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THE DYNAMIC ANALYSE OF A CONSTRUCTION WITH THE BASE INSULATION CONSISTING IN ANTI-SEISMIC DEVICES MODELLED AS A HOOKE-VOIGT-KELVIN LINEAR RHEOLOGICAL SYSTEM

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Abstract: *The paper presents the main results of theoretical and experimental researches regarding dynamic behaviour of constructions (building, bridge, or viaduct) with the base insulation, composed of series / parallel connection of elastomeric and dissipative anti-seismic devices. The purpose of the paper is to highlight the dynamic behaviour and the isolation capacity when the base insulation system can be constructed to respect the laws of the Hooke-Voigt-Kelvin linear rheological model. There were used the data of some rigid behaviour buildings, as well as the fundamental excitation of the 4th March 1977 earthquake. The carried out studies highlighted the possibility to assess the dynamic response and the isolation capacity at the action of fundamental spectral component of the 4th March 1977 earthquake.*

Key words: *isolation capacity, elastomeric and dissipative anti-seismic devices, laws of the Hooke-Voigt-Kelvin linear rheological model*

1. INTRODUCTION

The elastomeric and fluid dissipative anti-seismic devices currently manufactured in Europe production highlight a very good level of performances, with a wide range of parameters, and having all linear or nonlinear behavior.

For certain categories of constructions (rigid buildings, bridges, viaducts), technical solutions are sought in order to increase the insulation and dissipation capacity of the seismic energy depending on the high risk and significant vulnerability countries location. For this reason, some design solutions adopt the method of combining anti-seismic devices with predominantly elastic characteristic, with fluid dissipative anti-seismic devices, in a favorable connection following the Hooke-Voigt-Kelvin model.

In this case, the stiffness and damping characteristics of the system are determined on the basis of the in post-resonance dynamic isolation criterion, taking into account the corner period Tc and the period T of the zonal seismic motion.

The computation excitation of the system consists in selecting the fundamental component from the spectral composition of the earthquake specific for a seismic area belonging to a defined geographic area. In this case, in the horizontal direction of action of the seismic movement, the fundamental spectral component of the acceleration was adopted, as $a = \omega^2 X_0 \sin\omega t$, where: X_0 is the horizontal displacement amplitude of the ground, and ω is the angular frequency of the seismic movement, with the period $T = \omega / 2\pi$.

For the case analysis presented in the paper, the following values were adopted: $T = 2$ s, $F = 0{,}5$ Hz, $\omega = \pi$ rad/sec, $a = 3$ m/s^2, $X_0 = 0{,}3$ m. The used hypotheses are: rigid behavior of the construction, in linear domain behavior of the Hooke-Voigt-Kelvin viscous-elastic system, and translation motion of the rigid in relation with the absolute coordinate system related to considered fixed mark.

2. THE DYNAMIC SYSTEM RESPONSE IN RELATIVE INSTANTANEOUS DISPLACEMENTS

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The Hooke-Voigt-Kelvin dynamic linear model (Figure 1), with composed viscous-elastic link E – (E/V), represents the isolation at the base realized by a convenient arrangement of anti-seismic devices. Thus, elastomeric devices with predominantly elastic behavior are used, with the elastic constants k, Nk, where N is a real positive multiplier, as well as fluid dissipative devices with viscous damping constant c, all with linear behavior.

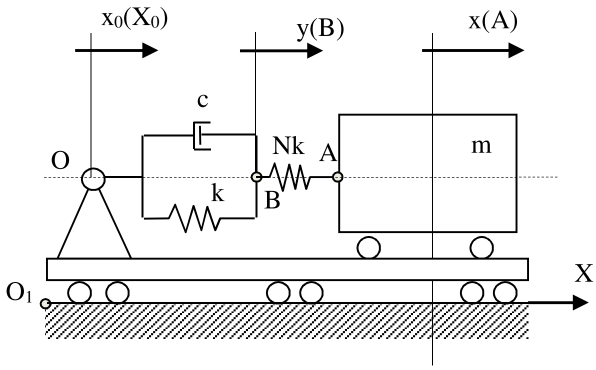

Fig. 1. Dynamic model with viscous-elastic link E – (E/V)

The coordinate system is adopted as follows: the mark O_1, considered fixed, linked to the motionless axis O_1X, defines the coordinates of the absolute motion; the mark O (x_o), linked to the ground defines the coordinates of the transport movement generated by earthquake, in the horizontal direction OX.

