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## PARAMETERS VARIATION OF THE VIBRATION MOVEMENT OF THE ELASTIC ARM-PALETTE SYSTEM AT THE MIXERS WITH VERTICAL AXIS, BY MODIFYING THE STIFFNESS IN ORDER TO IMPROVE THE HOMOGENIZATION PROCESS

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**Abstract:** The vertical axis mixers efficiency is closely related to the homogenization degree of the prepared concrete. The mixture homogenization process is influenced by the palletes vibrational movement, as a result of the alternative bending of the mixer arm. For mathematical modeling of the mixing process, it is considered a vertical axis mixer of 2.25 m<sup>3</sup>, for which the bending forces, the arrows, the pulsations and the amplitudes of the arm-palette systems were analyzed in four cases. In order to make the mixing more efficient, the stiffness of the arm-palette system are successively modified, aiming to obtain proper pulsations as close as possible to the system excitation pulsation and amplitudes of movement as high as possible, in conditions of high reliability.

**Key words:** mixer, concrete, elastic bar, vibrations, arm-palette.

### 1. INTRODUCTION

Arm-palette systems from vertical shaft mixers can be considered viscoelastic systems with a single degree of release. During the kneading process the movements of the arm-palette systems can be considered forced vibrations of some systems with viscous damping subjected to harmonic excitations [1].

The mixer arm can be assimilated with an embedded elastic bar subjected to the bending action, as a result of the concrete masses in front of the palette and the arm portion inside the concrete, to which are added the own masses of the palette and the arm, reduced at its end.

During the mixing process, on each arm-palette system of the mixer acts the harmonic force  $F_i = F_{0i} \sin \omega t$ , in which  $F_{0i}$  represents the amplitude of the force (maximum bending force) for the palette  $i$ , and  $\omega$  represents the pulsation system, given by the rotation speed of the mixer in a stable regime [1].

The vibrational process can be mathematically modeled by determining the bending arrows, its

own pulsations, the elastic constants and the amplitudes of the resulting oscillating motion, for each arm-palette system of the mixer.

The mixture homogenization efficiency is considered much better if the palletes pulsations have values closer to the excitation pulsation (of the rotor) and the movement amplitudes are higher. The ensuring of these requirements is conditioned by the arm-palette system elasticity degree, the system superior stiffness ensuring increased reliability in batching high consistency concrete, while leading to a decrease in mixing performances in the case of strength classes lower concrete, due to higher energy consumption.

In order to improve the system elasticity, the mixer arm can be successively modified, both in shape and diameter, resulting smaller resistance modules, pulsations closer and closer to the rotor pulsation and also higher and higher amplitudes. The calculations performed in four consecutive cases, for each constructive variant of the arm-palette system, followed by the graphical representation of the movement amplitudes have

to highlight: the movement amplitude depending on the working regim excitations, the pulsation values corresponding to the maximum amplitudes of the systems, and the reporting of the arm-pallete systems pulsations to the value of the rotor's own pulsation.

**2.MATHEMATICAL MODELING OF THE HOMOGENIZATION PROCESS AT THE VERTICAL SHAFT MIXER WITH PALLETES**

**2.1 Determination of bending arrows of arm-pallete systems for vertical shaft mixer**

For the vertical shaft mixer with palletes, shown schematically in figure 1, the arm-pallete system is considered as an embedded elastic bar and subjected to bending with composite force F (figure 2), resulting as an effect of the concrete masses in front of the pallete and the arm portion inside the concrete, to which are added the own masses of the pallete and of the respective arm portion [2-3].

The total bending mass of the arm-pallete system at the mixer shown in figure 1 can be determined with the relation:

$$M_1 = m_p + m_b + m_1 + m_2, \tag{1}$$

in which:

- $m_p$  is the palette mass;
- $m_b$  is the mass of the arm portion inserted into the material;
- $m_1$  represents the concrete mass in front of the palette, during the mixing process;
- $m_2$  represents the concrete mass in front of the pallet arm during the mixing process.

