TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue II, June, 2013

DYNAMIC CONTROL FUNCTION FOR A MOBILE ROBOT

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Abstract: *Unlike the serial robots, in the kinematics and dynamics of the mobile robots the mathematical models are different, in the first time due to nonholonomic links. As a result, the dynamics equations will be computed by means of the motion restrictions. In this paper will be applied the matrix differential transformations and exponentials for define the dynamics control functions corresponding to the structure of the mobile robots.*

Key words: *Mobile Robots, Kinematics, Dynamics Control.*

1. INTRODUCTION

The mobile robots, [1], are a category of robots that appeared as a result of the necessity to build new transport systems. The control function of the mobile robot allows it to describe certain motion trajectories, to displace from one point, considered as the initial position, to a programmed (final) position. In order to achieve this task, the control function generates a series of commands, realizing a continuous control of the locating parameters, velocities and accelerations, as well as generalized driving forces. As a result, the dynamic modeling for the structure of the mobile robots is fundamental.

In keeping with Fig. 1, the independent parameters in finite displacement are as:

$$\bar{X}'(t)=\begin{bmatrix} x_p(t) & y_p(t) & \theta(t) & \varphi_1(t) & \varphi_2(t) \end{bmatrix}^T . \quad (1)$$

The sliding constraint is given by the equation:

$$\dot{y}_R=-\dot{x}_p\cdot\sin\theta+\dot{y}_p\cdot\cos\theta=0 . \quad (2)$$

According to contact point velocities, it results:

$$\dot{x}_R=\dot{x}_p\cdot\cos\theta+\dot{y}_p\cdot\sin\theta=\frac{r}{2}\left(\dot{\varphi}_1+\dot{\varphi}_2\right); \quad (3)$$

$$\dot{\theta}=\frac{r}{2\cdot l}\left(\dot{\varphi}_1-\dot{\varphi}_2\right); \quad (4)$$

whence $\dot{\varphi}_1=\dot{\varphi}_2+\frac{2\cdot l}{r}\cdot\dot{\theta}$; $\dot{\varphi}_2=\dot{\varphi}_1-\frac{2\cdot l}{r}\cdot\dot{\theta}$. (5)

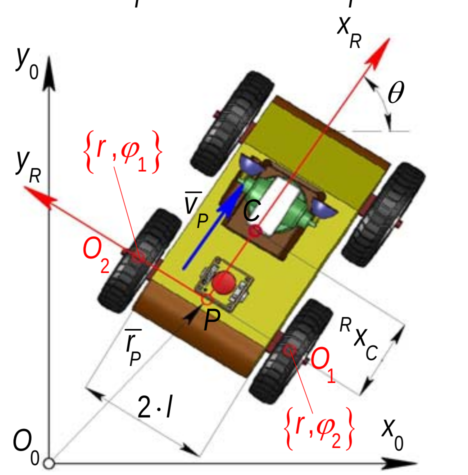

Fig. 1. The structure of the mobile robot

Substituting (5) in (3), another two restrictions of the wheel's contact point velocities is defined:

$$\left.\begin{matrix}\dot{x}_p\cdot\cos\theta+\dot{y}_p\cdot\sin\theta-r\cdot\dot{\varphi}_1+l\cdot\theta=0\\ \dot{x}_p\cdot\cos\theta+\dot{y}_p\cdot\sin\theta-r\cdot\dot{\varphi}_2-l\cdot\theta=0\end{matrix}\right\} . \quad (6)$$

The kinematic restrictions are as follows:

$$\begin{cases} -\sin\theta\cdot dx_p+\cos\theta\cdot dy_p+0\cdot d\theta+0\cdot d\varphi_1+0\cdot d\varphi_2=0 \\ \cos\theta\cdot dx_p+\sin\theta\cdot dy_p+l\cdot d\theta-r\cdot d\varphi_1+0\cdot d\varphi_2=0 \\ \cos\theta\cdot dx_p+\sin\theta\cdot dy_p-l\cdot d\theta+0\cdot d\varphi_1-r\cdot d\varphi_2=0 \end{cases} \quad (7)$$

The three link relations there are between the five displacements $\left(dx_p \;\; dy_p \;\; d\theta \;\; d\varphi_1 \;\; d\varphi_2\right)^T$. So, there are two independent parameters in elementary displacements, *nonholonomic links*.