The relative movement of points A and B is defined by the horizontal instantaneous displacements and in relation to the mobile mark O(x_o), namely *x(t)* and *y(t)*. The mobile mark O (xo) identifies the ground motion during the earthquake for the fundamental spectral component according to the law: $x_o = X_0 \sin \omega t$

The motion differential equations of the mass m, for the hypothesis $x_o > y > x$, can be written as follows:

$$\begin{cases} m \cdot \ddot{x} = Nk(y-x) \\ c\,(\dot{x}_0 - \dot{y}) + k\,(x_0 - y) = Nk\,(y-x) \end{cases} \quad (1)$$

and the complex formulation with the imaginary unit $j = \sqrt{-1}$:

$$\begin{cases} m \cdot \ddot{\tilde{x}} + Nk\tilde{x} - Nk\tilde{y} = 0 \\ c\,(\dot{\tilde{x}}_0 - \dot{\tilde{y}}) + k\,(\tilde{x}_0 - \tilde{y}) - Nk\,(\tilde{y} - \tilde{x}) = 0 \end{cases} \quad (2)$$

where: $\tilde{x}_0 = X_0 e^{j\omega t}$; $\tilde{x} = \tilde{A}e^{j\omega t}, with\, \tilde{A} = Ae^{j\theta_2}$; ; $\tilde{y} = \tilde{B}e^{j\omega t}, with \tilde{B} = Be^{j\theta_1}$

By replacing, in the relations (2) the complex measures and their derivatives in relation to time, it is we obtained the algebraic system in $\tilde{A}$ si $\tilde{B}$, as:

$$\begin{cases} (-m \cdot \omega^2 + Nk)\tilde{A} - Nk\tilde{x} - Nk\tilde{B} = 0 \\ Nk\,\tilde{A} - (jc\omega + k + Nk)\tilde{B} = -(jc\omega + k)X_0 \end{cases} \quad (3)$$

resulting the base system determinant $\tilde{D}$, as:

$$\tilde{D} = -[Nk^2 - km\omega^2(1+N)] - jc\omega(Nk - m\omega^2) \quad (4)$$

or:

$$\tilde{D} = -\gamma - j\delta \quad (5)$$

where:

$$\gamma = Nk^2 - km\omega^2(1+N) \quad (6)$$

$$\delta = c\omega(Nk - m\omega^2) \quad (7)$$

Noting: $\alpha = Nk - m\omega^2$ and $\beta = c\omega$, the following expressions are obtained:

$$\gamma = k(\alpha - m\omega^2 N) \quad (8)$$

$$\delta = \alpha\beta \quad (9)$$

The complex amplitude $\tilde{A}$, in system (3), is obtained as:

$$\tilde{A} = -X_0 Nk \frac{k + jc\omega}{\gamma + j\delta}$$

or:

$$\tilde{A} = -X_0 Nk \frac{1}{\gamma^2 + \delta^2}[(k\gamma + c\omega\delta) - j(k\delta - c\omega\gamma)] \quad (10)$$

From the equation (10), the instantaneous displacement amplitude *x(t),* is obtained as:

$$|A| = -X_0 Nk \sqrt{\frac{k^2 + c^2\omega^2}{\gamma^2 + \delta^2}} \qquad (11)$$

or:

$$|A| = A(c,\omega) = X_0 Nk \sqrt{\frac{k^2 + c^2\omega^2}{[Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2}} \qquad (12)$$

and the dephasing θ_1 expressed as:

$$tg\theta_1 = \frac{k\delta - c\omega\gamma}{k\gamma + cm\delta}$$

The complex amplitude $\tilde{B}$ is obtained as:

$$\tilde{B} = X_0 \alpha \frac{k + jc\omega}{\gamma + j\delta}$$

or:

$$\tilde{B} = X_0 \frac{\alpha}{\gamma^2 + \delta^2}[(k\gamma + c\omega\delta) - j(c\omega\gamma - k\delta)] \qquad (13)$$

Using the equation (13), the amplitude *B* of the instantaneous displacement *y(t)* and diphasing θ_2 s expressed as:

$$B = |B| = X_0 \sqrt{\frac{\alpha^2(k^2 + c^2\omega^2)}{\gamma^2 + \delta^2}} \qquad (14)$$

or:

$$B = |B| = B(c,\omega) = X_0 \sqrt{\frac{(Nk - m\omega^2)^2 \cdot (k^2 + c^2\omega^2)}{[Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2}} \qquad (15)$$