The calculation ratios for determining the four masses are as follows:

$$m_p = q_m \times S_p \times g_p \tag{2}$$

where:  $S_p$  și  $g_p$  represent the surface and thickness of the palette, and  $q_m$  the density of material (steel);

$$q_m = 7,85 \text{ kg/dm}^3$$

$$m_b = (h_m - h_p) \times \frac{\pi d^2}{4} \times q_m \tag{3}$$

in which:  $h_m$  is the height of the concrete layer in the mixer,  $h_p$  is the height of the pallete mixer and  $d$  is the diameter of the mixer arm;

$$m_1 = S_p \times q_b \times 2\pi \times R_i \tag{4}$$

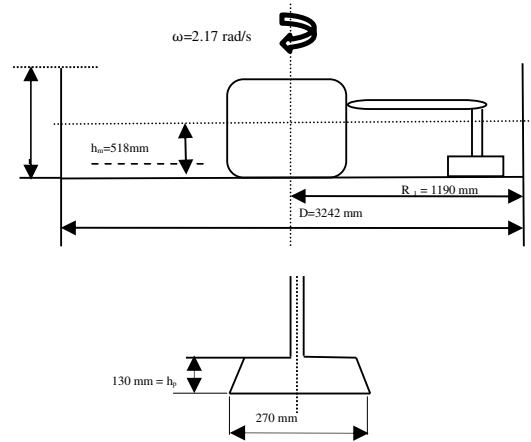
In which:  $q_b$  is the fresh concrete density and  $R_i$  is the radius of the pallete  $i$ ;  $q_b = 2200 \text{ kg/m}^3$

$$m_2 = (h_m - h_p) \times \frac{\pi d}{2} \times 2\pi R_i \times q_b \tag{5}$$

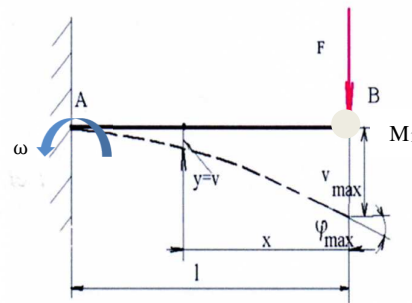
**a) Calculation of angular deformation and maximum arrow [2-3-4]**

For the case of the arm-pallete system considered as an embedded beam of length  $l$ , subjected to the action of force  $F$  produced by the bending masses (figure 2), the maximum angular deformation and the maximum arrow are determined by applying the following equations:

$$EI_z \frac{d^2\varphi}{dx^2} = EI_z \varphi = \int Fx dx + C_1 = \frac{Fx^2}{2} + C_1 \tag{6}$$



**Fig.1-** Vertical shaft mixer of 2.25 m<sup>3</sup>, schematic representation



**Fig.2-** The arm-pallete system, assimilated with an embedded elastic bar

The integration constant is determined from the boundary condition  $x=l, \varphi=0$ , that is it results:

$$C_1 = - \frac{Fl^2}{2} \tag{7}$$

Angular deformation  $\varphi$  can be written like this:

$$\varphi = \frac{1}{EI_z} \left( \frac{Fx^2}{2} - \frac{Fl^2}{2} \right) \quad (8)$$

Maximum rotation (angular deformation) occurs in the embedding, for  $x=0$ :

$$\varphi_{max} = \frac{Fl^2}{2EI_z} \quad (9)$$

### b) Maximum deflection calculation [2-3-4]

Integrating relation (8) we have:

$$EI_z v = \int \left( \frac{Fx^2}{2} - \frac{Fl^2}{2} \right) dx + C_2 = \frac{Fx^3}{6} - \frac{Fl^2 x}{2} + C_2 \quad (10)$$

The integration constant is determined from the boundary condition  $x=l, v=0$

$$0 = \frac{Fl^3}{6} - \frac{Fl^2}{2} + C_2, \text{ from it results } C_2 = \frac{Fl^3}{3} \quad (11)$$