2. THE DYNAMICS EQUATIONS

The dynamics equations are determined by means of the Lagrange-Euler *(LE)* equations for nonholonomic links [2], [3]:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_j}\right)-\frac{\partial E_c}{\partial q_j}+Q_g^j=Q_m^j+\sum_{i=1}^{3}\lambda_i\cdot a_{ij}\,. \quad (8)$$

In the above equations E_c represents the total kinetic energy of the robot, while Q_g^i and Q_m^i are the generalized gravitational and driving forces. In the same equations λ_i represent undetermined Lagrange parameters, while a_{ij} are considered the coefficients of the elementary displacements dq_j.

The equations (8) are completed with (7). The unknowns are symbolized in the following as:

$$\left\{q_j\,;\, j=1\rightarrow 5\right\};\left\{\lambda_i\,;\, i=1\rightarrow 3\right\}; \quad (9)$$

$$\psi_j=\left\{q_1=x_P\,;q_2=y_P\,;q_3=\theta;\; q_4=\varphi_1;\; q_5=\varphi_2\right\}$$

The total kinetic energy of the mobile robot is:

$$E_c=E_c^A+E_c^1+E_c^2\,. \quad (10)$$

In the above expression, E_c^A is the kinetic energy of the robot without wheels, while E_c^1 and E_c^2 are the kinetic energy of the driving wheels.

2.1 The Kinetic Energy

In the Fig. 1, the point C, $\left(PC={}^{R}x_C\right)$, is the mass center of the mobile robot which performs a parallel-plane motion. The Kőnig's theorem for the kinetic energy applied for robot is:

$$E_c^A=\frac{1}{2}\cdot M_A\cdot v_C^2+\frac{1}{2}\cdot I_{\Delta C}\cdot\dot{\theta}^2\,; \quad (11)$$

where M_A is the car mass, and I_{Δ_C} represents the mechanical inertia moment with respect to $C_z\equiv\Delta$ axis: $I_{\Delta_C}=I_{\Delta_P}-M_A\cdot{}^{R}x_C^2$ (Steiner theorem).

Remark. If $P\equiv C$ *then* ${}^{R}x_C=0$ *and* $I_{\Delta_C}=I_{\Delta_P}$.

The velocity of the mass center is determined:

$$v_C^2=\left\{\begin{array}{c}\dot{x}_P^2+\dot{y}_P^2+{}^{R}x_C^2\cdot\dot{\theta}^2- \\ -2\cdot{}^{R}x_C\cdot\dot{\theta}\cdot\left(\dot{x}_P\cdot\sin\theta-\dot{y}_P\cdot\cos\theta\right)\end{array}\right\}. \quad (12)$$

Substituting (12) in (11), the kinetic energy is:

$$E_C^A=\left\{\begin{array}{l}\frac{1}{2}\cdot M_A\cdot\left[-2\cdot{}^{R}x_C\cdot\dot{\theta}\cdot\left(\dot{x}_P\cdot\sin\theta-\dot{y}_P\cdot\cos\theta\right)\right]+ \\ +\frac{1}{2}\cdot M_A\cdot\left(\dot{x}_P^2+\dot{y}_P^2\right)+\frac{1}{2}\cdot I_{\Delta_P}\cdot\dot{\theta}^2\end{array}\right\} \quad (13)$$

The kinetic energy E_c^1 of the driving wheel 1 is determined by means of following expressions:

$$E_c^1=E_c^{1TR}+E_c^{1ROT}\,;\quad E_c^{1TR}=\frac{1}{2}\cdot M_1\cdot v_{O_1}^2\,; \quad (14)$$

$$E_c^{1ROT}=\frac{1}{2}\left(I_{xx}\cdot\omega_x^2+I_{yy}\cdot\omega_y^2+I_{zz}\cdot\omega_z^2\right). \quad (15)$$