(!) Note: by reporting expression (15) to expression (12) it is obtained the amplitudes ratio B/ A as:

$$\frac{B}{A} = \frac{N - \Omega^2}{N}, \text{ where: } \Omega^2 = \frac{\omega}{\omega_n} = \frac{m}{k}\omega \qquad (16)$$

Analysing the expression (16) the following cases of dynamic behaviour are considered

a. if N→0, then B/A →∞, meaning amplitude A≡0. In this case, the spring Nk has a very low stiffness, almost negligible, with "soft spring" effect, meaning that the motion from point B to point A cannot be transmitted.

b. if N→∞, then B/A →1, meaning that the motions of points A and B have the same amplitude. In this case, the spring Nk has such a high stiffness, that its deformation is negligible, meaning that the elastic link Nk behaves as a rigid.

c. if N =Ω^2 or $\Omega = \sqrt{N}$, then B/A =0, respectively the amplitude B becomes so small that it can be neglected, ie B≡0. In this case, the mass moves in relation to the "fixed point" B as if it were linked only at this point.

Figure 2 presents the amplitude A (c,ω) variation depending on the continuous variation of ω and the discrete variation of c

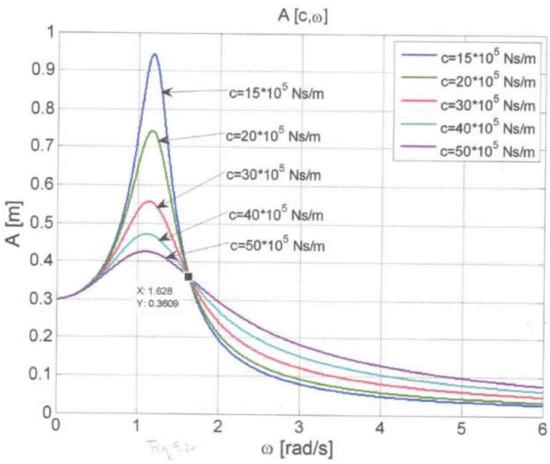

Fig. 2. The variation curves of the amplitude A depending on the discrete variation of c and the continuous variation of ω

Figure 3 presents the amplitude B variation depending on *c* and ω.

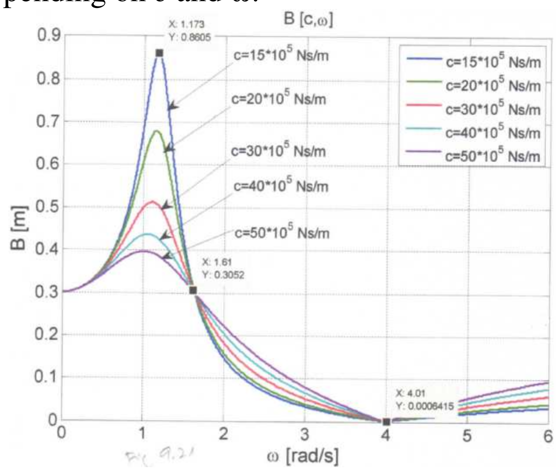

Fig. 3. The variation curves of the amplitude B depending on the discrete variation of c and the continuous variation of ω

3. THE DOFORMATIONS OF THE VISCOUS-ELASTIC SYSTEM E- (E/V)

There are analysed the instantaneous deformation process of the two systems series linked, namely the viscous-elastic system Voight-Kelvin, respectively the elastic system Hooke, with Nk stiffness.

3.1. The deformation of the viscous-elastic system Voigt-Kelvin (E/V)

Considering that the instantaneous deformations are $\tilde{v}_1 = \tilde{V}_1 e^{j\omega t}$ with $\tilde{V}_1 = V_1 e^{j\varphi_1}$, where V_1 is the amplitude of the viscous-elastic deformation (E/V). Taking into account the instantaneous displacements $x_0(t)$ and y(t) , or $\tilde{x}_0$, and $\tilde{y}$:

$$\tilde{v}_1 = \tilde{x}_0 - \tilde{y} \qquad (17)$$

or:

$$\tilde{V}_1 e^{j\omega t} = X_0 e^{j\omega t} - \tilde{B} e^{j\omega t}$$

obtaining:

$$\tilde{V}_1 = X_0 - \tilde{B} \qquad (18)$$

or:

$$\tilde{V}_1 = \frac{X_0}{S} km\omega^2(-\gamma + j\delta) \qquad (19)$$

where:

$$S = \gamma^2 - \delta^2 = [Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2 \qquad (20)$$