The deflection can be expressed like this:

$$f = v = \frac{1}{EI_z} \left( \frac{Fx^3}{6} - \frac{Fl^2 x}{2} + \frac{Fl^3}{3} \right) \quad (12)$$

The maximum arrow occurs at the free end, for  $x=0$ :

$$v_{max} = f = \frac{Fl^3}{3EI_z} \quad (13)$$

## 2.2 The parameters of forced vibrations

Characteristic physical sizes are:

- arm-palette system maximum deflection:

$$f = \frac{Fl^3}{3EI}$$

- disturbance force:  $F_i = M_i \omega_i^2 f_i$  (14)

- system elastic constant the:  $k_i = M_i \omega_i^2$  (15)

- own pulsation:  $\omega_i = \sqrt{\frac{g}{f_i}} = \sqrt{\frac{3EI}{M_i l_i^3}}$ , (16)

in which:  $M_i$  is the total bending mass and  $l = R_i$  (radius) for each palette,  $E$  – the modulus of elasticity of the steel  $E = 2,1 \times 10^5 \text{ N/mm}^2$ , and  $I = I_z$  – is the axial modulus of resistance, determined with the relation:

$$I = \frac{\pi d^4}{64}, \quad (17)$$

in which  $d$  is the diameter of mixer arm.

For each palette we consider the harmonic forces  $F_i = F_{0i} \sin \omega t$ , and the amplitude of the force for the palette  $i$ , is  $F_{0i} = M_i g$  in which  $M_i$  represents the total bending mass.

The amplitude calculation formula for each palette is as follows [1]:

$$A_i = \frac{M_i \omega_i^2 f_i}{\sqrt{(k_i - M_i \omega_i^2)^2 + c_i^2 \omega_i^2}} \quad (18)$$

where we have:

$$c_i = 2\xi \sqrt{k_i M_i} \quad (19)$$

in which  $\xi = 0,2$  is the viscoelastic system damping factor [1].

## 3. DYNAMIC MODELING OF THE OSCILLATOR SYSTEM FOR THE CASE OF THE VERTICAL AXIS MIXER OF 2.25 M<sup>3</sup>

### Case 1

It is consider the case of the mixer with vertical axis of capacity 2,25 m<sup>3</sup>, shown schematically in figure 1 and constructively in Figure 3, having the following technical and technological characteristics [13]:

- standard dimensions:  $D = 3242 \text{ mm}$ ;  $B = h_m = 518 \text{ mm}$ ;  $H = 1043 \text{ mm}$ ;
- engine power 90 kW;
- rotor speed  $n_r = 20,7 \text{ rot/min}$ ;
- number of palletes: 5 pieces;
- the diameter of mixer arm:  $d = 58 \text{ mm}$ ;
- axial resistance modulus,  $I_z = 55,52 \times 10^4 \text{ mm}^4$
- the radiuses of the palletes:  $R_1 = 1365 \text{ mm}$ ;  $R_2 = 1250 \text{ mm}$ ;  $R_3 = 1135 \text{ mm}$ ;  $R_4 = 1020 \text{ mm}$ ;  $R_5 = 905 \text{ mm}$ ;
- the palette surface:  $S_p = 27 \times 13 = 351 \text{ cm}^2$ ;
- palette mass calculated with relation (2), considering  $g_p = 2 \text{ cm}$ , it results  $m_p = 5,5 \text{ kg}$ .

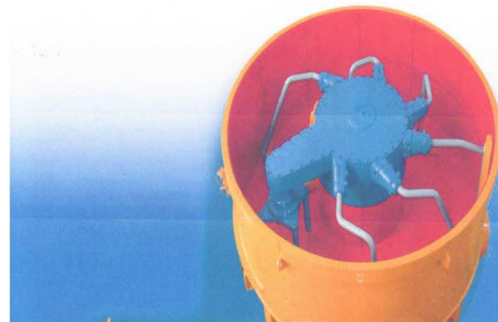


Fig. 3 - The mixer with vertical axis and palletes with a capacity of 2.25 m<sup>3</sup>

The rotor excitation pulsation is calculated with relation:

$$\omega_r = \frac{\pi n_r}{30} \quad (20)$$

from which it results  $\omega_r = 2,17$  rad/s.