In the above expressions are substituted the terms:

$$v_{O_1}^2=\left(\dot{x}_p+l\cdot\dot{\theta}\cdot\cos\theta\right)^2+\left(\dot{y}_p+l\cdot\dot{\theta}\cdot\sin\theta\right)^2; \quad (16)$$

$$\omega_x=\dot{\varphi}_1;\quad \omega_y=0;\quad \omega_z=\dot{\theta}\,; \quad (17)$$

$$I_{xx}=\frac{M_1\cdot r^2}{2};\qquad I_{zz}=\frac{M_1\cdot r^2}{4}\,. \quad (18)$$

The expression of the kinetic energy is:

$$E_c^1=\left\{\begin{array}{l}\frac{1}{2}\cdot M_1\cdot\left[\left(\dot{x}_p+l\cdot\dot{\theta}\cdot\cos\theta\right)^2+\left(\dot{y}_p+l\cdot\dot{\theta}\cdot\sin\theta\right)^2\right]+ \\ +\frac{M_1\cdot r^2}{4}\cdot\left(\dot{\varphi}_1^2+\frac{1}{2}\cdot\dot{\theta}^2\right)\end{array}\right\} \quad (19)$$

In the same way, the kinetic energy E_c^2 is:

$$E_c^2=\left\{\begin{array}{l}\frac{1}{2}\cdot M_2\cdot\left[\left(\dot{x}_p-l\cdot\dot{\theta}\cdot\cos\theta\right)^2+\left(\dot{y}_p-l\cdot\dot{\theta}\cdot\sin\theta\right)^2\right]+ \\ +\frac{M_2\cdot r^2}{4}\cdot\left(\dot{\varphi}_2^2+\frac{1}{2}\cdot\dot{\theta}^2\right)\end{array}\right\} \quad (20)$$

Substituting (13), (19) and (20) in (10) the total kinetic energy of the mobile robot is defined as:

$$E_c = \begin{Bmatrix} \frac{1}{2}\cdot M\cdot\left(\dot{x}_p^2+\dot{y}_p^2\right)- \\ -M_A\cdot{}^R x_c\cdot\dot{\theta}\cdot\left(\dot{x}_p\cdot\sin\theta-\dot{y}_p\cdot\cos\theta\right)+ \\ +\frac{1}{2}\cdot I_R\cdot\dot{\theta}^2+\frac{M_r\cdot r^2}{4}\left(\dot{\varphi}_1^2+\dot{\varphi}_2^2\right) \end{Bmatrix} \quad (21)$$

where I_R is the mechanical inertial moment of the mobile robot with respect to P_z axis, that is:

$$I_R = I_{\Delta_p}+\frac{M_r\cdot r^2}{2}+2\cdot M_r\cdot l^2; \quad (22)$$

$$M = M_A+2\cdot M_r;\ M_r = M_1 = M_2. \quad (23)$$

Taking into consideration de notations from (9), the total kinetic energy is re-written as follows:

$$E_c = \begin{Bmatrix} \frac{1}{2}\cdot M\cdot\left(\dot{q}_1^2+\dot{q}_2^2\right)- \\ -M_A\cdot{}^R x_c\cdot\dot{q}_3\cdot\left(\dot{q}_1\cdot\sin q_3-\dot{q}_2\cdot\cos q_3\right)+ \\ +\frac{1}{2}\cdot I_R\cdot\dot{q}_3^2+\frac{M_r\cdot r^2}{4}\left(\dot{q}_4^2+\dot{q}_5^2\right) \end{Bmatrix}. \quad (24)$$

2.2 The Differential Motion Equations

Considering [2], [3], the generalized gravitational forces are determined as:

$$Q_g^j = \sum_{i=1}^{5} M_i\cdot{}^0\overline{g}^T\cdot\frac{\partial\overline{r}_i}{\partial q_j}. \quad (25)$$