From the expressions (6), (7), (19) and (20) the amplitude of the deformation V_1(c,ω) and the dephasing φ_1 can be obtained as:

$$V_1(c,\omega) = X_0 \frac{km\omega^2}{\sqrt{[Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2}} \qquad (21)$$

$$tg\varphi_1 = -\frac{\delta}{\gamma}$$

or:

$$tg\varphi_1 = \frac{c\omega(Nk - m\omega^2)}{Nk^2 - km\omega^2(1+N)} \qquad (22)$$

Figure 4 presents the amplitude variation of the system E/V system, depending on the discrete variation of C and continuous variation of pulsation ω

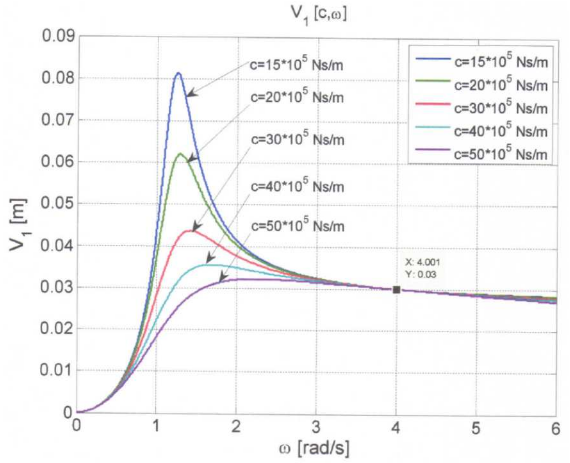

Fig. 4 The variation curves of the deformation V_1 depending on the discrete variation of dumping c and the continuous variation of excitation pulsation ω

3.2. The deformation of the elastic system Hooke (E)

For the spring with the stiffness Nk, where N>0, the instantaneous elastic deformation is expressed as:

$$\tilde{v}_2 = \tilde{y} - \tilde{x}, \qquad (23)$$

or:

$$\tilde{V}_2 = \tilde{B} - \tilde{A} \qquad (24)$$

By expressing $\tilde{A}$ and $\tilde{B}$ in expression (24), results:

$$\tilde{V}_2 = \frac{X_0}{S}(\alpha - Nk)[(k\gamma + \beta\delta) + j(\beta\gamma - k\delta)] \qquad (25)$$

$$tg\varphi_2 = \frac{\beta\gamma - k\delta}{k\gamma + \beta\delta} \qquad (26)$$

from where the amplitude V_2 of the elastic deformation and the dephasing φ_2, can be obtained as:

$$V_2(c,\omega) = (\alpha - Nk)X_0\sqrt{\frac{k^2 + \beta^2}{\gamma^2 + \delta^2}} \qquad (27)$$

Taking into account the expressions (6), (7) and (8), the equations (26) and (27) can be written as:

$$V_2(c,\omega) = -m\omega X_0\sqrt{\frac{k^2 + c^2\omega^2}{[Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2}} \qquad (28)$$

$$tg\varphi_2 = -\frac{ck\omega^2 mN}{k^2[Nk - m\omega^2(1+N)] + c^2\omega^2(Nk - m\omega^2)} \qquad (29)$$

Figure 5 presents the variation of the maximum elastic variation V_2 depending on the discrete variation of c and continuous variation of ω.

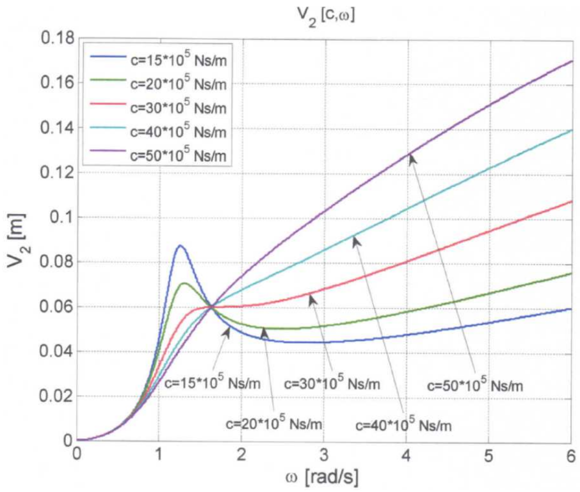

Fig.5 The variation curves of the amplitude V_2 of the elastic deformation depending on c and the continuous variation of ω

4. DYNAMIC INSULATION CAPACITY

The assessment of the dynamic insulation capacity is performed, as follows:

a. by determining the maximum transmitted force Q_o from the construction support invariably linked to the ground under seismic motion towards the construction having the mass m, by means of the base insulation system;

b. by establishing the calculus relation of the transmissibility T of the motion from the mobile mark O to the point A belonging to the mass m. In this case, the degree of seismic isolation is calculated with the relation I = 1- T.