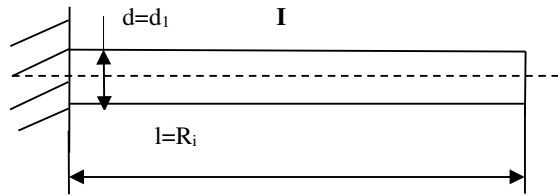


Fig.4- Constructive shape of the arm in the initial version - case 1

Applying the relations (1)- (19) for the mixer of 2.25 m<sup>3</sup> capacity in the initial version, considered as case 1 for determining the vibration movement parameters, we obtain the values from table 1.

Table 1  
Parameter values for the arm-pallette system in the initial version- case 1

Pallette no.	M <sub>i</sub> (kg)	ω <sub>i</sub> rad/s)	k <sub>i</sub> (N/m)	A <sub>i</sub> (m)
1	1341.78	10.12	137417.6	0.24
2	1229.7	12.06	178852	0.17
3	1117.9	14.62	238944	0.11
4	1006.07	18.09	329234	0.07
5	894.1	22.97	471745.8	0.04

#### 4.MODIFICATION OF THE ARM-PALLETE SYSTEM STIFFNESS, BY SHAPE MODIFICATION OF THE ARM

##### Case 2

The arm-pallette system is modified by redesigning the arm as in figure 5, in which the arm element of diameter d<sub>2</sub> and length l / 2 has the axial resistance modulus equal to half the value I.

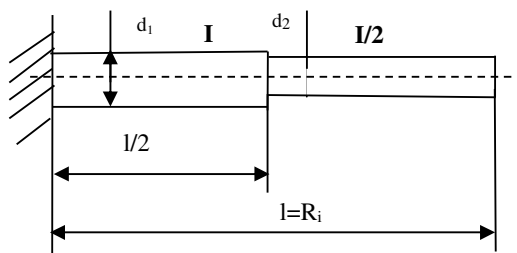


Fig. 5- Constructive shape of the arm in modified version- case 2

This change in arm stiffness leads to the recalculation of the new arrow of the arm-pallette system (composed of the sum of the arrows of the two elements of equal lengths l/2 and having resistance modules I and I/2), according to the relation:

$$f = f_1 + f_2 = \frac{F(l/2)^3}{EI} + \frac{F(l/2)^3}{EI/2} = \frac{3Fl^3}{8EI} \quad (21)$$

in which  $I = I_z = 55,52 \times 10^4 \text{ mm}^4$

For the expression of the own pulsation it results:

$$\omega = \sqrt{\frac{8EI}{3 M_i l_i^3}} \quad (22)$$

From the relation for the axial modulus of resistance (17) we obtain: d<sub>2</sub>=48 mm

Applying the relations (1)-(22), for the five pallettes of the mixer, we obtain the values figured in table 2.

Table 2  
Parameter values for the arm-pallette system in modified version- case 2

Pallette no.	M <sub>i</sub> (kg)	ω <sub>i</sub> rad/s)	k <sub>i</sub> (N/m)	A <sub>i</sub> (m)
1	1224.36	9.99	122191.2	0.25
2	1221.9	11.91	178852	0.18
3	1019.5	14.42	211991.2	0.12
4	909.2	17.86	292711.6	0.08
5	815.4	22.68	419422.3	0.05

##### Case 3

The arm-pallette system is modified by redesigning the arm as in figure 6, in which the arm element of diameter d<sub>2</sub> and length 2l / 3 has the axial resistance modulus equal to one third of the value I.