But $\overline{r}_i = x_i\cdot\left(q_j\right)\cdot\overline{x}_0+y_i\cdot\left(q_j\right)\cdot\overline{y}_0+r\left(const.\right)\cdot\overline{z}_0$, hence:

$$Q_g^j = 0 \text{ where } j = 1\to 5. \quad (26)$$

The coefficients a_{ij} of the elementary displacements dq_j, from (8), are obtained from kinematics restrictions (7). About the kinetic energy (24) are applied the partial derivatives and then time derivatives. Following this calculus the differential motion equations of second order for the mobile robot are obtained as follows:

$$\begin{gathered} M\cdot\ddot{q}_1-M_A\cdot{}^R x_c\cdot\left(\ddot{q}_3\cdot\sin q_3+\dot{q}_3^2\cdot\cos q_3\right) \\ = Q_m^1-\lambda_1\cdot s\theta+\left(\lambda_2+\lambda_3\right)\cdot\cos\theta; \\ M\cdot\ddot{q}_2-M_A\cdot{}^R x_c\cdot\left(\ddot{q}_3\cdot\cos q_3-\dot{q}_3^2\cdot\sin q_3\right) = \\ Q_m^2-\lambda_1\cdot\cos\theta+\left(\lambda_2+\lambda_3\right)\cdot\sin\theta; \\ I_R\cdot\ddot{q}_3-M_A\cdot{}^R x_c\cdot\begin{pmatrix} \ddot{q}_1\cdot\sin q_3-\ddot{q}_2\cdot\cos q_3+ \\ +\dot{q}_1\cdot\dot{q}_3\cdot\cos q_3+ \\ +\dot{q}_2\cdot\dot{q}_3\cdot\sin q_3 \end{pmatrix}+ \\ +M_A\cdot{}^R x_c\cdot\dot{q}_3\cdot\begin{pmatrix} \dot{q}_1\cdot\cos q_3- \\ -\dot{q}_2\cdot\sin q_3 \end{pmatrix} = Q_m^3+\left(\lambda_2-\lambda_3\right)\cdot l; \\ \frac{M_r\cdot r^2}{2}\cdot\ddot{q}_4 = Q_m^4-\lambda_2\cdot r; \\ \frac{M_r\cdot r^2}{2}\cdot\ddot{q}_5 = Q_m^5-\lambda_3\cdot r; \\ -\dot{q}_1\cdot\sin q_3+\dot{q}_2\cdot\cos q_3 = 0; \\ \dot{q}_1\cdot\cos q_3+\dot{q}_2\cdot\sin q_3+l\cdot\dot{q}_3-r\cdot\dot{q}_4 = 0; \\ \dot{q}_1\cdot\cos q_3+\dot{q}_2\cdot\sin q_3-l\cdot\dot{q}_3-r\cdot\dot{q}_5 = 0. \end{gathered} \quad (27)$$

The unknowns in the (27) system could be $\left\{q_j;\ j=1\to 5\right\}$ and $\lambda_i;\ i=1\to 3$ in the case of the direct problem. For control, the unknowns are:

$$\left\{Q_m^j;\ j=1\to 5\right\} \text{ and } \lambda_i;\ i=1\to 3; \quad (28)$$

where $Q_m^1 = Q_m^2 = Q_m^3 = 0$. The above equations are applied for define the dynamic robot control.

3. THE DYNAMIC ROBOT CONTROL

The mobile robot (Fig.2) is moving between two points: $O_0\left(\tau_0\right)$ initial position and $O_G\left(\tau_f\right)$ the final position and orientation.