4.1. The maximum transmitted force

The maximum transmitted force Q_o is determined as the link force in the region of the spring with multiple stiffness Nk for the Hooke modelled elastic line (E). Thus, the expression is:

$$Q_0 = NkV_2 \qquad (30)$$

taking into account the expression (28), it is obtained:

$$Q_0(c,\omega) = km\omega^2 X_0 N \sqrt{\frac{k^2 + c^2\omega^2}{[Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2}} \qquad (31)$$

with the graphic representation in figure 6.

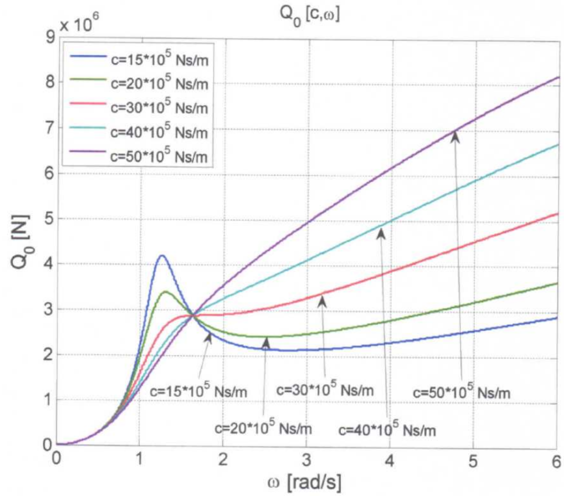

Fig. 6 The variation curves of the maxiumum transmitted force Q_o depending on c and ω

4.2 Motion transmissibility

The motion transmissibility T from the excitation point Q_o to the receiving point A(x) is assessed using the expression:

$$T = \frac{A}{x_0} \qquad (32)$$

where by expressing A as in expression (12), it is obtained:

$$T(c,\omega) = kN \sqrt{\frac{k^2 + c^2\omega^2}{[Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2}} \qquad (33)$$

with the graphic representation in figure 7.

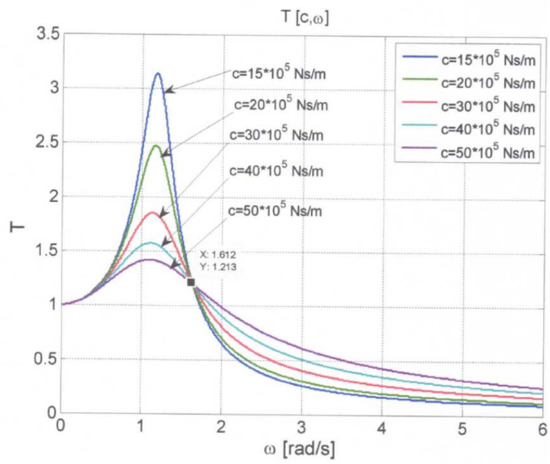

Fig. 7 The variation curves of the transmissibility T depending on c and ω

5. THE DISSIPATED ENERGY

The amount of dissipated energy depends essentially on the equivalent viscous damping constant of the entire viscous fluidic dissipative system and on the amplitude of the deformation of the viscous-elastic system V1, Voigt-Kelvin modelled. Thus, the dissipated energy W_d can be expressed as:

$$W_d = \pi c \omega V_1^2 \qquad (34)$$

where, by expressing V_1 as in expression (21), it is obtained:

$$W_d(c,\omega) = \pi X_0^2 \frac{k^2 m^2 c \omega^5}{[Nk^2 - km\omega^2(1+N)]^2 + c^2\omega^2(Nk - m\omega^2)^2} \qquad (35)$$

with the graphic representation in figure 8.