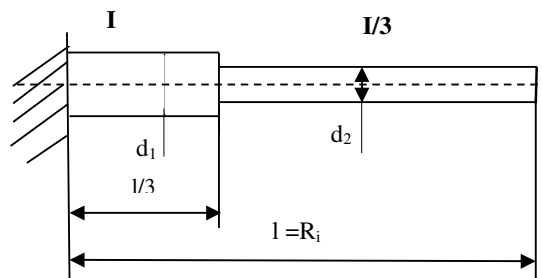


Fig. 6- Constructive shape of the arm in modified version - case 3

This change in arm stiffness leads to the recalculation of the new arrow of the arm-pallette system (composed of the sum of the arrows of the two elements of lengths  $l / 3$  and  $2l / 3$  and having resistance modules  $I$  and  $I / 3$ ), after the relation:

$$f = f_1 + f_2 = \frac{F(l/3)^3}{EI} + \frac{F(2l/3)^3}{EI/3} = \frac{25Fl^3}{27EI} \quad (23)$$

The relation for pulsation calculation becomes:

$$\omega = \sqrt{\frac{27EI}{25 M_i l_i^3}} \quad (24)$$

From the relation for the axial modulus of resistance (17) we obtain:  $d_2=44$  mm

Applying the relations (1)- (24), we obtain the values figured in table 3.

Table 3

Parameter values for the arm-pallette system in modified version – case 3

Pallette no.	$M_i$ (kg)	$\omega_i$ rad/s)	$k_i$ (N/m)	$A_i$ (m)
1	1177.52	6.48	49444.5	0.59
2	1079.2	7.72	64318.6	0.42
3	980.4	9.37	86076.1	0.28
4	875.03	11.64	118557	0.18
5	784.12	14.72	169901.8	0.13

**Case 4**

The arm-pallette system is modified by redesigning the arm as in figure 7, in which the arm element of diameter  $d_2$  and length  $3l / 4$  has the axial resistance modulus equal to a quarter of the value  $I$ .

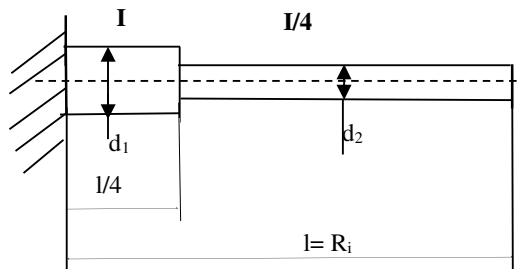


Fig. 7- Constructive shape of the arm in modified version- case 4

This change in arm stiffness leads to the recalculation of the new arrow of the arm-pallette system (composed of the sum of the arrows of

the two elements of lengths  $l / 4$  and  $3l / 4$  and having resistance modules  $I$  and  $I / 4$ ), after the relation:

$$f = f_1 + f_2 = \frac{F(l/4)^3}{EI} + \frac{F(3l/4)^3}{EI/4} = \frac{109Fl^3}{64EI} \quad (25)$$

$$\omega_i = \sqrt{\frac{64EI}{109 M_i l_i^3}} \quad (26)$$

From the relation for the axial modulus of resistance (17) we have:  $d_2=41$  mm.

For the five palletes of mixer we obtained the values presented in table 4.

Table 4

Parameter values for the arm-pallette system in modified version – case 4

Pallette no.	$M_i$ (kg)	$\omega_i$ rad/s)	$k_i$ (N/m)	$A_i$ (m)
1	1142.5	4.85	26874.4	1.06
2	1046.82	5.78	34972.5	0.74
3	951.1	7.01	46737	0.50
4	855.4	8.68	64447.9	0.33
5	760.6	11.02	92367.5	0.20

**5. AMPLITUDE VARIATION OF THE MOVEMENT IN RELATION TO THE PALETTES PULSATION, FOR THE MIXER WITH VERTICAL AXIS**

In the figure 8 is represented the movement amplitude variation, depending on the arm-pallette pulsation for the mixer with vertical axis of  $2.25 \text{ m}^3$ , in the initial version - case 1. The graph shows the motion damping mode, starting from the pallette number one - the farthest from the rotor, with a maximum amplitude of 0.249 m and up to the one closest to it (the fifth), with a maximum amplitude of only 0.048 m.