According to [2], [6] a trapezoidal variation law is imposed for the velocities (see Fig.3).The following notations are implemented:

$$\dot{\psi}_{ji} = \begin{Bmatrix} \dot{q}_{1i} = \dot{x}_{Pi} & \dot{q}_{2i} = \dot{y}_{Pi} & \dot{q}_{3i} = \dot{\theta}_i \\ \dot{q}_{4i} = \dot{\varphi}_{1i} & \dot{q}_{5i} = \dot{\varphi}_{2i} & \end{Bmatrix}; \quad (29)$$

where $i=1 \to 7$ represent the trajectory segments between $O_0 \to O_G$ (initial and final).

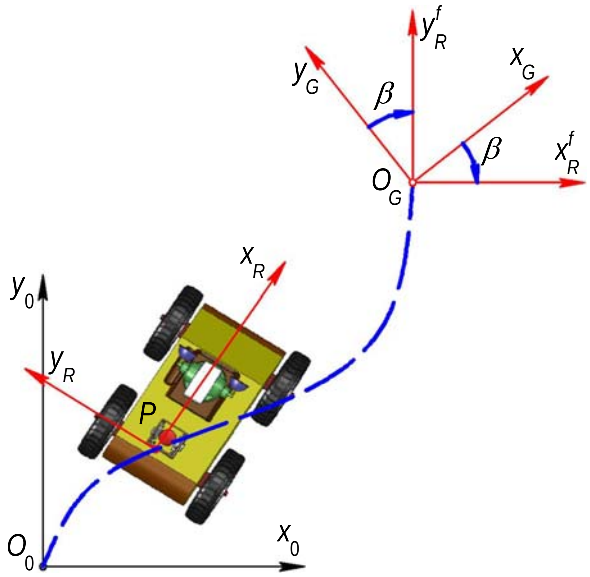

Fig.2 The motion of the mobile robot

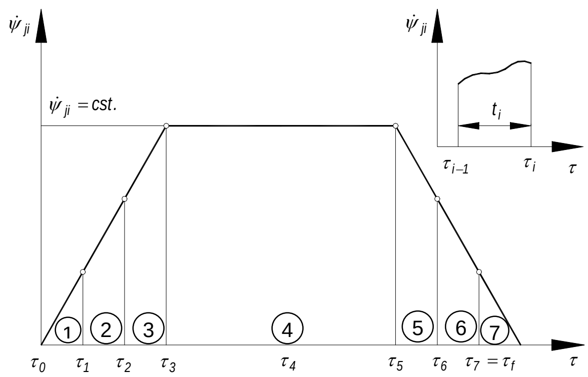

Fig.3 The trapezoidal variation law

Considering the trapezoidal variation law of the velocities a polynomial linear interpolation function is imposed, according to following:

$$\dot{\psi}_{ji} = \frac{\tau_i - \tau}{t_i} \cdot \dot{\psi}_{ji}\left(\tau_{i-1}\right) + \frac{\tau - \tau_{i-1}}{t_i} \cdot \dot{\psi}_{ji}\left(\tau_i\right); \quad (30)$$

where τ is the real variable of the time. The equation (29) is time derived. As a result, the interpolation functions for accelerations shows:

$$\ddot{\psi}_{ji} = -\frac{1}{t_i} \cdot \dot{\psi}_{ji}\left(\tau_{i-1}\right) + \frac{1}{t_i} \cdot \dot{\psi}_{ji}\left(\tau_i\right). \quad (31)$$

The equation (29) is integrated and the next parameters are established:

$$\psi_{ji} = \left\{ \begin{matrix} \left[q_{1i} = x_{Pi} \quad q_{2i} = y_{Pi} \quad q_{3i} = \theta_i \right]^T \\ \left[q_{4i} = \varphi_{1i} \quad q_{5i} = \varphi_{2i} \right]^T \end{matrix} \right\}^T ; \quad (32)$$

$$\psi_{ji} = -\frac{\left(\tau_i - \tau\right)^2}{2 \cdot t_i} \cdot \dot{\psi}_{ji}\left(\tau_{i-1}\right) + \frac{\left(\tau - \tau_{i-1}\right)^2}{2 \cdot t_i} \cdot \dot{\psi}_{ji}\left(\tau_i\right) + \psi_{ji}^{(0)};$$

where the integrating constant $\psi_{ji}^{(0)}$ is obtain from condition: $\psi_{ji}\left(\tau_{i-1}\right) = \psi_{ji-1}$ or $\psi_{ji}\left(\tau_i\right) = \psi_{ji}$.

