\beta \equiv -\theta_B = -A\tan 2\left(\frac{\Delta y}{\rho}\ ;\ \frac{\Delta x}{\rho}\right). \quad (37)$$

Because $\theta_A \equiv \beta_A$ the angle α is determined as:

$$\alpha = -\left(\beta + \theta_A\right) = -\theta_A + A\tan 2\left(\frac{\Delta y}{\rho}\ ;\ \frac{\Delta x}{\rho}\right). \quad (38)$$

From the direct kinematic modeling is obtained:

$$\dot{\rho}_{1,2} = \frac{1}{r}\left(v \pm l \cdot \dot{\theta}\right). \quad (39)$$

Substituting (39) in the *direct kinematical model* equations in polar coordinates (34), they are changed as follows:

$$\begin{cases} \dot{\rho} = -\frac{r}{2}\left(\dot{\varphi}_1 + \dot{\varphi}_2\right)\cdot \cos\alpha; \\ \dot{\alpha} = \frac{r}{2\cdot\rho}\left(\dot{\varphi}_1 + \dot{\varphi}_2\right)\cdot \sin\alpha - \frac{r}{2\cdot l}\left(\dot{\varphi}_1 + \dot{\varphi}_2\right); \\ \dot{\beta} = -\frac{1}{2\cdot\rho}\left(\dot{\varphi}_1 + \dot{\varphi}_2\right)\cdot \sin\alpha; \end{cases} \quad (40)$$

$$\begin{cases} \dot{\rho} = -\frac{r}{2}\left(\dot{\varphi}_1 + \dot{\varphi}_2\right)\cdot \cos\alpha; \\ \dot{\alpha} + \dot{\beta} = -\frac{r}{2\cdot l}\left(\dot{\varphi}_1 + \dot{\varphi}_2\right). \end{cases} \quad (41)$$

The angular rotation velocities of the wheels are determined as follows:

$$\dot{\varphi}_{1,2} = -\frac{\dot{\rho}}{r\cdot\cos\alpha} \pm \frac{l}{r}\cdot\dot{\theta}. \quad (42)$$

The displacement between the states A and B is considered as elementary and finite. As a result, the relation (42) is changed for kinematic control:

$$\frac{\Delta\varphi_{1,2}}{\Delta t} = \frac{1}{\Delta t}\left(-\frac{\Delta\rho}{r\cdot\cos\alpha} \pm \frac{l}{r}\cdot\Delta\theta\right); \quad (43)$$

$$\Delta\rho = \sqrt{\Delta x^2 + \Delta y^2}\ ;\Delta\theta = \theta_B - \theta_A = -\left(\beta + \theta_A\right).$$

The equation (43) represents the kinematic control functions of the two wheels. The steps are corresponding to the rotation and translation.

1. For the robot rotation (orientation) is applied:

$$\Delta\rho = 0,\quad \frac{\Delta\varphi_{1,2}}{\Delta t} = \pm\frac{1}{\Delta t}\cdot\frac{l}{r}\cdot\Delta\theta. \quad (44)$$

2. For the robot translation from A to B is used:

$$\Delta\theta = 0,\quad \frac{\Delta\varphi_{1,2}}{\Delta t} = -\frac{1}{\Delta t}\cdot\frac{\Delta\rho}{r\cdot\cos\alpha}. \quad (45)$$

The direct kinematics with the control functions stay at the basis of the establishment of the dynamics control functions for the mobile robot.

3. CONCLUSIONS

Unlike the serial robots, in the kinematics and dynamics of the mobile robots the mathematical models are different, in the first time due to nonholonomic links. As a result, the kinematics equations have been computed by means of the motion restrictions. In this paper have been applied matrix transformations and exponentials for define the forward kinematics modeling in the Cartesian and polar coordinates. In the same time, the algorithm of the kinematic control functions of the mobile robot has been presented.

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Comandă cinematică pentru un robot mobil

Rezumat: Spre deosebire de roboții seriali, modelele matematice ale cinematicii și dinamicii structurilor de roboți mobili sunt diferite, datorită legăturilor nonholonome, caracteristice acestor structuri mecanice. Ca urmare, ecuațiile cinematicii și dinamicii se vor calcula pe baza restricțiilor de mişcare. Așadar, in această lucrare se vor aplica transformări matriceale și exponențiale pentru definirea modelului direct al cinematicii, precum și funcțiile de control, corespunzătoare roboților mobili.

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