The graph also shows the excitation pulsation (rotor pulsation  $\omega_r$ ) to highlight the amplitude values when passing through this pulsation value. It is observed the amplitudes decrease with the position of the pallettes close to rotor, at the same time the pulsations increase at values more and more distant from the excitation pulsation.

Figure 9 shows graphically the motion amplitude variation for case 2, which highlights how the arm-pallette system lower rigidity influences the motion damping degree, starting

from the pallette number one- farthest from the rotor, having the maximum amplitude of 0.2557 m and up to the one closest to it, having a maximum amplitude of 0.054m. There is a slight increase in the movement amplitudes, as well as a slight decrease in the pulsations values, compared to the previously presented version.

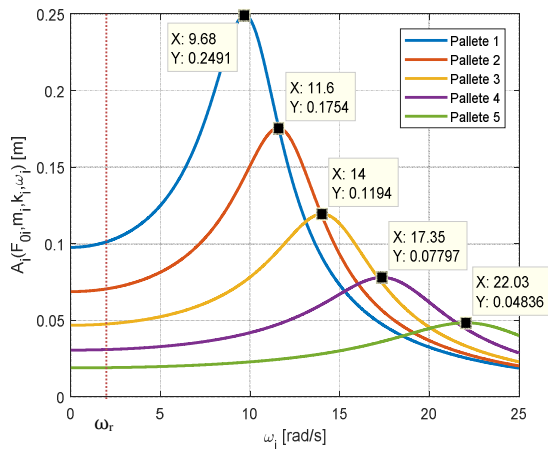


Fig. 8. -Variation of amplitude depending on the arm – pallette pulsation in case 1

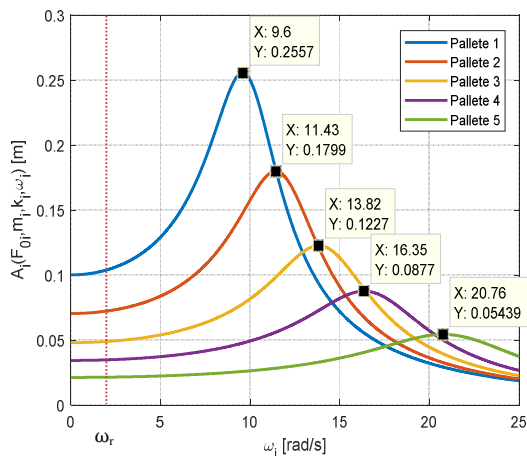


Fig. 9- Variation of amplitude depending on the arm – pallette pulsation in case 2

Figure 10 shows the graphical representation of the motion amplitude variation for case 3, which highlights the improved effect of the arm-pallette system elasticity increasing on the motion damping degree, starting from the pallette number one- farthest from the rotor, with a maximum amplitude value of 0,6076 m, clearly

superior to the previously presented case and up to the pallette closest to it (the fifth), with the maximum amplitude of 0,1178 m. There is an obvious increase of the amplitudes in relation to the previous version, as well as decrease of the own pulsations in the sense of approaching of the excitation pulsation.

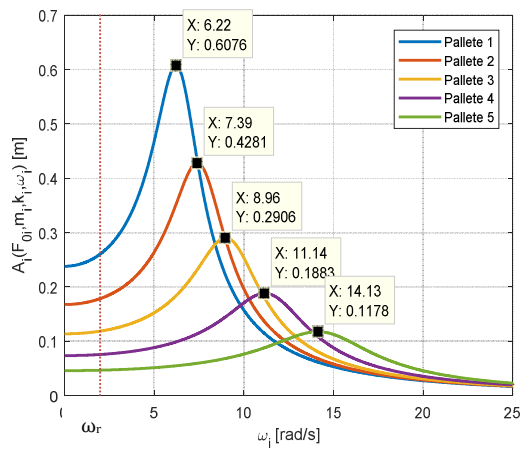


Fig. 10- Variation of amplitude depending on the arm – pallette pulsation in case 3