The continuity condition is applied for every point, $i = 1 \to 7$, from the motion trajectory:

$$\psi_{ji-1}\left(\tau_{i-1}\right) = \psi_{ji}\left(\tau_{i-1}\right). \quad (33)$$

From (33) it results $\dot{\psi}_{ji}\left(\tau_i\right)$ for every $i = 1 \to 7$.

***Remark**: The expressions (30), (31), (32) and (33) are applied for $j = 1 \to 5$ and $i = 1 \to 7$. The results are substituted in (27), whence the unknowns (28) are deduced for the demanding control.*

3.1 The Dynamic Control Algorithm

According to the kinematic control, the robot is oriented with the angle $\beta = -\theta_B$.

- $\dot{x}_P = \dot{q}_1 = 0$; $\dot{y}_P = \dot{q}_2 = 0$ are imposed.
- For the orienting function, $\dot{\theta} = \dot{\theta}(t) = \dot{q}_3(t)$, a polynomial variation law is imposed, having a trapezoidal form (see Fig. 3).
- On the interval $\left[\tau_0, \tau_3\right]$ and $\left[\tau_4, \tau_7\right]$ there is:

$$\dot{\theta}_i(\tau) = \dot{q}_{3i}(\tau) = \mp \frac{\tau_i - \tau}{t_i} \cdot \dot{\theta}_{i-1} + \frac{\tau_i - \tau_{i-1}}{t_i} \cdot \dot{\theta}_i . \quad (34)$$

- $[\tau] = \left(\tau_i ;\ i = 0 \to 7\right)^T$ time matrix is imposed
- The angle $\theta_B - \theta_A$ is divided, according to:

$$\bar{\theta} = \left(\theta_i ; i = 0 \to 7\right)^T ; \theta_0 = \theta_A ; \theta_7 = \theta_B = -\beta . \quad (35)$$

- The acceleration variation law is defined by:

$$\ddot{\theta}_i = \ddot{q}_{3i} = \pm \frac{1}{t_i} \cdot \dot{\theta}_{i-1} + \frac{1}{t_i} \cdot \dot{\theta}_i = \frac{1}{t_i}\left(\dot{\theta}_i \pm \dot{\theta}_{i-1}\right) = cst. \quad (36)$$

➢ The variation law of the control functions is:

$$\begin{Bmatrix} \theta_i(\tau) \\ q_{3i}(\tau) \end{Bmatrix} = \begin{Bmatrix} \theta_{i-1} \pm \dfrac{(\tau_i - \tau)^2}{2 \cdot t_i} \cdot \dot{\theta}_{i-1} + \\ + \dfrac{(\tau - \tau_{i-1})^2}{2 \cdot t_i} \cdot \dot{\theta}_i \pm \dfrac{1}{2} \cdot t_i \dot{\theta}_{i-1} \end{Bmatrix} . (37)$$

➢ The above expressions are implemented in the system of differential equations (27). The unknowns become $\{Q_m^4 \,;\, Q_m^5\}$ and $\lambda_i \,;\, i = 1 \to 3$.