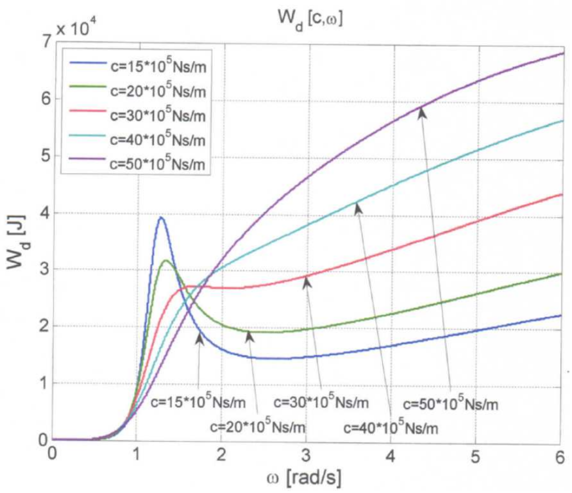

Fig. 8 The variation curves of the dissipated energy W_d depending on the viscous dumping c and the excitation pulsation ω

The initial data used for the case of study and for the plotting of the variation curves of the response parameters are as follows: m=3Mkg , k=4,8M N/m; N =10; c = (1,5 ; 2 ; 3 ; 4 ; 5) MN·s/m; a =3m/s^2 la ω=π rad/s and X_o =0,3m.

6. CONCLUSIONS

The technical solutions for increasing the base dynamic isolation capability are realized in a wide variety of combinations, with serial / parallel connection of the anti-seismic devices with linear / non-linear behaviour.

In order to establish the strategy for the realization of a system composed of elastomeric anti-seismic devices in series / parallel connection, with fluid dissipative anti-seismic devices it is necessary a systemic analysis for the components selection, so that the final objective of the dynamic isolation efficiency to be achieved.

For this, the most reliable and rational way is to establish parametric analytical relations in order to provide the possibility to assess the dynamic response to kinematic excitations of the seismic movement on the fundamental spectral component.

In this context, analytical and experimental researches were carried out in order to establish the efficiency of the rheological composed model E-(E/V) type Hooke-Voigt-Kelvin. The assessment scenario for the analytical approach was based on the initial numerical data presented in the paper, and the results of the study can be synthesized as follows:

a. dynamic response, to the given excitation is expressed by the instantaneous displacement x(t) = Asin(ωt+θ_2) where the amplitude A = A (c,ω) is represented by a family of curves parameterized by the viscous damping c to the continuous variation of the excitation pulsation ω.

b. instantaneous deformations v_1(t) and v_2(t) have the amplitudes V1 and V2 corresponding to the viscous-elastic system (E/V) type Voigt-Kelvin and respectively the elastic system (E) type Hooke. The analytical expressions of the amplitudes V_1(c,ω) si V_2(c,ω) are useful in the assessment and calculation of the dissipated energy W_d on the viscous linear damper c, and accordingly for the calculation of the maximum transmitted force Q_o on the elastic system with the multiplied stiffness Nk.

c. dynamic isolation capability is assessed both by the maximum transmitted force Q_o, as well as by the motion transmissibility T = T (c,ω) depending on the discrete variations of c and the continuous variation of ω.

d. dissipated energy W_d in the damper c with fluid dissipation is expressed analytical form and graphically represented by a family of parameterized curves.

Essentially, the assessment of dynamic behaviour using the analytical approach for a dynamic base isolation system is a necessary and significant calculation method in the pre-stage of the parametric dynamic analysis. As a result, the parametric relations established for the dynamic response parameters, for the deformations of the composed system Hooke-Voigt-Kelvin, as well as for the isolation and dissipation capacity can be used in the calculations of dynamic analysis and for the dimensioning of the base insulation systems for constructions exposed to seismic actions.

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Analiza dinamica a unei constructii cu izolare la baza alcatuita din dispozitive antiseismice modelate ca sistem reologic liniar Hooke-Voigt-Kelvin

Rezumat: In lucrare se prezinta principalele rezultate ale unor cercetari teoretice si experimentale privind comportarea dinamica a unei constructii (cladire, pod, viaduct) cu izolare la baza, aceasta fiind alcatuita din conexiunea in serie/ paralel a dispozitivelor antiseismice elastomerice si disipative. Scopul lucrarii este acela al evidentierii comportarii dinamice si a capacitatii de izolare atunci cand sistemul de izolare la baza poate fi alcatuit astfel incat sa respecte legitatile modelului reologic liniar Hooke-Voigt-Kelvin. Pentru aceasta au fost utilizate datele tehnice ale unor cladiri cu comportament de rigid cat si excitatia fundamentala a cutremurului de pamant din Romania de la data de 4 martie 1977. Asamblarea dispozitivelor elastomerice si disipative din productia existenta in Europa a permis stabilirea unei solutii tehnice de sistem de izolare dinamica modelat Hooke-Voigt-Kelvin. Ca urmare studiile efectuate au evidentiat posibilitatea evaluarii analitice a raspunsului dinamic si a capacitatii de izolare la actiunea componentei fundamentale spectrale a cutremurului din Romania ce s-a produs la 4 martie 1977

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