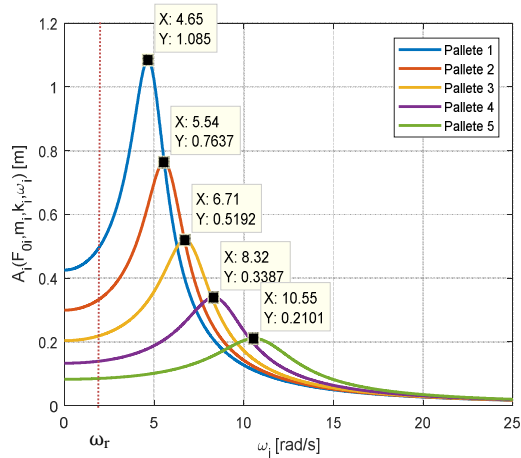


Fig. 11- Variation of amplitude depending on the arm – pallette pulsation in case 4

Figure 11 represents the movement amplitude variation depending on the palletes pulsation for case 4, in which the influence of the pronounced elasticity increase of the system on the damping phenomenon attenuation is clearly observed, starting from the pallette number one- farthest from the rotor, having the maximum amplitude reaching at 1.085 m and up to the pallette closest

to it, with a maximum amplitude of 0.2101 m. The pulsation values are reduced to half of the values obtained in the initial case (the mixer without arm-pallette system constructive modifications), approaching even more much of the rotor pulsation and the amplitudes of the palletes close to the rotor increase to values comparable to the initial ones for the palletes located away from the rotor.

## 6. CONCLUSIONS

In accordance with the results presented in tables 1-4 and in the graphs from figures 8-11, the following conclusions can be summarized:

- in case 1, the arm-pallette systems vibration movement for the mixer with vertical axis of 2.25 m<sup>3</sup> is characterized by pulsation high values in relation to the excitation pulsation value, as well as relatively low amplitude values, indicating the motion damping high degree;
- the redesign of the mixer arm presented in case 2 brings an improvement of the system elasticity, in the sense that the amplitude values increase slightly, and the own pulsations are reduced a little, approaching the excitation pulsation;
- the change of the system stiffness presented in case 3 leads to an obvious movement amplitudes increase and to palletes own pulsations significantly closer to the rotor pulsation, which can bring a materials homogenization improvement at mixing;
- in case 4 the palletes pulsations are even closer to the excitation pulsation and the amplitude values indicates a very good homogenization at mixing, also the arm-pallette system resistance module ensures in this conditions sufficient operational reliability, even in the case of high strength concrete mixing;
- the successive mixer arm constructive shape modification, with the effect of increasing the arm-pallette system elasticity especially in case 4, considered the best redesigning arm solution, led to

the decreasing of palletes movement damping degree, allowing the material mixing performances increase, with maintaining optimal wear resistance in the event of severe operating conditions.

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**VARIAȚIA PARAMETRILOR MIȘCĂRII VIBRAȚIILOR SISTEMULUI  
ELASTIC ARM-PALETTE LA MIXERELE CU AXĂ VERTICALĂ, PRIN  
MODIFICAREA RIGIDITĂȚII ÎN VEDEREA ÎMBUNĂTĂȚIRII  
PROCESULUI DE OMOGENIZARE**

Rezumat: Eficiența mixerelor cu ax vertical este strâns legată de gradul de omogenizare a betonului preparat. Procesul de omogenizare a amestecului este influențat de mișcarea vibrațională a paleților, ca urmare a îndoirii alternative a brațului mixerului. Pentru modelarea matematică a procesului de amestecare, este considerat un mixer cu ax vertical de 2,25 m<sup>3</sup>, pentru care au fost analizate forțele de îndoire, săgețile, pulsațiile și amplitudinile sistemelor de braț-palet în patru cazuri. Pentru a face amestecarea mai eficientă, rigiditatea sistemului de braț-paleti este modificată succesiv, urmărind obținerea unor pulsații adecvate cât mai aproape posibil de pulsația de excitație a sistemului și amplitudinile de mișcare cât mai mari posibil, în condiții de fiabilitate ridicată.

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