The final expressions of the generalized driving forces are shown in the following variants:

$$Q_m^{4i} = \frac{1}{2} \cdot M_r \cdot r^2 \cdot \ddot{q}_{4i} - \frac{1}{2} \cdot r \left(\frac{1}{l} \cdot I_R \cdot \ddot{q}_{3i} - M_A \cdot {}^R x_C \cdot \dot{q}_{3i}^2 \right);$$

$$Q_m^{5i} = \frac{1}{2} \cdot M_r \cdot r^2 \cdot \ddot{q}_{5i} + \frac{1}{2} \cdot r \left(\frac{1}{l} \cdot I_R \cdot \ddot{q}_{3i} + M_A \cdot {}^R x_C \cdot \dot{q}_{3i}^2 \right);$$

$$\ddot{q}_{4,5i} = \pm \frac{l}{r} \cdot \frac{1}{t_i} \left(\dot{\theta}_i \pm \dot{\theta}_{i-1} \right) = \pm \frac{l}{r} \cdot \frac{1}{t_i} \left(\dot{\theta}_i + \sigma_i \cdot \dot{\theta}_{i-1} \right)$$

$$Q_m^i = \begin{Bmatrix} \Delta_i \cdot \dfrac{1}{2} \cdot M_r \cdot r \cdot \dfrac{1}{t_i} \left(\dot{\theta}_i + \sigma_i \cdot \dot{\theta}_{i-1} \right) + \Delta_i \cdot \dfrac{1}{2} \cdot \dfrac{r}{l} \cdot I_R \cdot \ddot{q}_{3i} + \\ + \dfrac{1}{2} \cdot M_A \cdot {}^R x_C \left(-\sigma_i \cdot \dfrac{\tau_i - \tau}{t_i} \cdot \dot{\theta}_{i-1} + \dfrac{\tau - \tau_{i-1}}{t_i} \cdot \dot{\theta}_i \right)^2 \end{Bmatrix};$$

Where $\Delta_i = \left\{ \{1; Q_m^i = Q_m^{4i}\}; \{-1; Q_m^i = Q_m^{5i}\} \right\}$; (38)

$$\sigma_i = \left\{ \{+1 \,;\, \tau_0 \le \tau \le \tau_3\} \,;\, \{-1 \,;\, \tau_4 \le \tau \le \tau_7\} \right\};$$

$$Q_m^i = \begin{Bmatrix} \Delta_i \cdot \dfrac{1}{2} \cdot M_r \cdot l \cdot r \cdot \dfrac{1}{t_i} \cdot \left(\dot{\theta}_i + \sigma_i \cdot \dot{\theta}_{i-1} \right) - \\ - \Delta_i \cdot \dfrac{1}{2} \dfrac{r}{l} \cdot I_R \cdot \dfrac{1}{t_i} \cdot \left(\dot{\theta}_i + \sigma_i \cdot \dot{\theta}_{i-1} \right) + \\ + \dfrac{1}{2} \cdot M_A \cdot r \cdot {}^R x_C \cdot \begin{pmatrix} -\sigma_i \cdot \dfrac{\tau_i - \tau}{t_i} \cdot \dot{\theta}_{i-1} + \\ + \dfrac{\tau - \tau_{i-1}}{t_i} \cdot \dot{\theta}_i \end{pmatrix}^2 \end{Bmatrix} (39)$$

The generalized driving forces (torques), for the two driving wheels, are applied in order to assure the needed orientation in the input data.

4. CONCLUSIONS

Unlike the serial robots, in the dynamics of the mechanical structure for the mobile robots the mathematical modeling is different, in the first time due to non-holonomic links. As a result, the dynamics equations will be computed by means of the motion restrictions. In this paper have been applied the matrix differential transformations and exponentials for define the dynamics control functions corresponding to the structure of the mobile robots. In the same time the dynamic control algorithm devoted to establishment the generalized driving forces has been presented.

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Comanda dinamică pentru un robot mobil

Rezumat: Spre deosebire de roboţii seriali, abordarea modelelor matematice ale cinematicii şi dinamicii structurilor de roboţi mobili sunt diferite, datorită legăturilor nonholonome, ce caracterizează aceste structuri mecanice. Ca urmare, determinarea ecuaţiilor modelului dinamic pentru o structură mobilă, vor avea la baza restricţiile de mişcare. Așadar, in această lucrare se vor aplica transformări matriceale şi exponenţiale pentru a defini funcţiile de control dinamic corespunzătoare unui model mecanic de robot mobil.

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